Coulombic Quantum Liquids in Spin-1/2 Pyrochlores

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Natal, December 13th, 2011

Collaborators

$Yb_2Ti_2O_7$ project

Leon Balents

(KITP, UCSB)

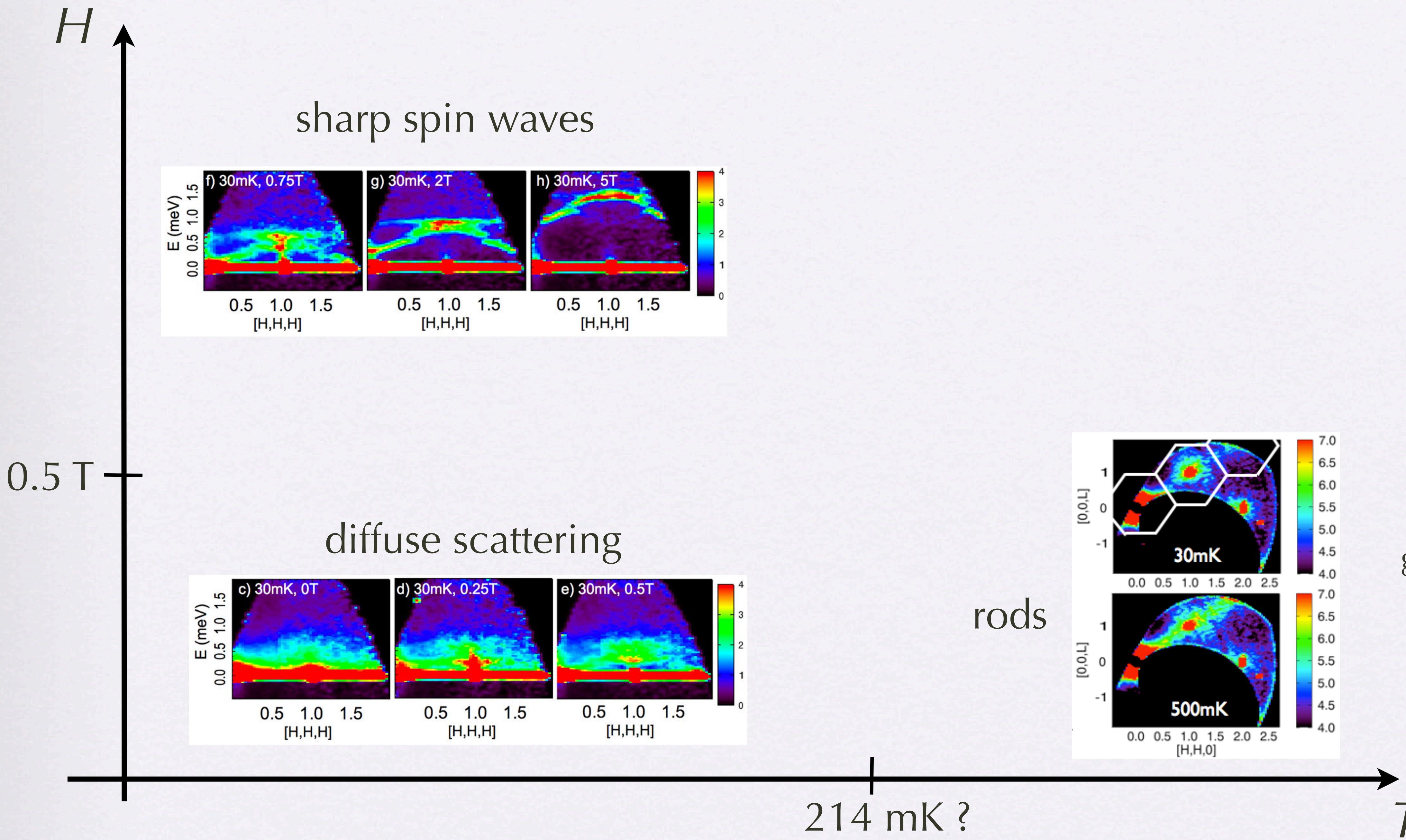
Kate Ross

Bruce Gaulin

(experiments, Mc Master)

Special thanks to Benjamin Canals and Peter Holdsworth.

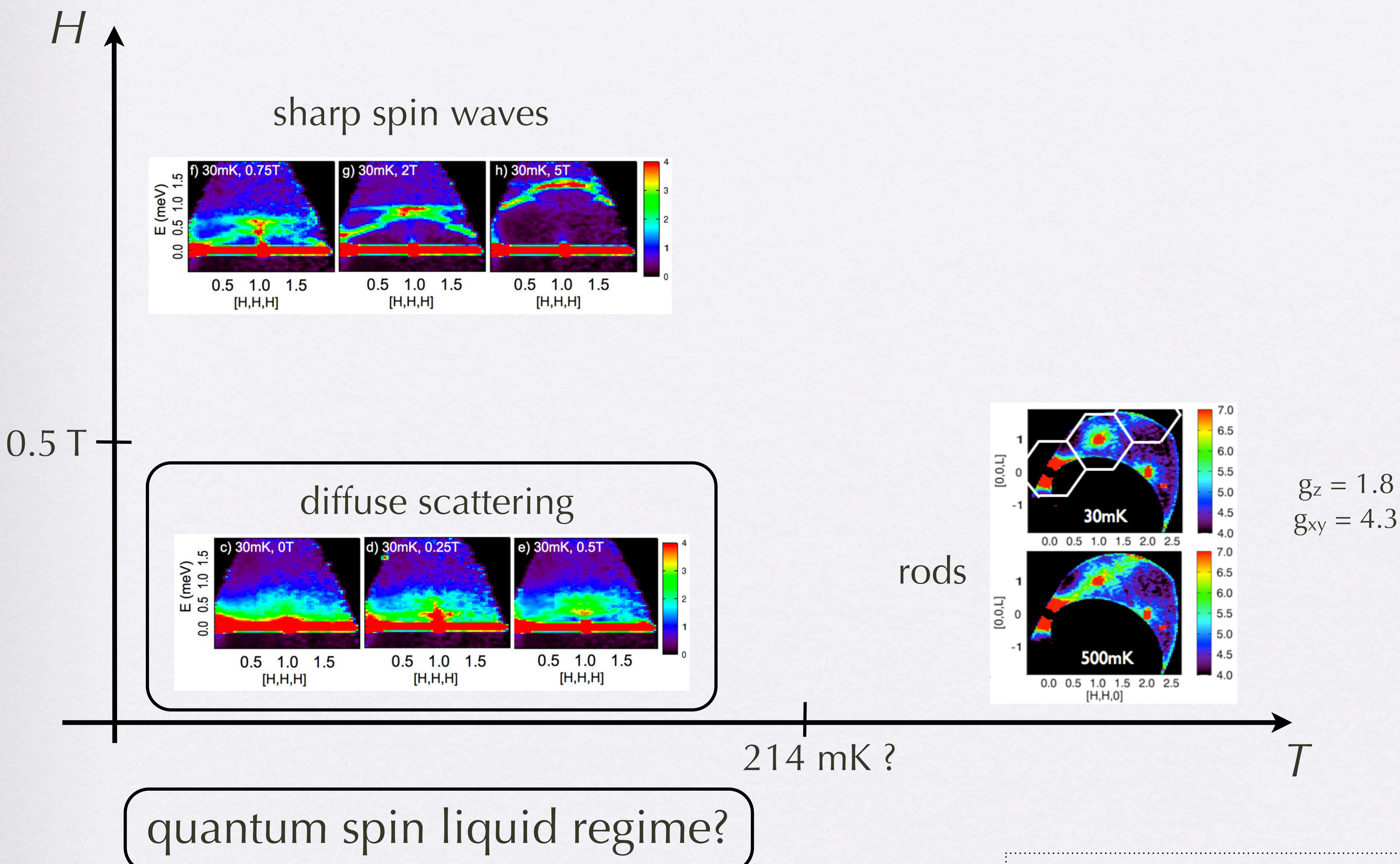
$Yb_2Ti_2O_7$: puzzling experimental features



$g_z = 1.8$
$g_{xy} = 4.3$

Ross *et al.*, PRL 2009, PRB 2011

$Yb_2Ti_2O_7$: puzzling experimental features

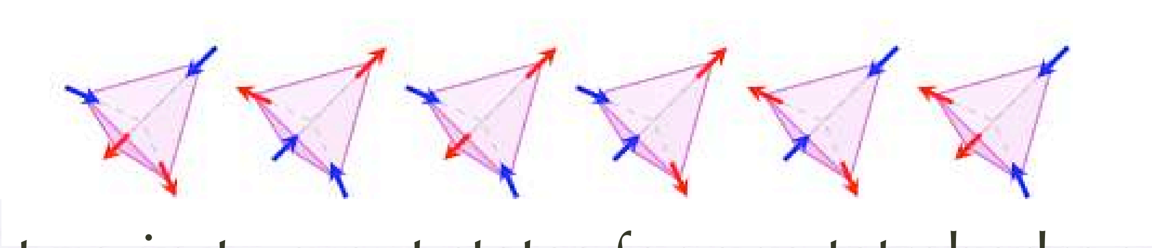


Ross *et al.*, PRL 2009, PRB 2011

What we know

- What spin ice is -- Michel's talk this morning

$$H_{\rm SI} = J_{zz} \sum_{\langle i,j \rangle} \mathsf{S}_i^z \mathsf{S}_j^z \qquad J_{zz} > 0$$



two-in-two-out states for one tetrahedron

- What the generic definition of a quantum spin liquid is (and that it would be *very* nice to find one in nature) -- Leon's talk yesterday afternoon
  - looks trivial: $\langle \vec{\mathsf{S}} \rangle = \vec{0}$ *history
  - but non-trivial correlations & fractional excitations!

} quantum entanglement

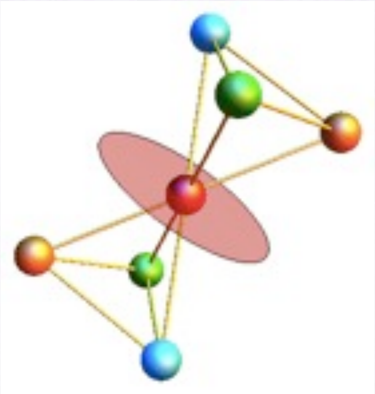
- Good place to look for QSLs: frustrated magnets

picture from Jaubert 2009

Rare-earth pyrochlores

- grown rare-earth pyrochlores: $Ho_2Ti_2O_7$, $Dy_2Ti_2O_7$, $Ho_2Sn_2O_7$, $Dy_2Sn_2O_7$, $Er_2Ti_2O_7$, $Yb_2Ti_2O_7$, $Tb_2Ti_2O_7$, $Er_2Sn_2O_7$, $Tb_2Sn_2O_7$, $Pr_2Sn_2O_7$, $Nd_2Sn_2O_7$, $Gd_2Sn_2O_7$, ...

- grown rare-earth B-site spinels: $CdEr_2S_4$, $CdEr_2Se_4$, $CdYb_2S_4$, $CdYb_2Se_4$, $MgYb_2S_4$, $MgYb_2S_4$, $MnYb_2S_4$, $MnYb_2Se_4$, $FeYb_2S_4$, $CdTm_2S_4$, $CdHo_2S_4$, $FeLu_2S_4$, $MnLu_2S_4$, $MnLu_2Se_4$, ...



lots of room for diverse behaviors!

Gardner, Gingras, Greedan, RMP 2010, Lago *et al.* PRL 2010

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spin ices

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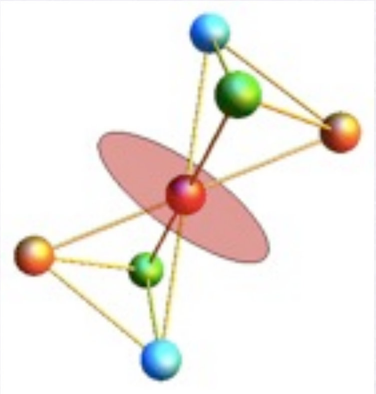

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quantum AFM

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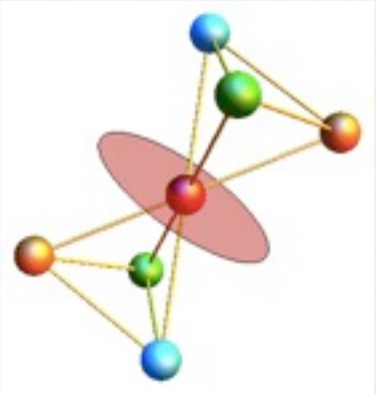

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quantum spin liquids ?

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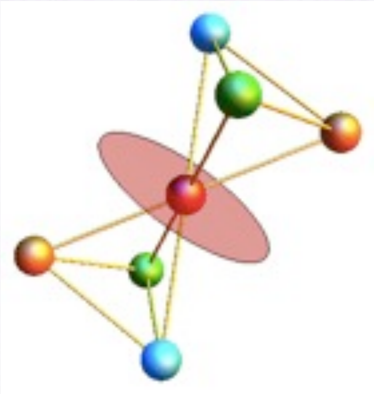

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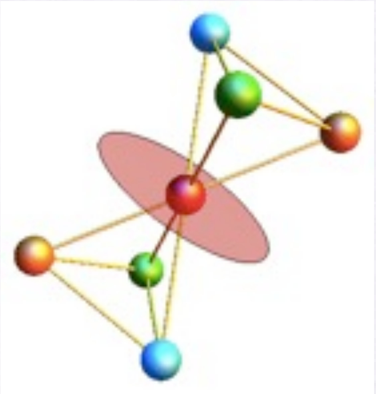

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Outline

- method
- results
- experimental signatures
- materials

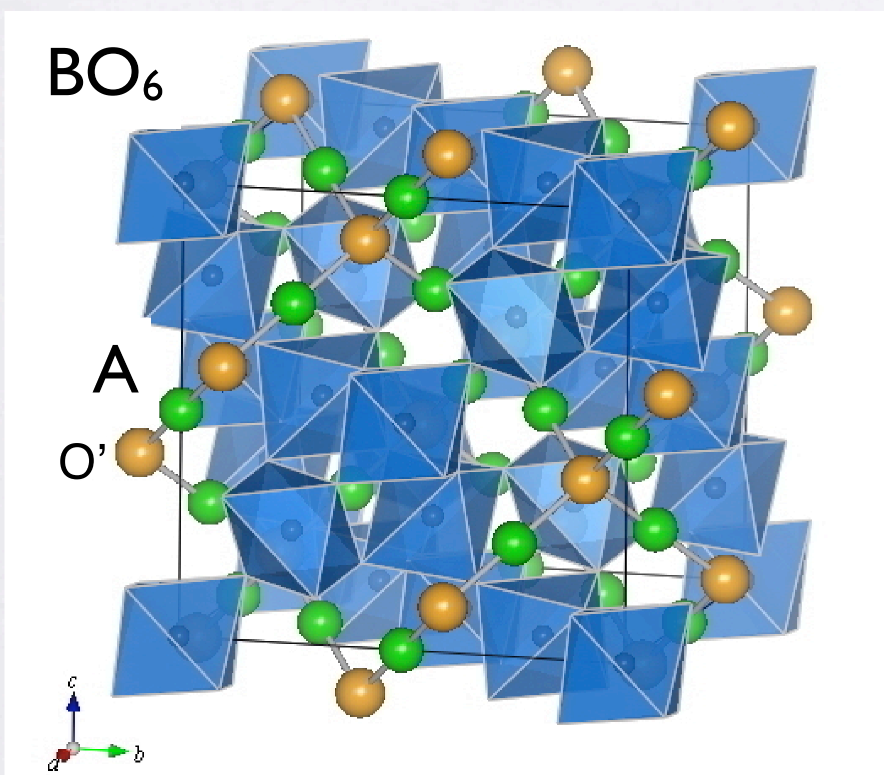
based on Savary and Balents arXiv 1110.2185 - to appear in PRL (Dec. 2011)

Symmetries of the Hamiltonian

rare-earths : intrinsic strong spin-orbit coupling

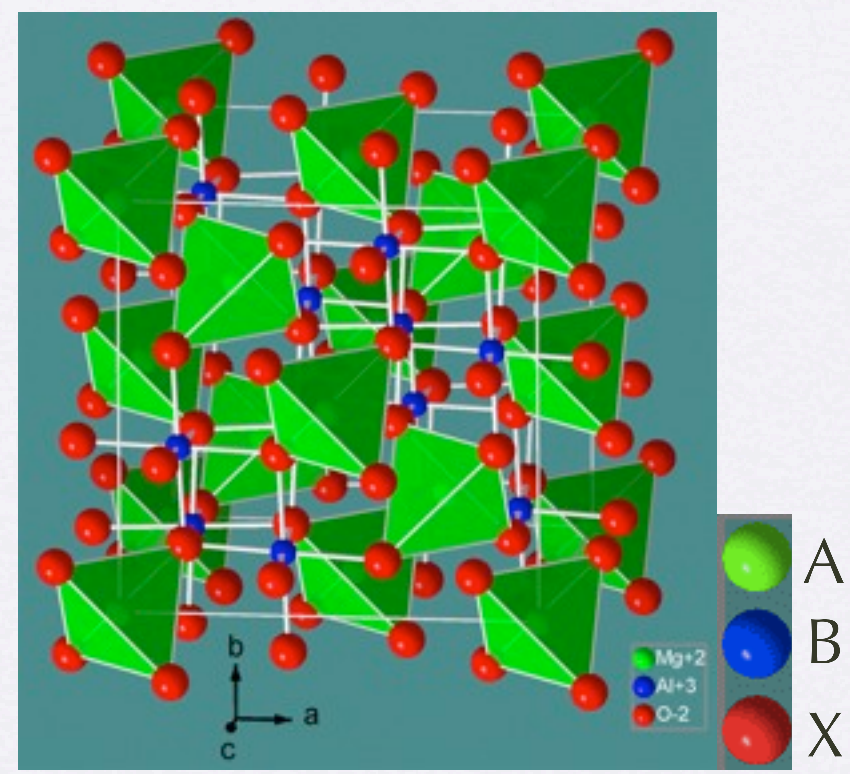
⟶ *discrete* cubic symmetries *only*

space group: Fd-3m, i.e. #227 :



$\mathbf{A}_2B_2O_7$

"pyrochlore oxides"



$A\mathbf{B}_2X_4$

spinels

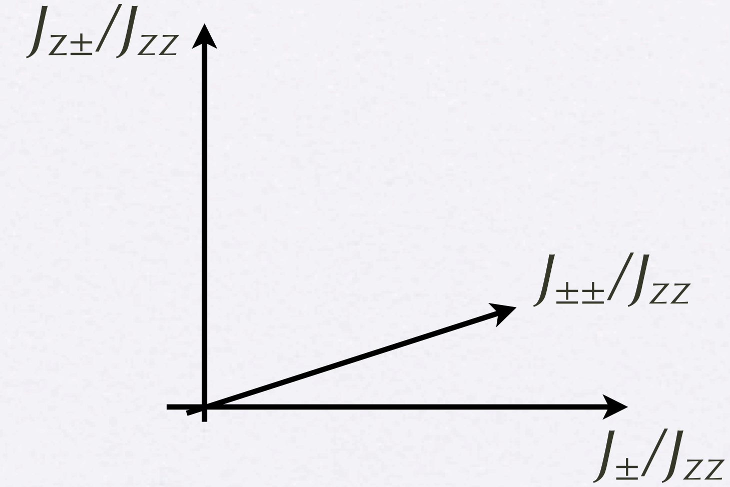
General NN exchange Hamiltonian for effective spins 1/2

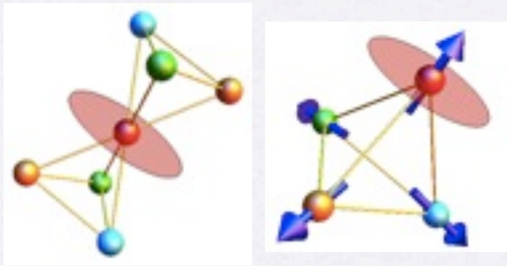
$$H = \sum_{\langle ij \rangle} \Big[\quad J_{zz} \mathsf{S}_i^z \mathsf{S}_j^z$$

$$- J_{\pm} (\mathsf{S}_i^+ \mathsf{S}_j^- + \mathsf{S}_i^- \mathsf{S}_j^+)$$

$$+ J_{z\pm} \left[\mathsf{S}_i^z (\zeta_{ij} \mathsf{S}_j^+ + \zeta_{ij}^* \mathsf{S}_j^-) + i \leftrightarrow j \right]$$

$$+ J_{\pm\pm} \left[\gamma_{ij} \mathsf{S}_i^+ \mathsf{S}_j^+ + \gamma_{ij}^* \mathsf{S}_i^- \mathsf{S}_j^- \right] \Big]$$





local axes, specific local bases

to each material corresponds a set of *J*'s

Curnoe PRB 2008, Ross Savary Gaulin Balents PRX 2011

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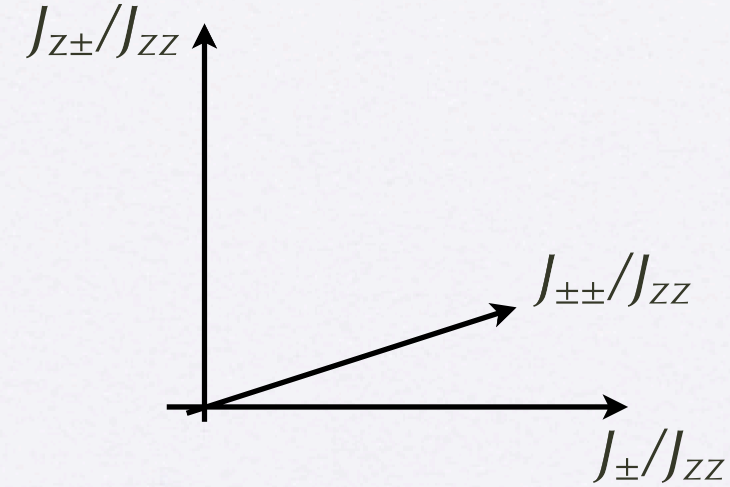

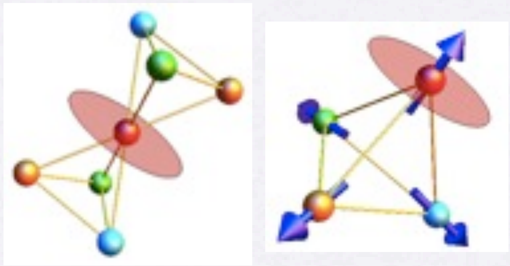

local axes, specific local bases

to each material corresponds a set of *J*'s

What is the phase diagram ?

Are there any exotic phases there ?

Curnoe PRB 2008, Ross Savary Gaulin Balents PRX 2011

The Hermele *et al*. QSL

$$H = \sum_{\langle ij \rangle} \Big[\quad J_{zz} \mathsf{S}_i^z \mathsf{S}_j^z$$
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Hermele Fisher Balents, PRB 2004

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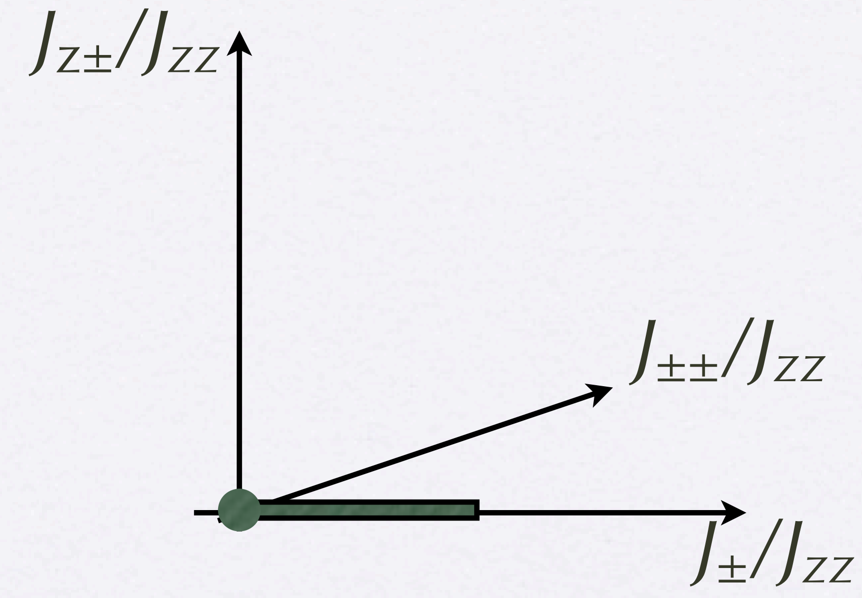
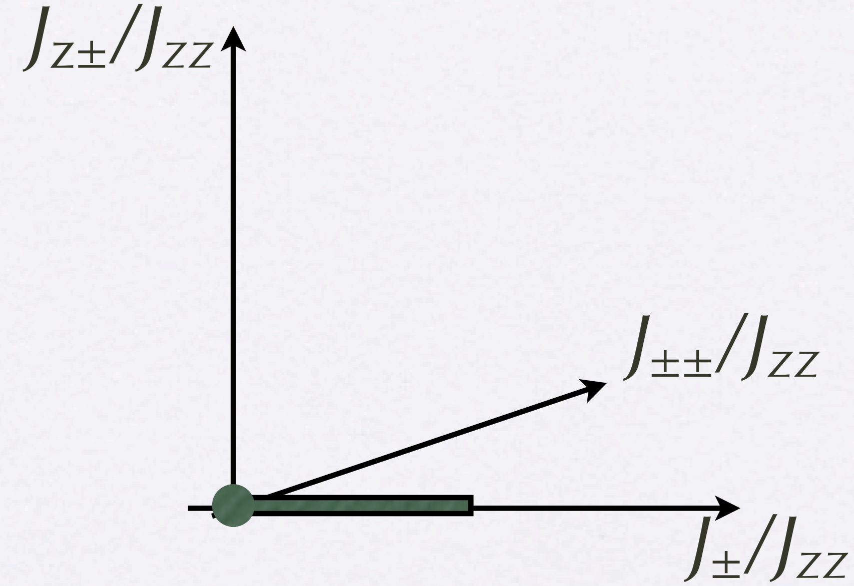


perturbation theory in $J_{\pm}/J_{zz}$ →

Hermele Fisher Balents, PRB 2004

The Hermele *et al*. QSL

$$H = \sum_{\langle ij \rangle} \Big[\quad J_{zz} \mathsf{S}_i^z \mathsf{S}_j^z - J_{\pm} \left(\mathsf{S}_i^+ \mathsf{S}_j^- + \mathsf{S}_i^- \mathsf{S}_j^+ \right) \Big]$$



perturbation theory in $J_\pm/J_{zz}$ → quantum electrodynamics $H \sim H_{\rm QED} \sim E^2 + B^2$

Hermele Fisher Balents, PRB 2004

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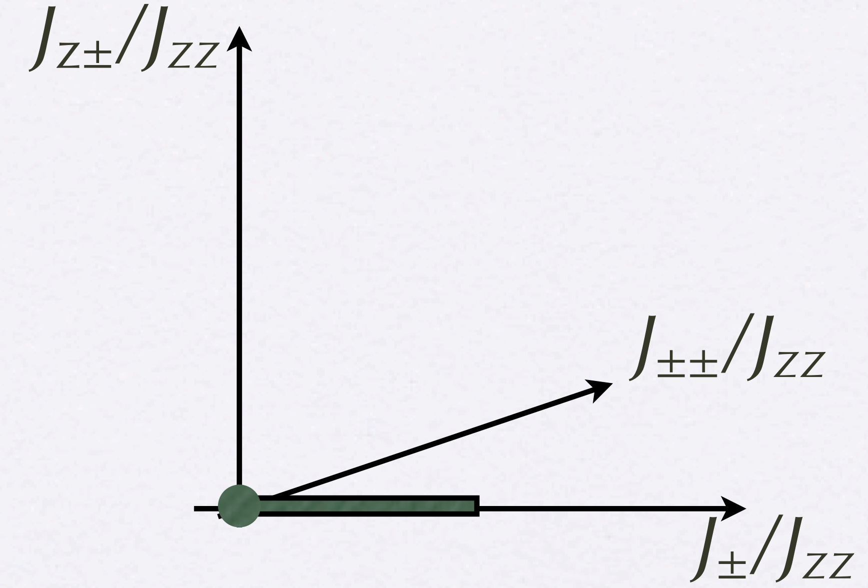


$\xrightarrow{\text{perturbation theory in } J_{\pm}/J_{zz}}$ quantum electrodynamics $H \sim H_{\mathrm{QED}} \sim E^2 + B^2$

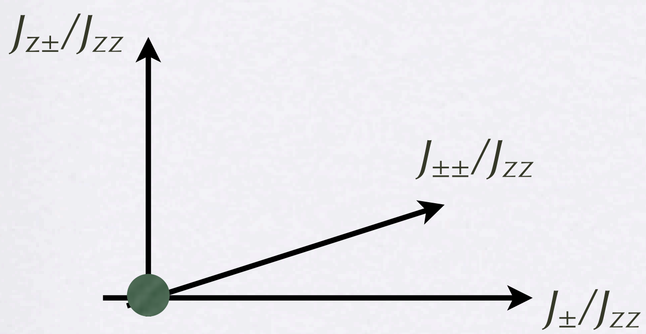
$\Longrightarrow$ photon (gapless and linear)

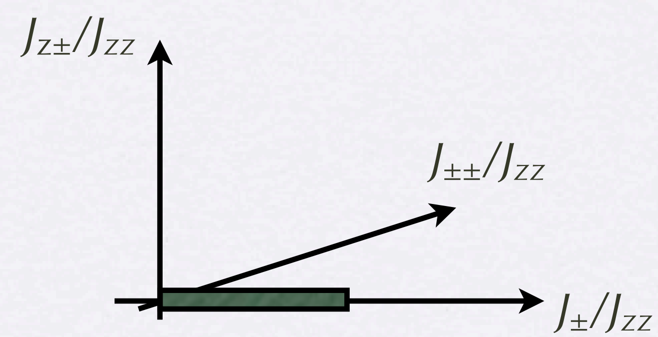
particle-hole excitations (gapped)

Hermele Fisher Balents, PRB 2004

Relation to classical spin ice

classical spin ice	U(1) quantum spin liquid
thermal spin liquid	quantum spin liquid
extensively many degenerate ground states	*one* entangled ground state (= vacuum)
magnetic monopoles = spinons	spinons "electric" monopoles gapless photon





Gingras, Introduction to Frustrated Magnetism (2011), Hermele Fisher Balents, PRB 2004

How we do this: compact abelian lattice Higgs theory

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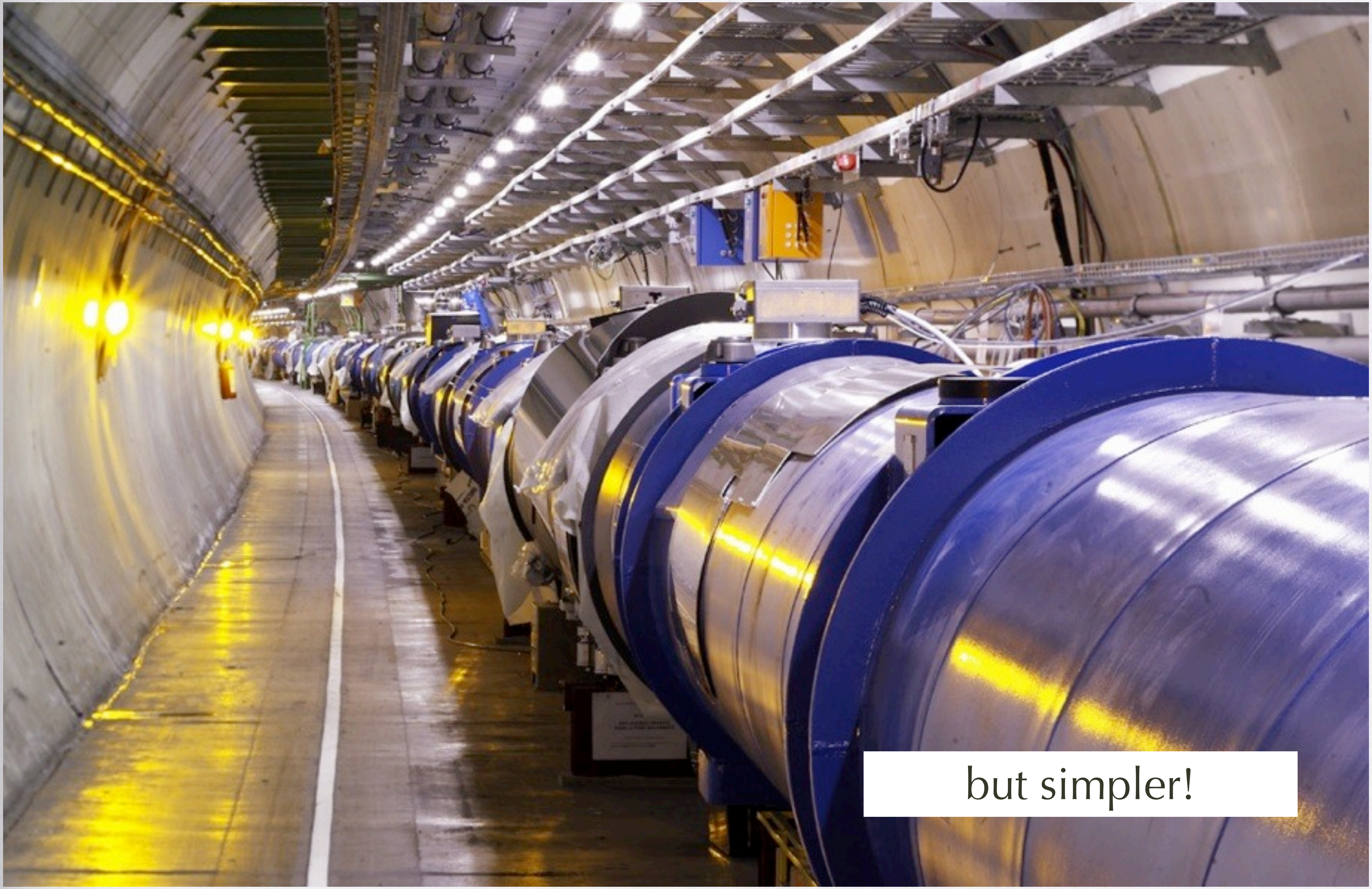
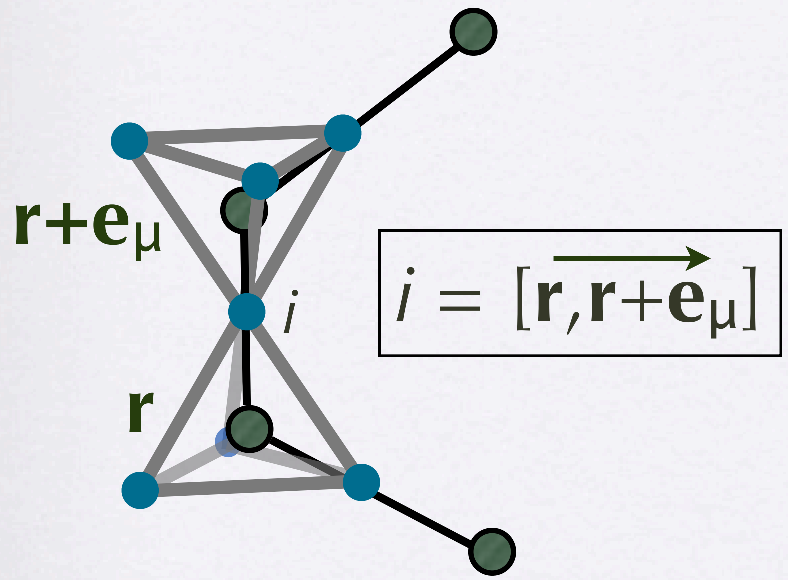
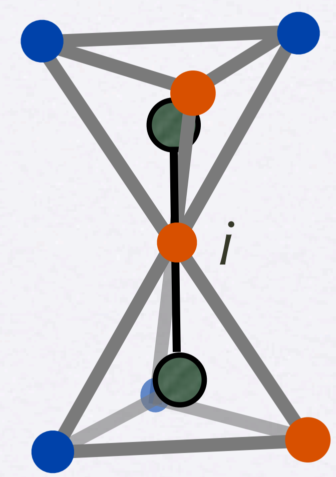


but simpler!

How we do this: compact abelian lattice Higgs theory



dual diamond lattice



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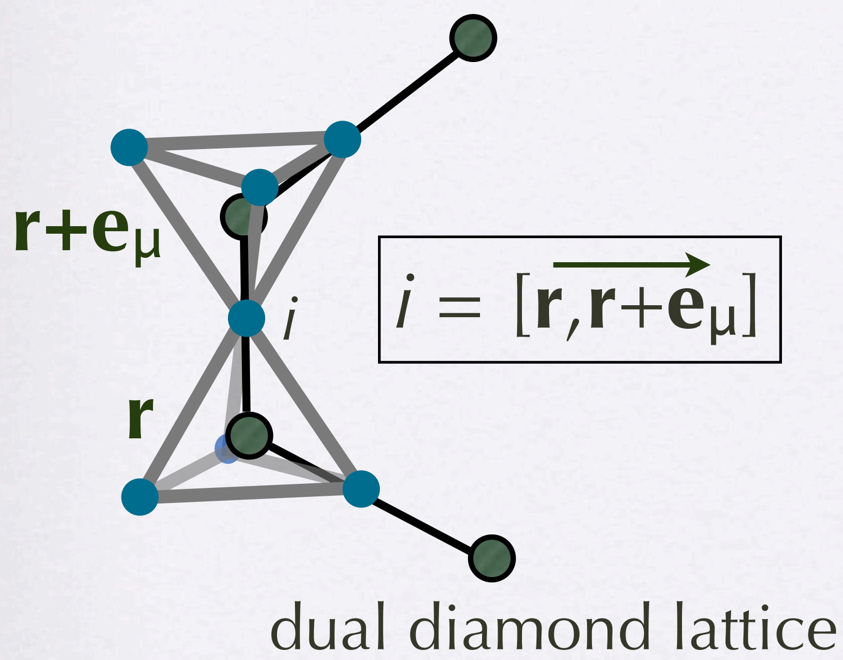

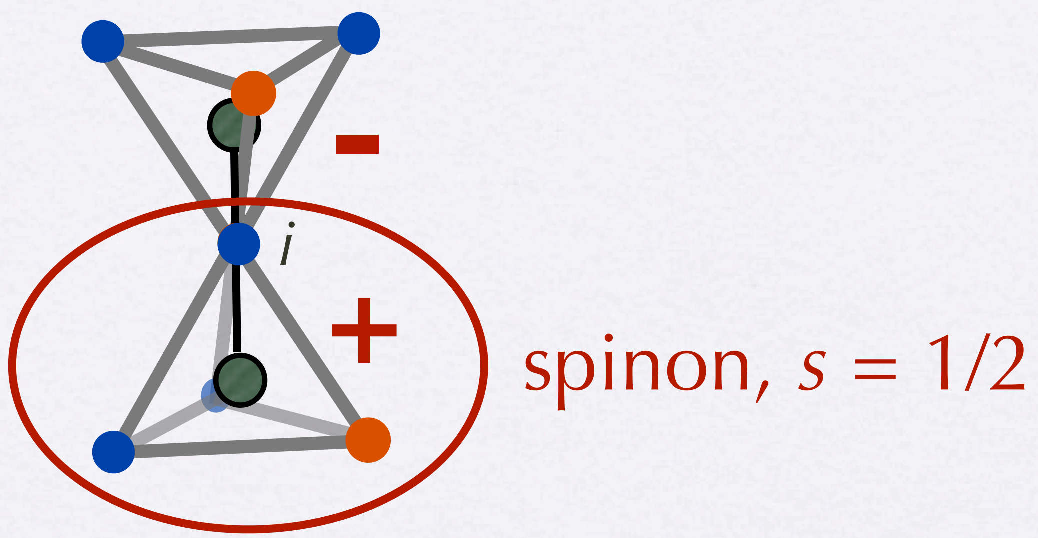

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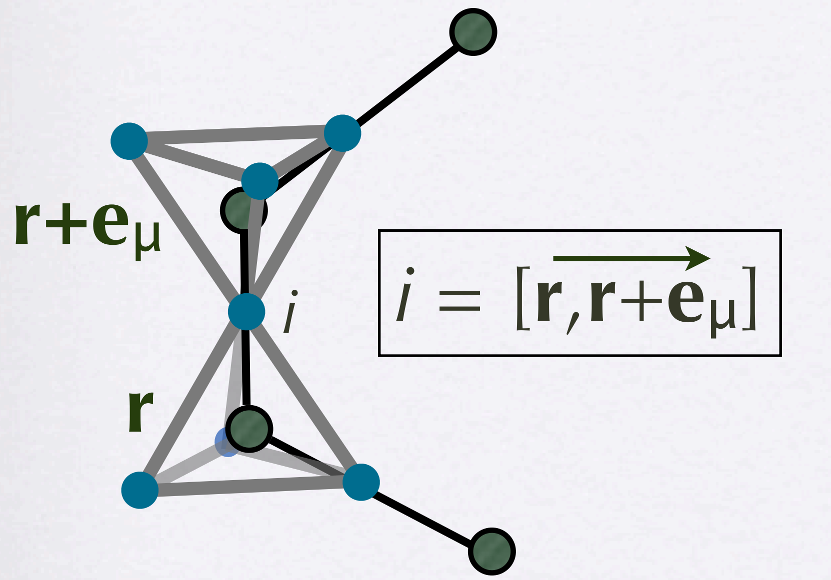

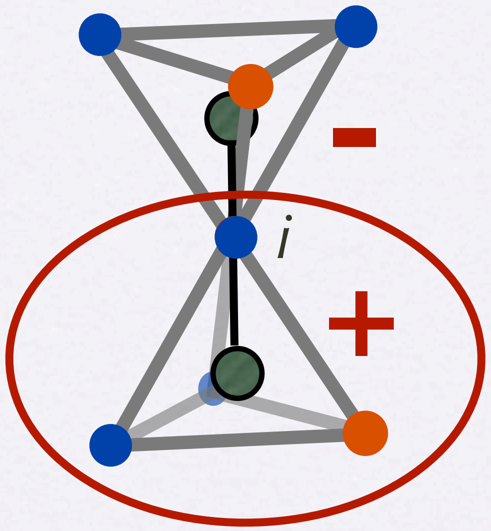

$$\mathsf{S}^{+}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu} = \Phi^{\dagger}_{\mathbf{r}}\, \mathsf{s}^{+}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu}\, \Phi_{\mathbf{r}+\mathbf{e}_\mu}$$

$$\mathsf{S}^{z}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu} = \mathsf{s}^{z}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu}$$

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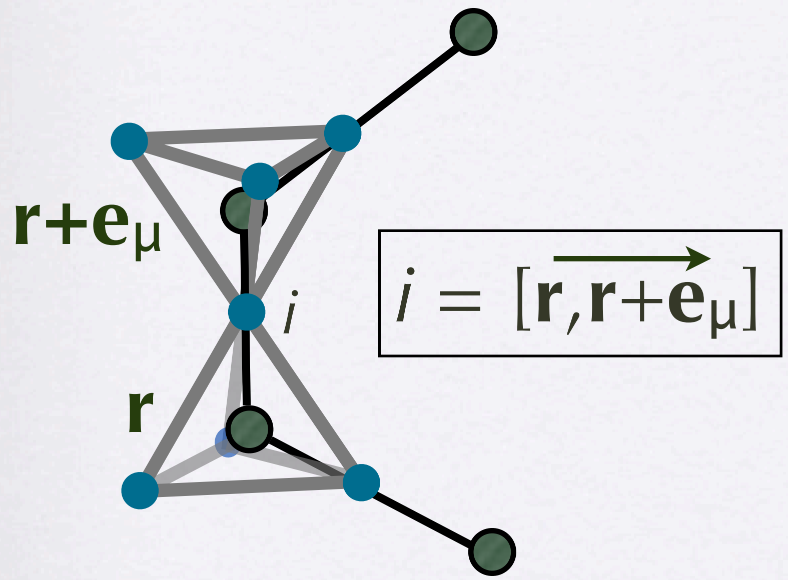

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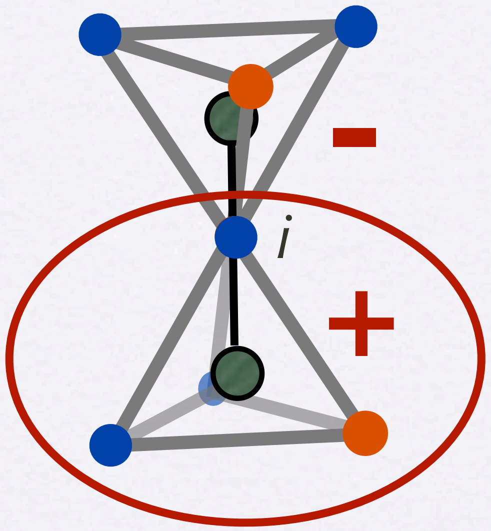


spinon, $s = 1/2$

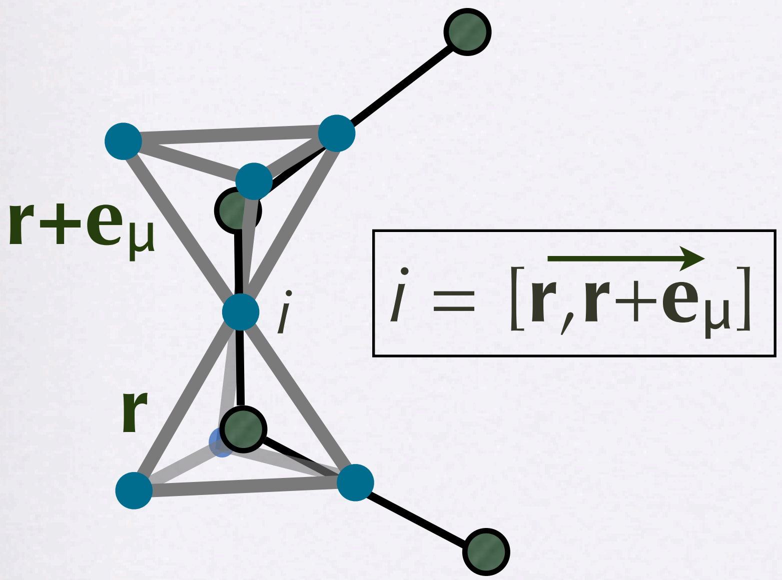
$$S^{+}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu} = \Phi^{\dagger}_{\mathbf{r}}\, s^{+}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu}\, \Phi_{\mathbf{r}+\mathbf{e}_\mu}$$

$$S^{z}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu} = s^{z}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu}$$

$$\begin{cases} \Phi_{\mathbf{r}} \to \Phi_{\mathbf{r}}\, e^{-i\chi_{\mathbf{r}}} \\ s^{\pm}_{\mathbf{r}\mathbf{r}'} \to s^{\pm}_{\mathbf{r}\mathbf{r}'}\, e^{\pm i(\chi_{\mathbf{r}'}-\chi_{\mathbf{r}})} \end{cases}$$

U(1) gauge symmetry

How we do this: compact abelian lattice Higgs theory



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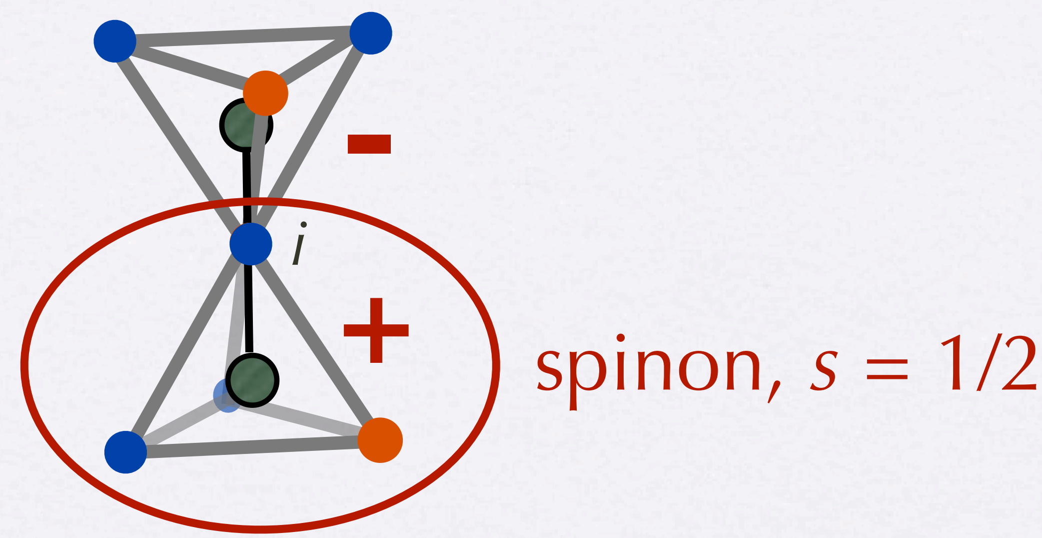

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U(1) gauge symmetry

the slave particles have a simple interpretation

How we do this: compact abelian Higgs U(1) lattice gauge theory

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$$\mathsf{S}^{z}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu} = \mathsf{s}^{z}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu}$$

$$|\Phi_{\mathbf{r}}| = 1$$

$$Q_{\mathbf{r}} = \pm \sum_{\mu} \mathsf{s}^{z}_{\mathbf{r},\mathbf{r}\pm\mathbf{e}_\mu}$$

$$\mathsf{s}^{z}_{\mathbf{r}\mathbf{r}'} = E_{\mathbf{r}\mathbf{r}'}$$

$$\mathsf{s}^{\pm}_{\mathbf{r}\mathbf{r}'} = e^{\pm i A_{\mathbf{r}\mathbf{r}'}}$$

our spinons are bosons

How we do this: compact abelian Higgs U(1) lattice gauge theory

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our spinons are bosons $\Longrightarrow$ they can condense

$\langle\Phi\rangle$	phase
$\neq 0$	conventional
$= 0$	exotic

How we do this: compact abelian Higgs U(1) lattice gauge theory

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$$H = \sum_{\mathbf{r}\in \mathrm{I,II}} \frac{J_{zz}}{2} Q_{\mathbf{r}}^2 - J_\pm \left\{ \sum_{\mathbf{r}\in\mathrm{I}} \sum_{\mu,\nu\neq\mu} \Phi^\dagger_{\mathbf{r}+\mathbf{e}_\mu} \Phi_{\mathbf{r}+\mathbf{e}_\nu} \mathsf{s}^-_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu} \mathsf{s}^+_{\mathbf{r},\mathbf{r}+\mathbf{e}_\nu} + \sum_{\mathbf{r}\in\mathrm{II}} \sum_{\mu,\nu\neq\mu} \Phi^\dagger_{\mathbf{r}-\mathbf{e}_\mu} \Phi_{\mathbf{r}-\mathbf{e}_\nu} \mathsf{s}^+_{\mathbf{r},\mathbf{r}-\mathbf{e}_\mu} \mathsf{s}^-_{\mathbf{r},\mathbf{r}-\mathbf{e}_\nu} \right\}$$

$$-J_{z\pm} \left\{ \sum_{r\in\mathrm{I}} \sum_{\mu,\nu\neq\mu} \left(\gamma^*_{\mu\nu} \Phi^\dagger_{\mathbf{r}} \Phi_{\mathbf{r}+\mathbf{e}_\nu} \mathsf{s}^z_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu} \mathsf{s}^+_{\mathbf{r},\mathbf{r}+\mathbf{e}_\nu} + \mathrm{h.c.} \right) + \sum_{\mathbf{r}\in\mathrm{II}} \sum_{\mu,\nu\neq\mu} \left(\gamma^*_{\mu\nu} \Phi^\dagger_{\mathbf{r}-\mathbf{e}_\nu} \Phi_{\mathbf{r}} \mathsf{s}^z_{\mathbf{r},\mathbf{r}-\mathbf{e}_\mu} \mathsf{s}^+_{\mathbf{r},\mathbf{r}-\mathbf{e}_\nu} + \mathrm{h.c.} \right) \right\} + \mathrm{const.}$$

$J_{\pm\pm} = 0$

How we do this: compact abelian Higgs U(1) lattice gauge theory

$$\mathsf{S}^{+}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu} = \Phi^{\dagger}_{\mathbf{r}}\, \mathsf{s}^{+}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu} \Phi_{\mathbf{r}+\mathbf{e}_\mu}$$

$$\mathsf{S}^{z}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu} = \mathsf{s}^{z}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu}$$

$$|\Phi_{\mathbf{r}}| = 1$$

$$Q_{\mathbf{r}} = \pm \sum_{\mu} \mathsf{s}^{z}_{\mathbf{r},\mathbf{r}\pm\mathbf{e}_\mu}$$

$$\mathsf{s}^{z}_{\mathbf{r}\mathbf{r}'} = E_{\mathbf{r}\mathbf{r}'}$$

$$\mathsf{s}^{\pm}_{\mathbf{r}\mathbf{r}'} = e^{\pm i A_{\mathbf{r}\mathbf{r}'}}$$

our spinons are bosons $\Longrightarrow$ they can condense

$\langle \Phi \rangle$	phase
$\neq 0$	conventional
$= 0$	exotic

vacuum: quantum superposition of two-in-two-out states

H = hopping Hamiltonian for spinons in fluctuating background

gauge Mean Field Theory (gMFT)

$$\Phi^\dagger \Phi \, \mathsf{s}\, \mathsf{s} \to \Phi^\dagger \Phi \langle \mathsf{s} \rangle \langle \mathsf{s} \rangle + \langle \Phi^\dagger \Phi \rangle \mathsf{s} \langle \mathsf{s} \rangle + \langle \Phi^\dagger \Phi \rangle \langle \mathsf{s} \rangle \mathsf{s} - 2 \langle \Phi^\dagger \Phi \rangle \langle \mathsf{s} \rangle \langle \mathsf{s} \rangle$$

gauge Mean Field Theory (gMFT)

$$\Phi^\dagger \Phi\, \mathsf{s}\, \mathsf{s} \to \Phi^\dagger \Phi \langle \mathsf{s} \rangle \langle \mathsf{s} \rangle + \langle \Phi^\dagger \Phi \rangle \mathsf{s} \langle \mathsf{s} \rangle + \langle \Phi^\dagger \Phi \rangle \langle \mathsf{s} \rangle \mathsf{s} - 2 \langle \Phi^\dagger \Phi \rangle \langle \mathsf{s} \rangle \langle \mathsf{s} \rangle$$

$$H_{\mathsf{s}}^{\mathrm{MF}} = -\sum_{\mathbf{r}} \sum_{\mu} \vec{\mathsf{h}}_{\mathrm{eff},\mu}^{\mathrm{MF}} \cdot \vec{\mathsf{s}}_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu}$$

$$H_{\Phi}^{\mathrm{MF}} = -\sum_{\mathbf{r}} \sum_{\mu \neq \nu} \left[t_\mu^{\mathrm{MF}} \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r}+\mathbf{e}_\mu} + {t'}_{\mu\nu}^{\mathrm{MF}} \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r}+\mathbf{e}_\mu-\mathbf{e}_\nu} + \mathrm{h.c.} \right]$$

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$$\Phi^\dagger \Phi\, \mathsf{s}\, \mathsf{s} \to \Phi^\dagger \Phi \langle \mathsf{s} \rangle \langle \mathsf{s} \rangle + \langle \Phi^\dagger \Phi \rangle \mathsf{s} \langle \mathsf{s} \rangle + \langle \Phi^\dagger \Phi \rangle \langle \mathsf{s} \rangle \mathsf{s} - 2 \langle \Phi^\dagger \Phi \rangle \langle \mathsf{s} \rangle \langle \mathsf{s} \rangle$$

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free (but self-consistent) "spins"

$$H_{\Phi}^{\mathrm{MF}} = -\sum_{\mathbf{r}} \sum_{\mu \neq \nu} \left[t_{\mu}^{\mathrm{MF}} \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r}+\mathbf{e}_\mu} + {t'}_{\mu\nu}^{\mathrm{MF}} \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r}+\mathbf{e}_\mu - \mathbf{e}_\nu} + \mathrm{h.c.} \right]$$

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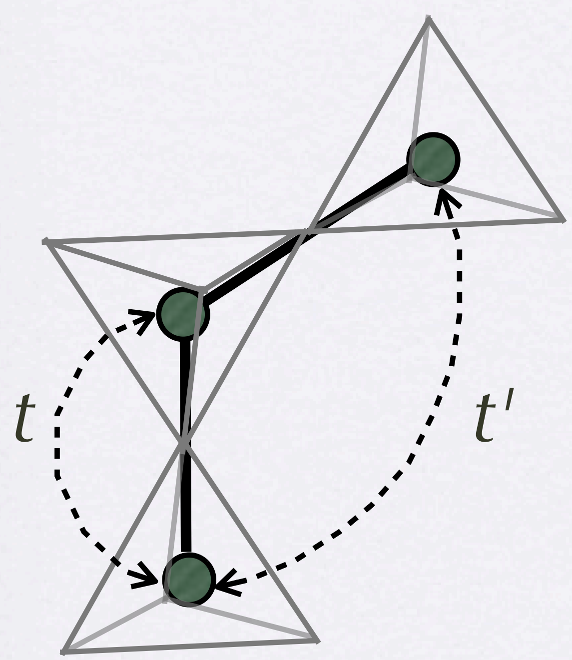
hopping Hamiltonian for spinons in fixed (but self-consistent) background



gauge Mean Field Theory (gMFT)

$$\Phi^\dagger \Phi \, \mathsf{s}\, \mathsf{s} \to \Phi^\dagger \Phi \langle \mathsf{s} \rangle \langle \mathsf{s} \rangle + \langle \Phi^\dagger \Phi \rangle \mathsf{s} \langle \mathsf{s} \rangle + \langle \Phi^\dagger \Phi \rangle \langle \mathsf{s} \rangle \mathsf{s} - 2 \langle \Phi^\dagger \Phi \rangle \langle \mathsf{s} \rangle \langle \mathsf{s} \rangle$$

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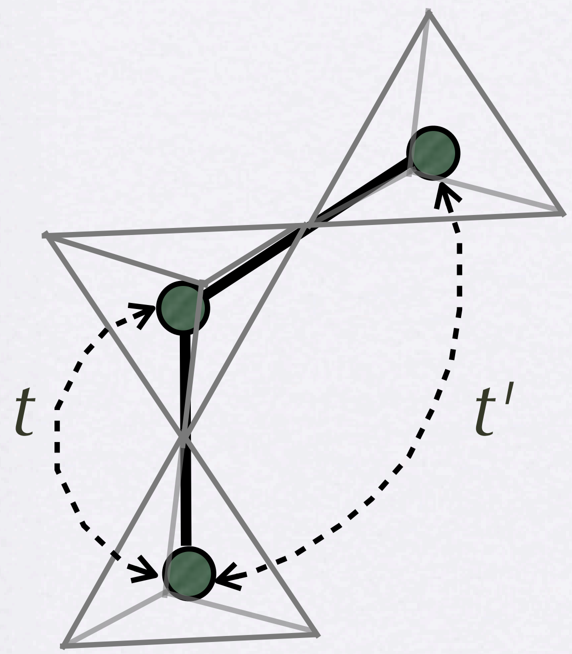
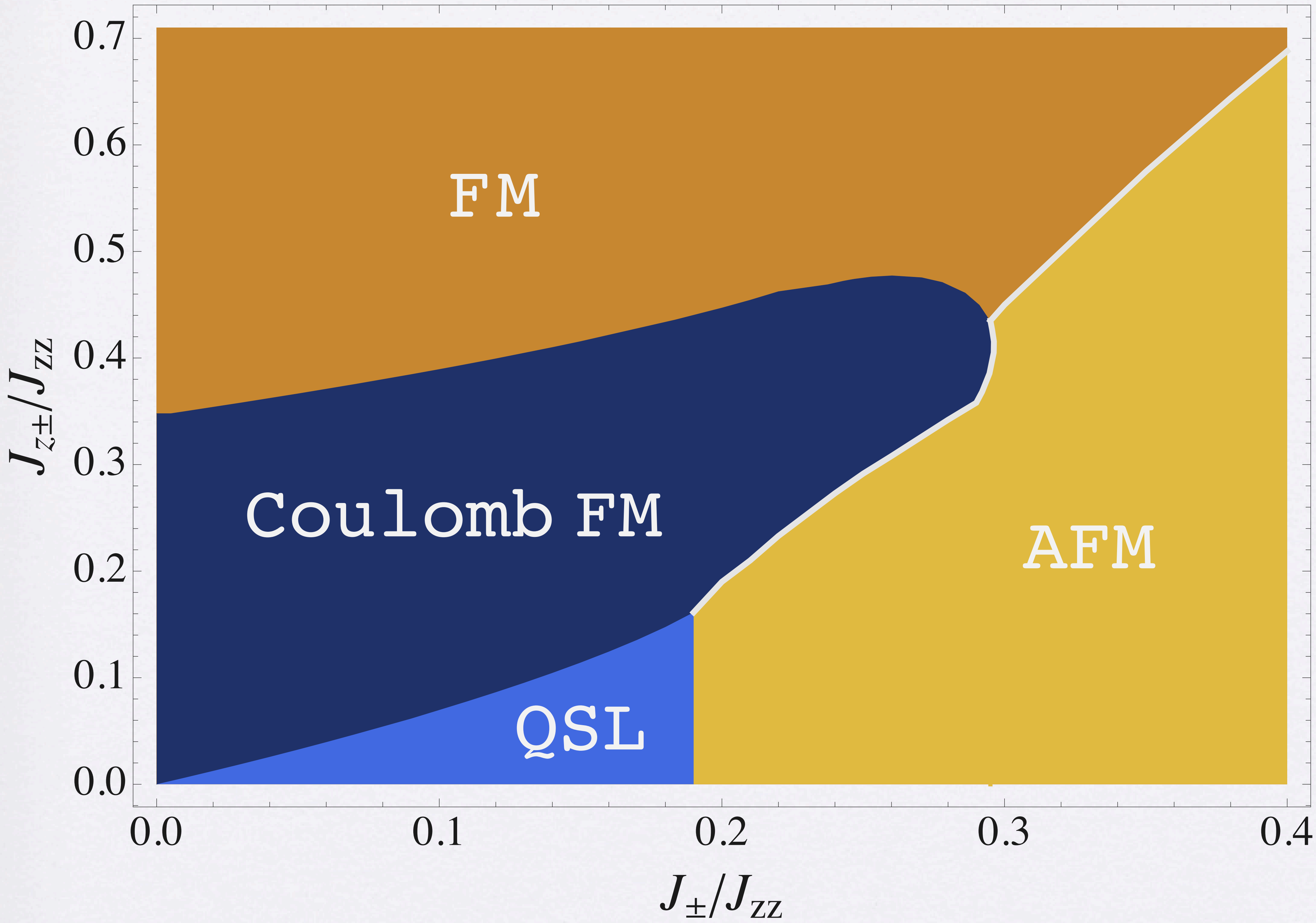


Solve the consistency equations

Now is when you tune back in for

Quantum spin liquids in quantum spin ices

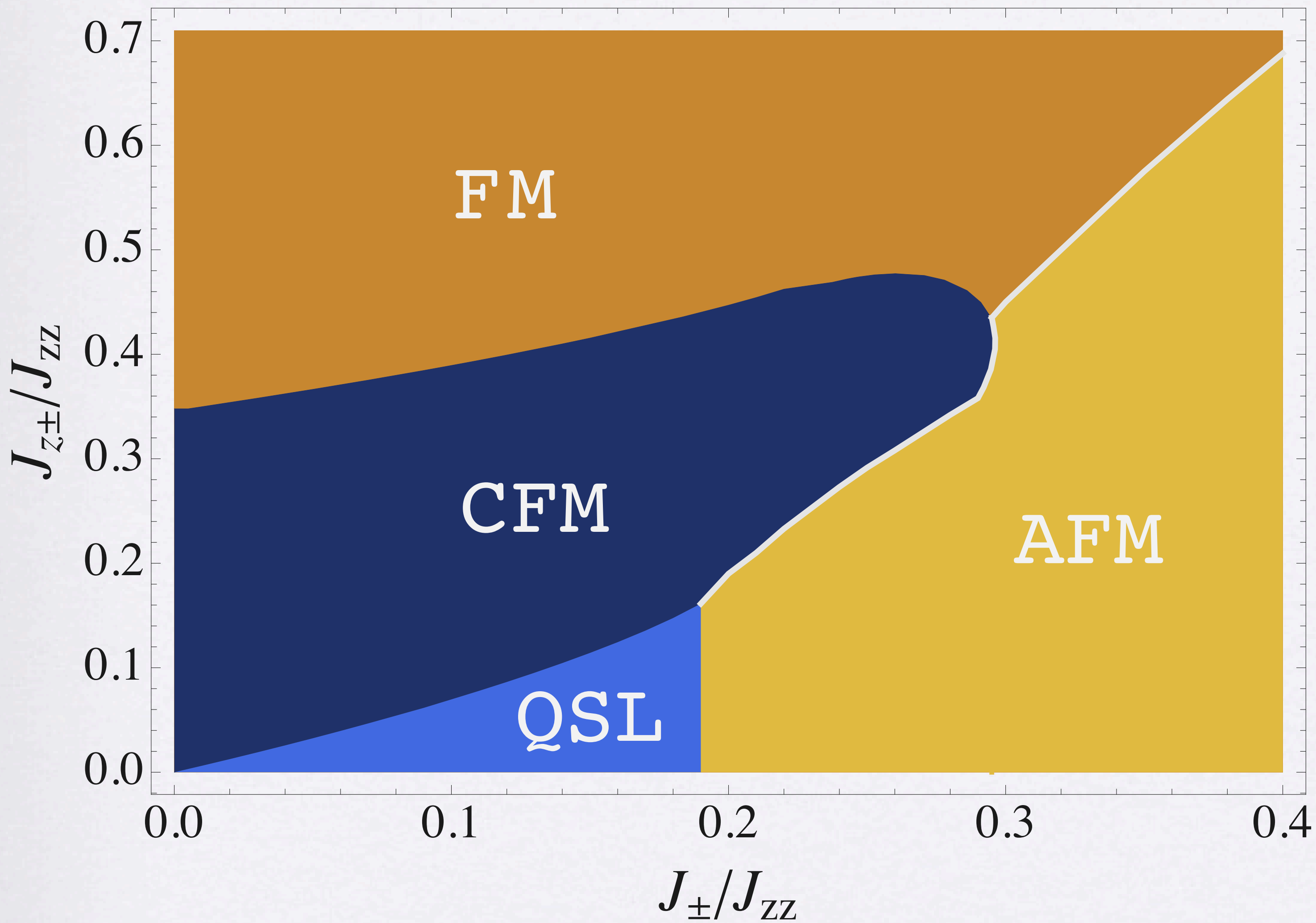
Phase diagram



$J_{\pm\pm} = 0$

Phase diagram

$J_{\pm\pm} = 0$



$\langle\Phi\rangle$	$\langle S^z\rangle$	phase
$\neq 0$	$= 0$	AFM
$\neq 0$	$\neq 0$	FM
$= 0$	$= 0$	QSL
$= 0$	$\neq 0$	CFM

Phase diagram

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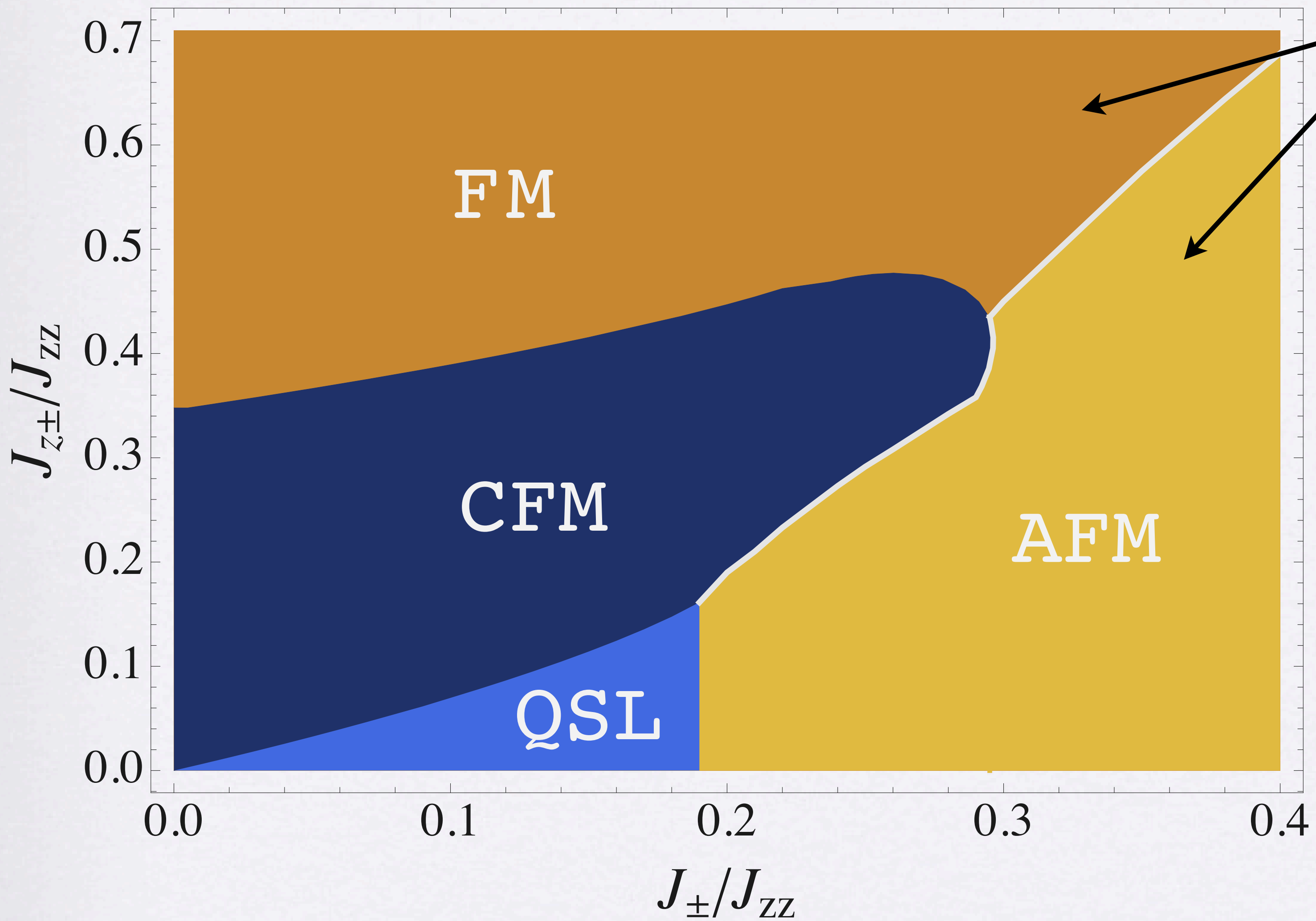

Higgs
= gauge symmetry breaking
= condensed
= **conventional phases**

$\langle \Phi \rangle \neq 0$

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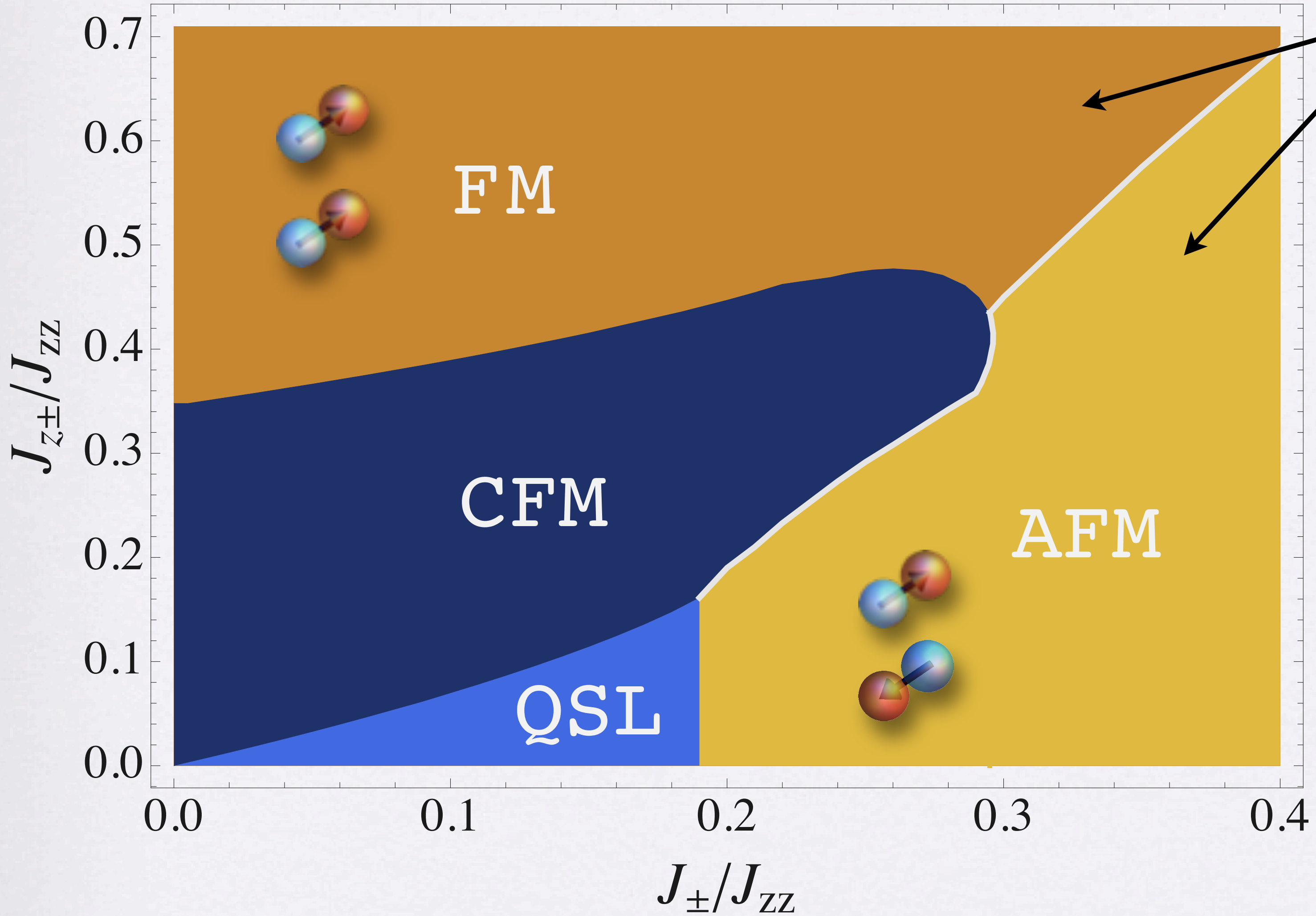

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Phase diagram

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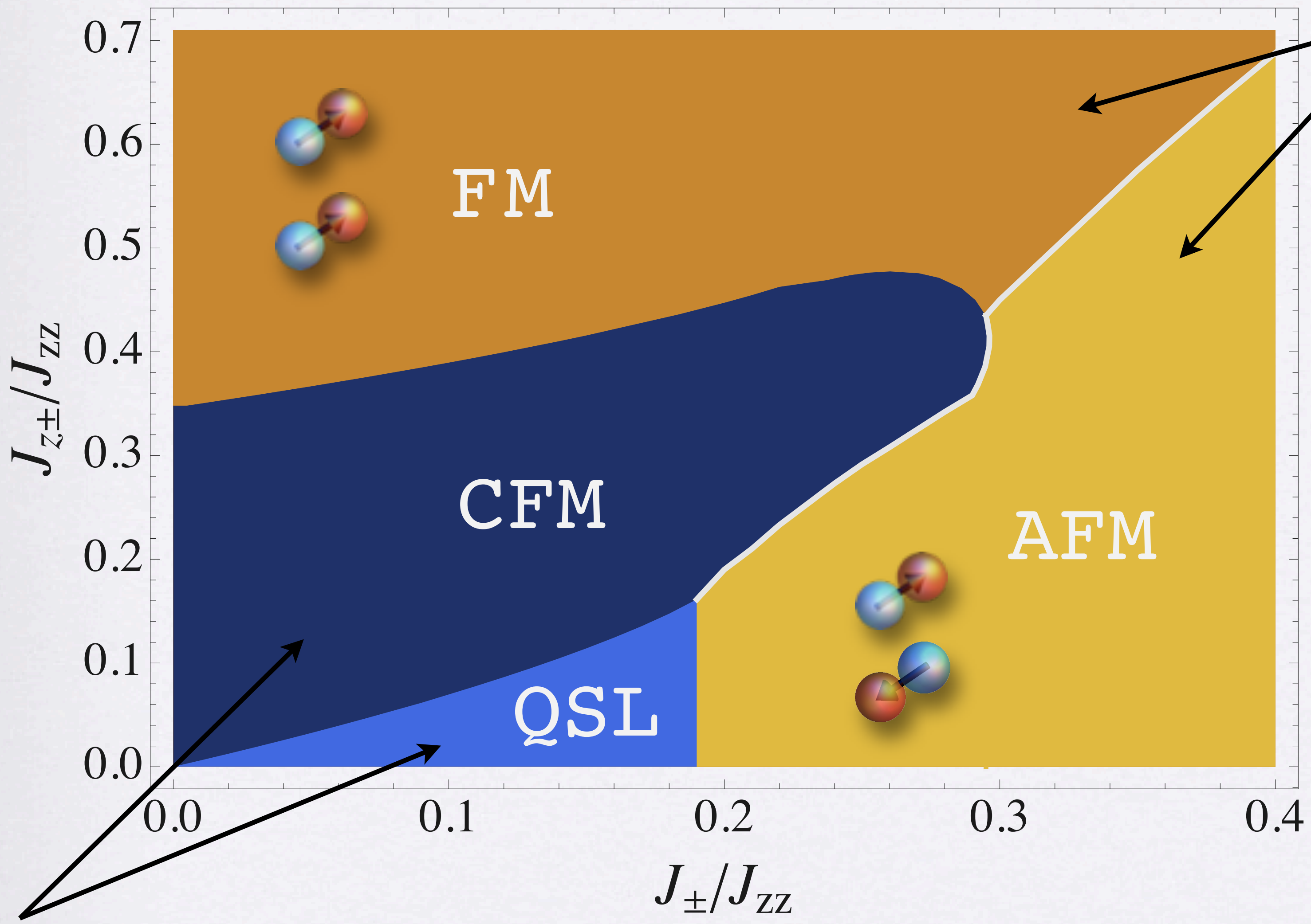

Higgs
= gauge symmetry breaking
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= **conventional phases**

$\langle \Phi \rangle \neq 0$

deconfined
= uncondensed
= **exotic**

$\langle \Phi \rangle = 0$

$\langle \Phi \rangle$	$\langle S^z \rangle$	phase
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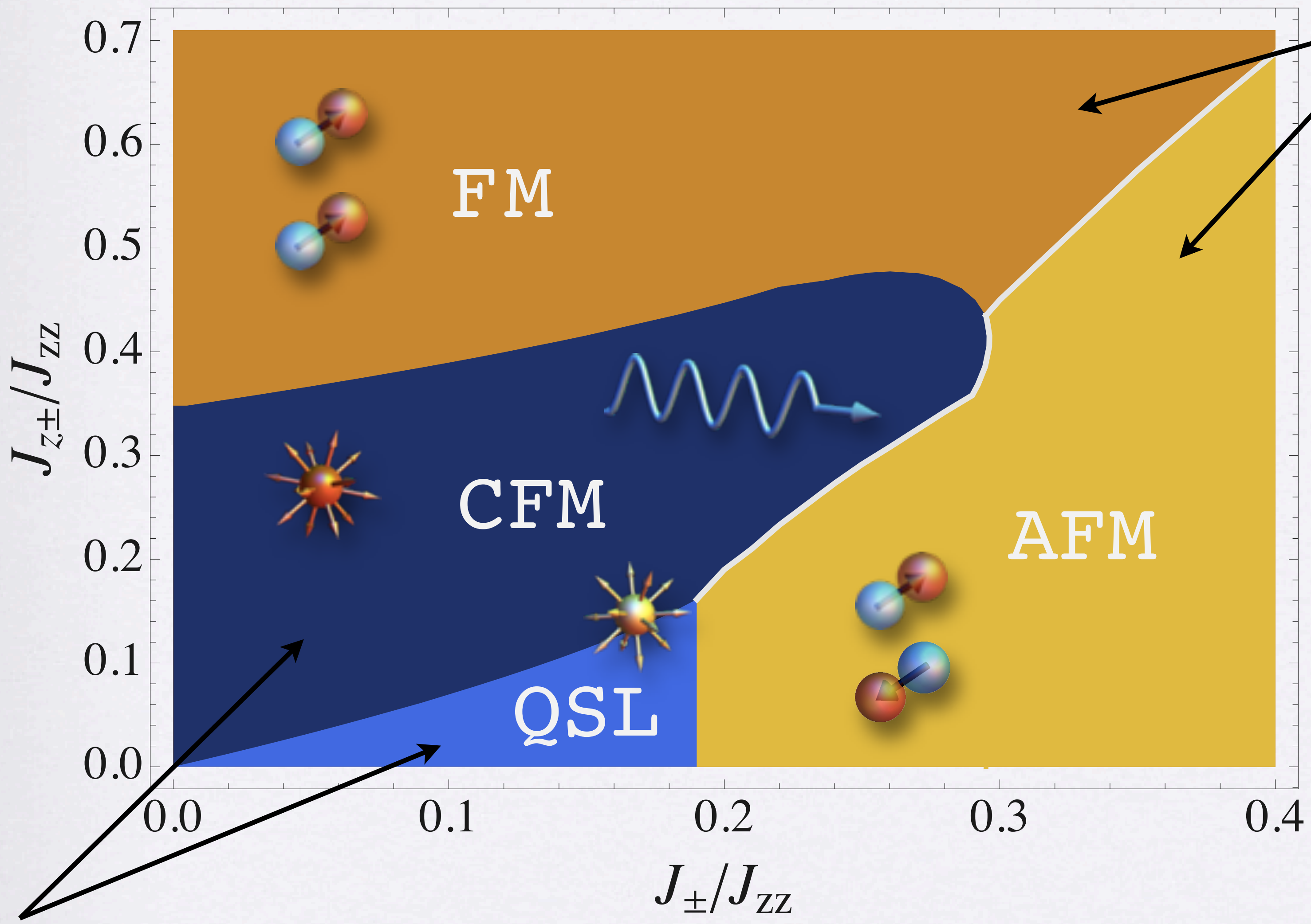
Phase diagram

$J_{\pm\pm} = 0$



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Insight into the exotic phases

- superposition of states

$$|\psi\rangle \sim \text{equal-weight quantum superposition of 2-in-2-out states}$$

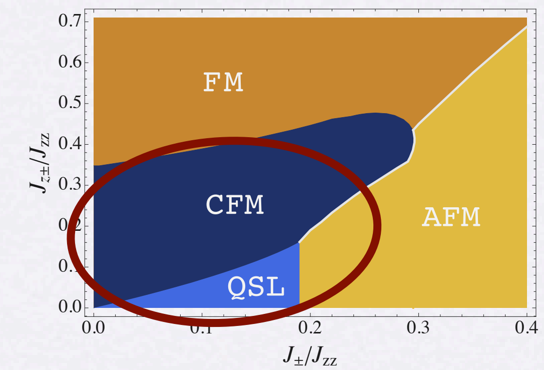
- inelastic structure factor $\mathcal{S}(\mathbf{k},\omega) = \sum_{\mu,\nu}\left[\delta_{\mu\nu} - (\hat{\mathbf{k}})_\mu(\hat{\mathbf{k}})_\nu\right]\sum_{a,b}\left\langle m_a^\mu(-\mathbf{k},-\omega) m_b^\nu(\mathbf{k},\omega)\right\rangle$

$\langle \mathsf{S}^z\mathsf{S}^z\rangle$ contribution $\longleftrightarrow$ photon mode

$\langle \mathsf{S}^+\mathsf{S}^-\rangle$ contribution $\longleftrightarrow$ spinon mode

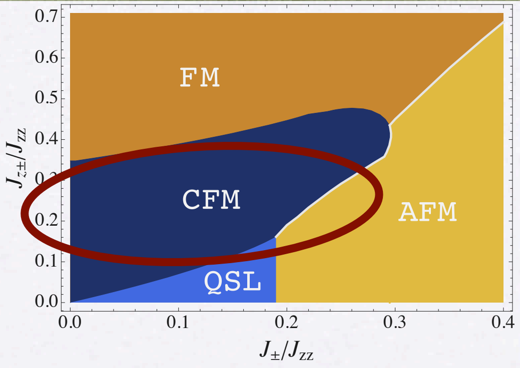
$$\mathsf{S}^z|\psi\rangle = |1 \text{ photon} + \text{vacuum}\rangle$$

$$\mathsf{S}^+|\psi\rangle = |2 \text{ spinons} + \text{vacuum}\rangle$$



The Coulomb ferromagnet

(secretly a quantum spin liquid!)



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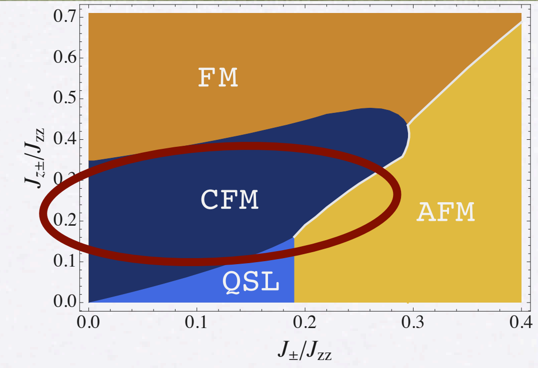

- magnetized

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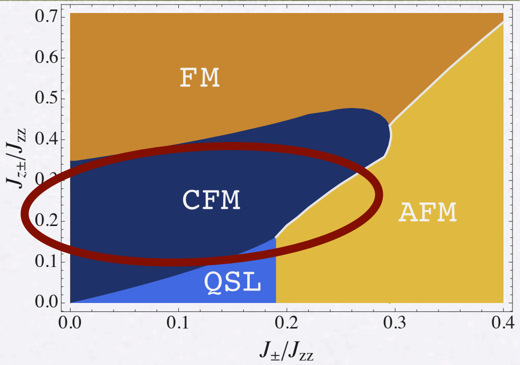
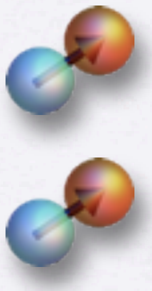


(secretly a quantum spin liquid!)

- magnetized

$$\langle S^z \rangle \neq 0$$



spins with non-zero expectation value

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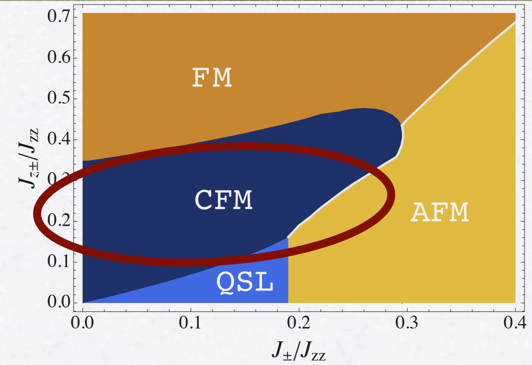

- magnetized

$$\langle S^z \rangle \neq 0$$

$$\langle S^z \rangle < 1/2$$

spins with non-zero expectation value

$\langle \Phi \rangle$	$\langle S^z \rangle$	phase
$\neq 0$	$= 0$	AFM
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The Coulomb ferromagnet

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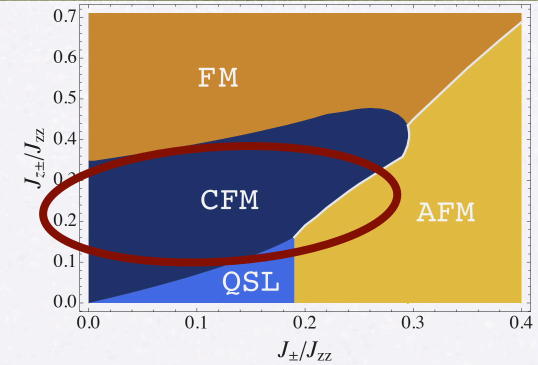

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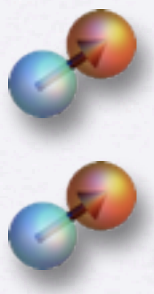
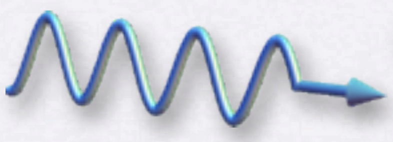


spins with non-zero expectation value

$$\langle S^z \rangle < 1/2$$

- supports exotic excitations



gapless photon

$$\langle \Phi \rangle = 0$$

spinon

$$\langle S^\pm \rangle = 0$$

"electric" monopole

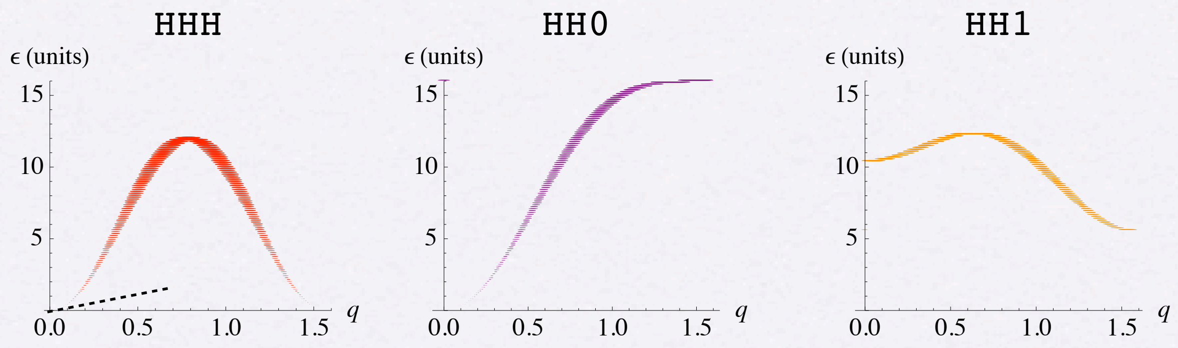
Signatures of the deconfined phases

- inelastic neutron scattering:
  - photon
  - spinon
- specific heat:
  - photon

Signatures of the deconfined phases

- inelastic neutron scattering:

  - photon



$$\mathcal{S}(\mathbf{k} \sim \mathbf{0}, \omega) \sim \omega\, \delta[\omega - v|\mathbf{k}|]$$

  - spinon

- specific heat:

  - photon

Signatures of the deconfined phases

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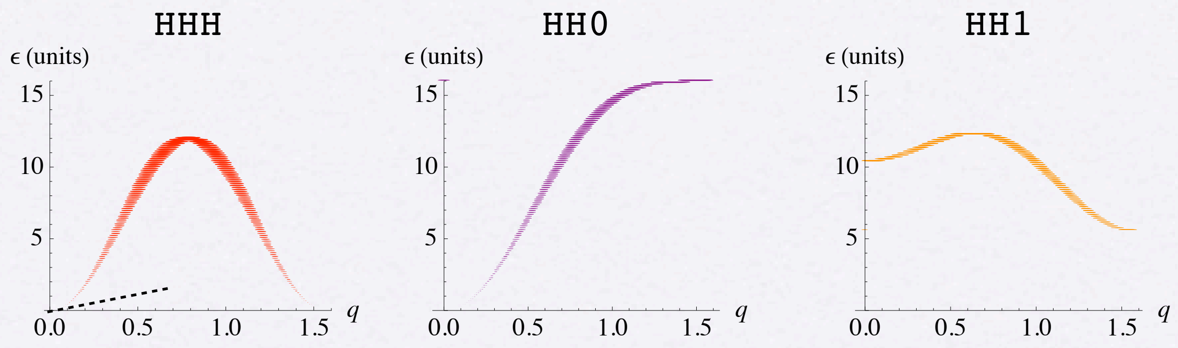

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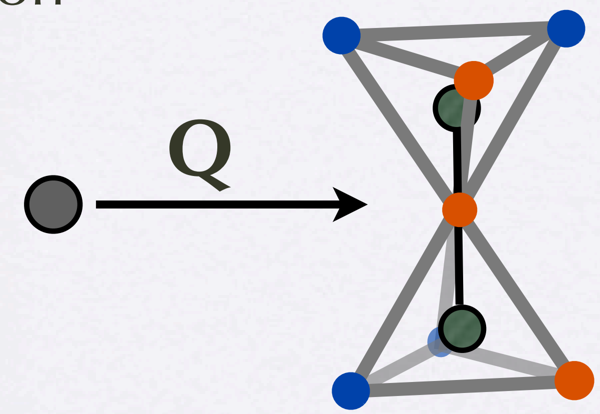

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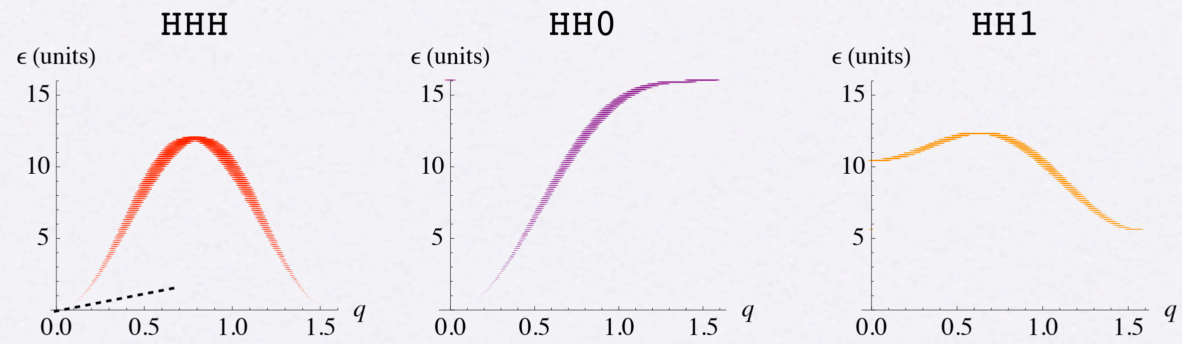
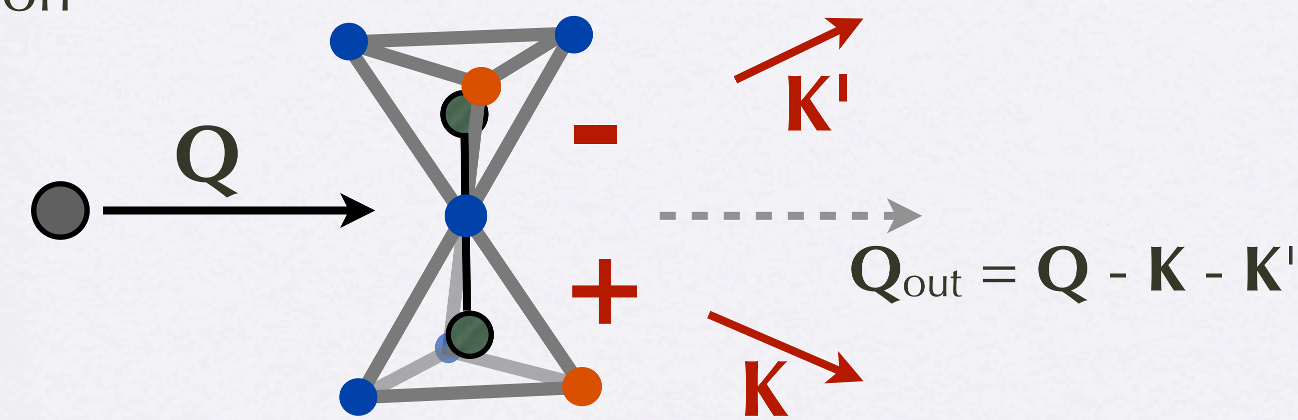


$$\mathcal{S}(\mathbf{k} \sim \mathbf{0}, \omega) \sim \omega\, \delta[\omega - v|\mathbf{k}|]$$

  - spinon



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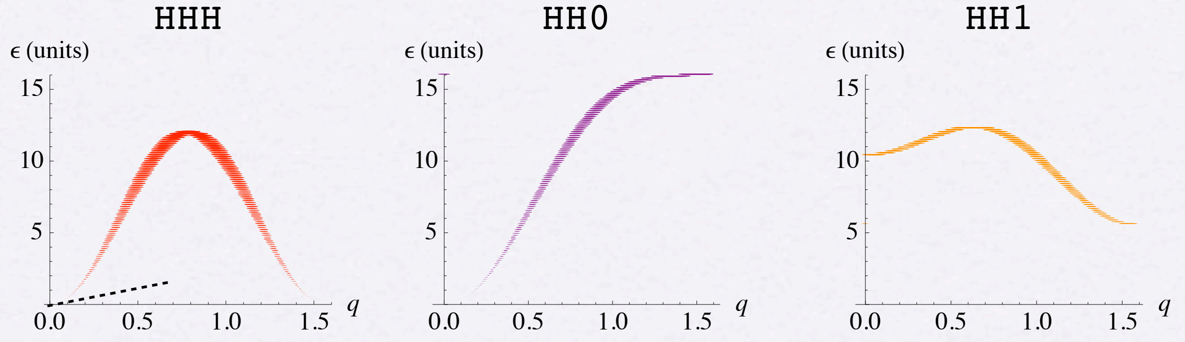
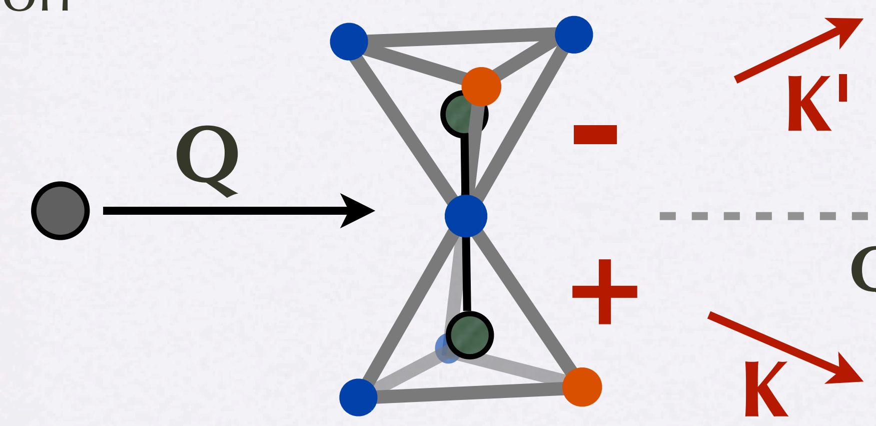
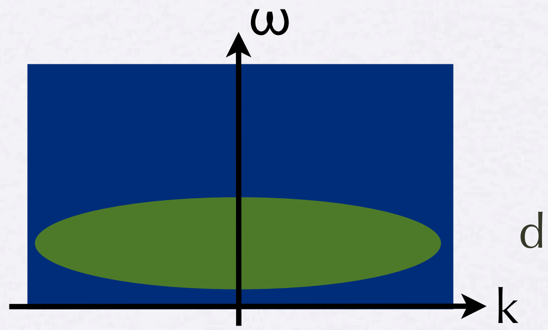


$$\mathcal{S}(\mathbf{k} \sim \mathbf{0}, \omega) \sim \omega\, \delta[\omega - v|\mathbf{k}|]$$

  - spinon



$Q_{out} = Q - K - K'$

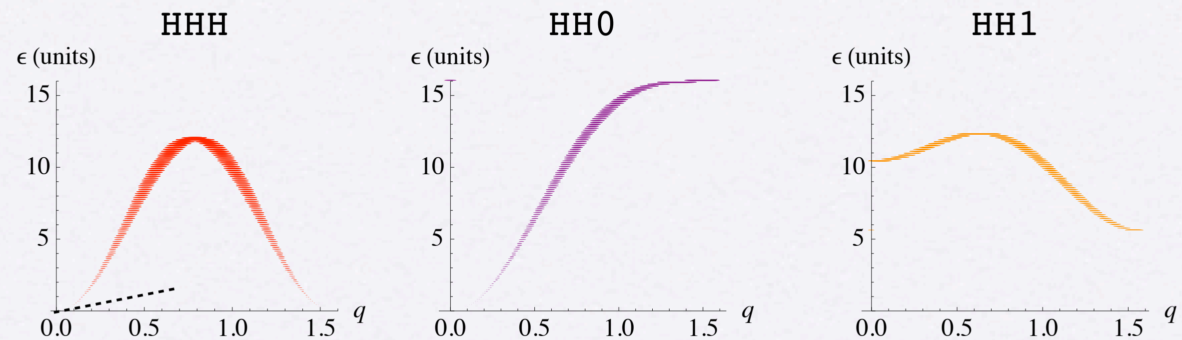


diffuse scattering

- specific heat:
  - photon

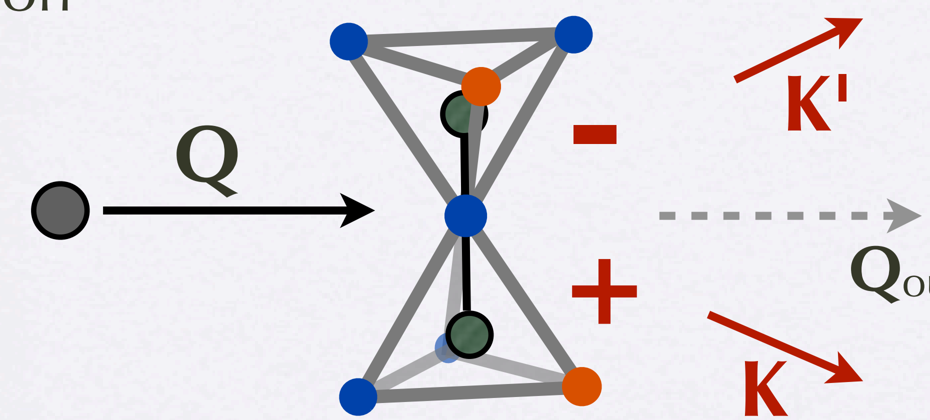
Signatures of the deconfined phases

- inelastic neutron scattering:
  - photon

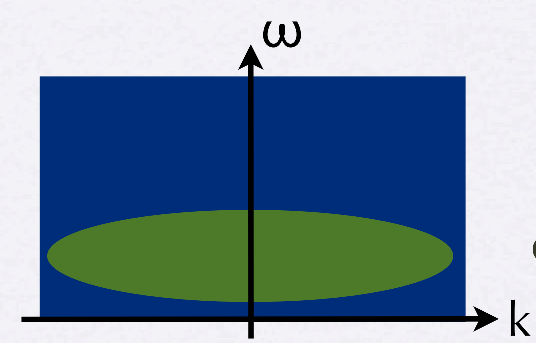


$$\mathcal{S}(\mathbf{k} \sim \mathbf{0}, \omega) \sim \omega\, \delta[\omega - v|\mathbf{k}|]$$

  - spinon



$\mathbf{Q}_{\text{out}} = \mathbf{Q} - \mathbf{K} - \mathbf{K}'$

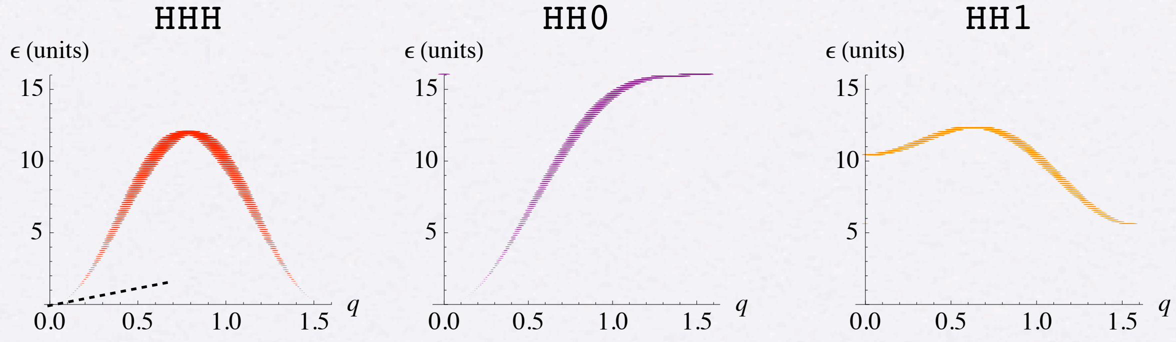


diffuse scattering

- specific heat:
  - photon $C_v^{T\approx 0} \sim B_{\text{photon}} T^3 + B_{\text{phonon}} T^3$

Signatures of the deconfined phases

- inelastic neutron scattering:
  - photon



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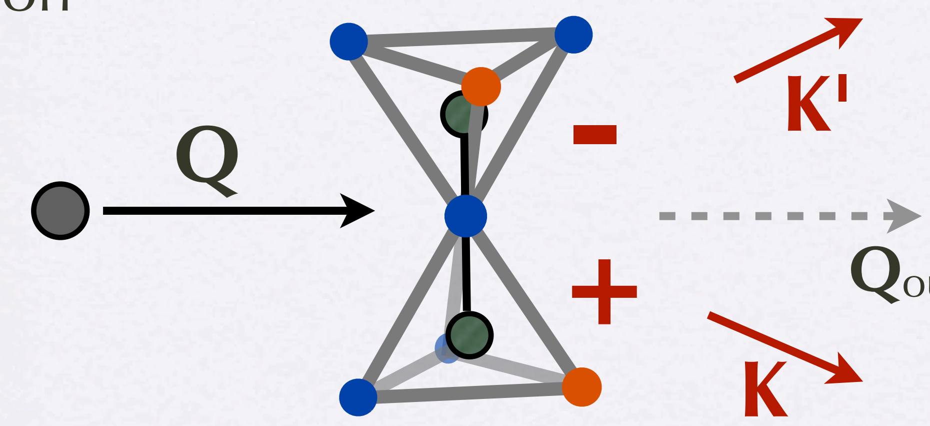
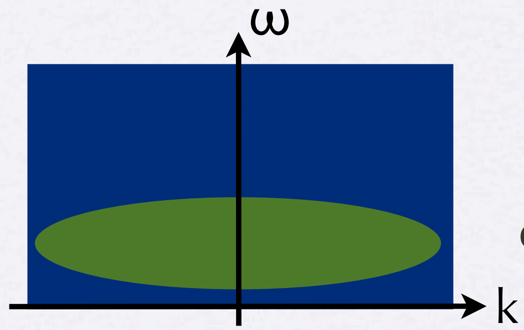
  - spinon





diffuse scattering

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$$\boxed{B_{\text{photon}} \sim 1000\, B_{\text{phonon}}}$$

Signatures of the deconfined phases

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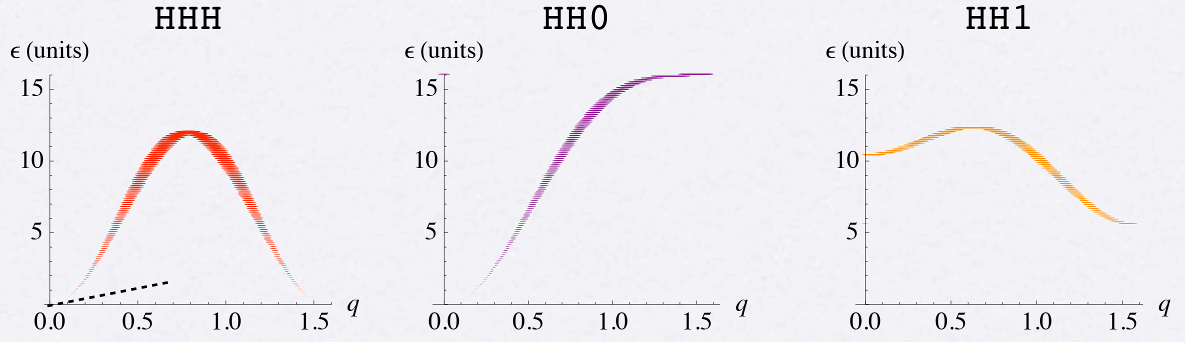

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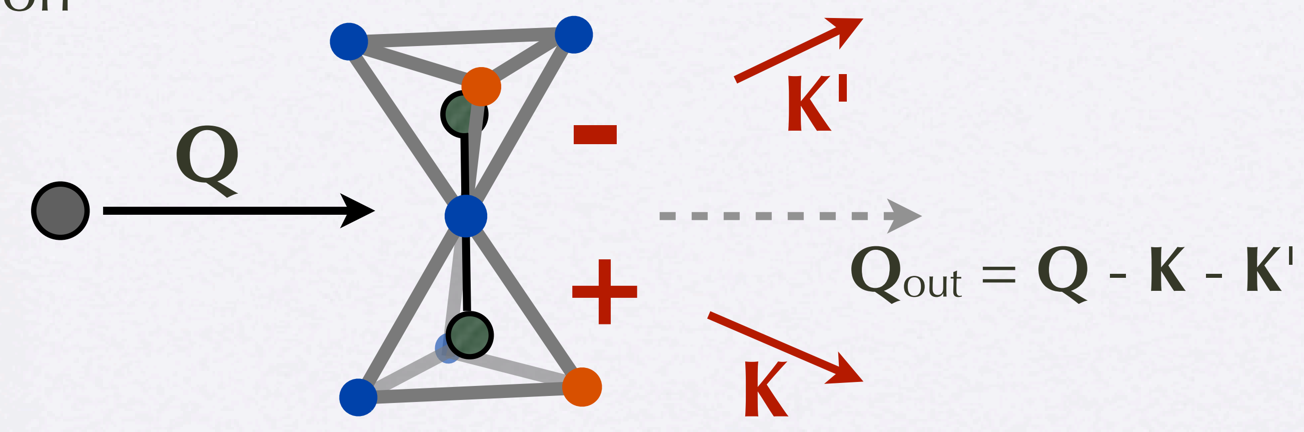

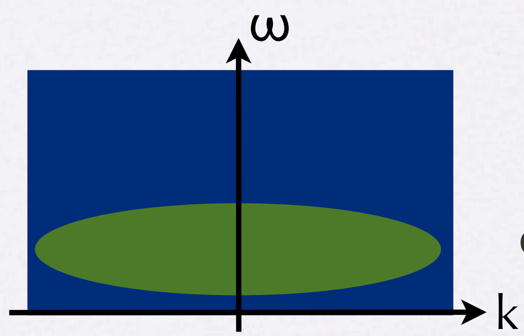

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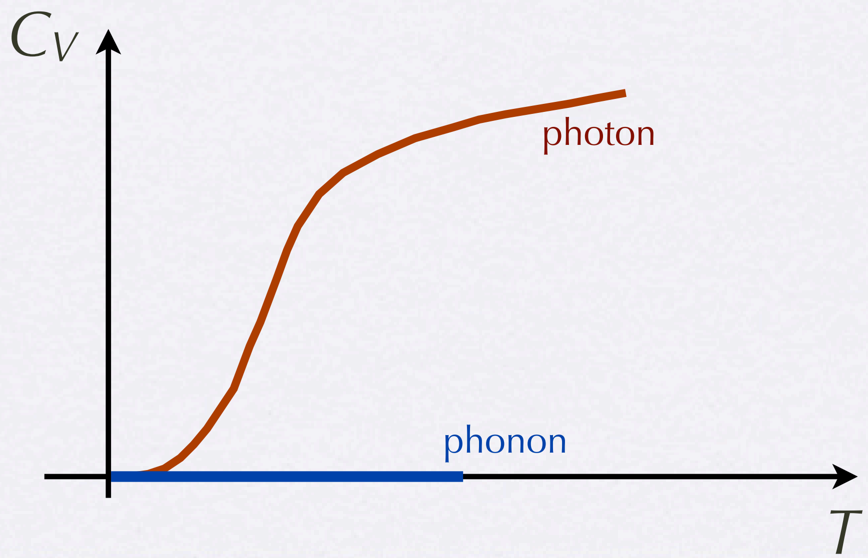
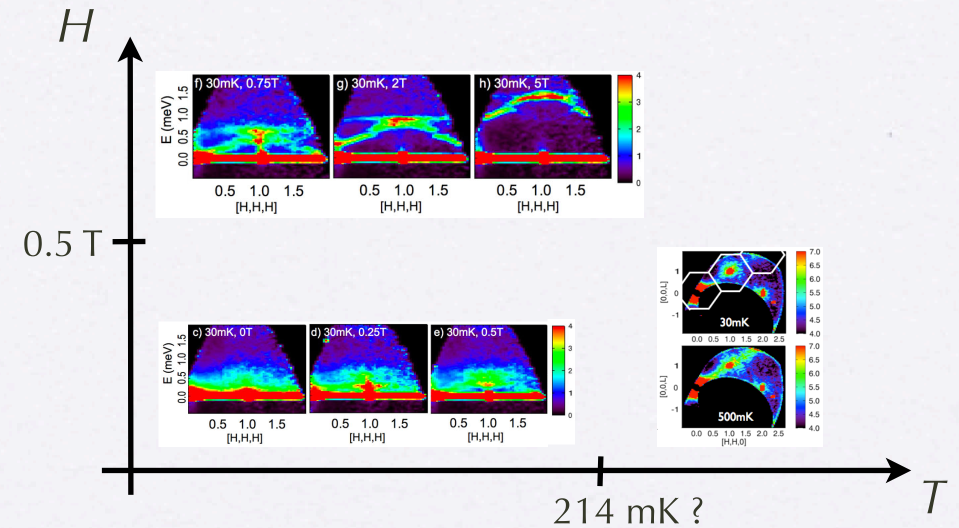
  - photon $C_v^{T \approx 0} \sim B_{\mathrm{photon}} T^3 + B_{\mathrm{phonon}} T^3$

$$\boxed{B_{\mathrm{photon}} \sim 1000\, B_{\mathrm{phonon}}}$$

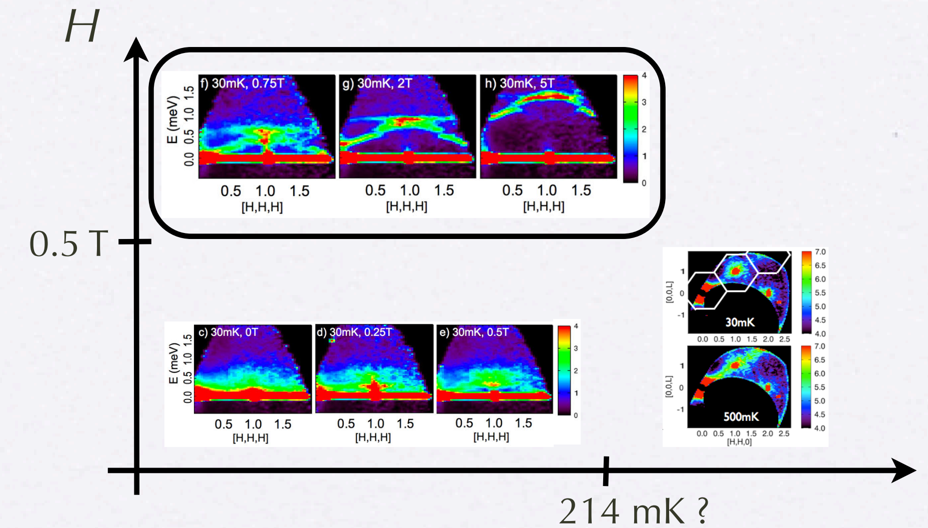


$Yb_2Ti_2O_7$



Ross, Savary, Gaulin, Balents PRX 2011

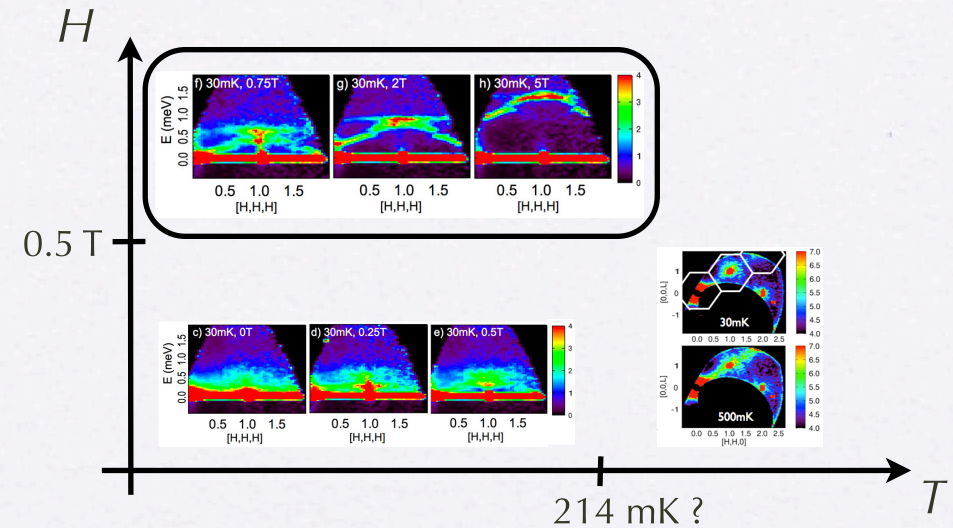
$Yb_2Ti_2O_7$



high field H = 5T

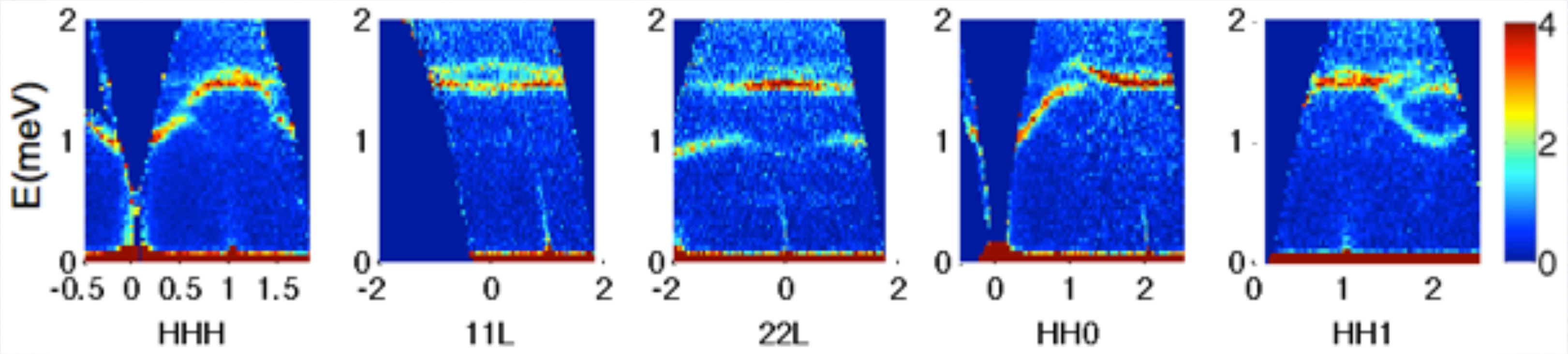
Ross, Savary, Gaulin, Balents PRX 2011

$Yb_2Ti_2O_7$



high field H = 5T

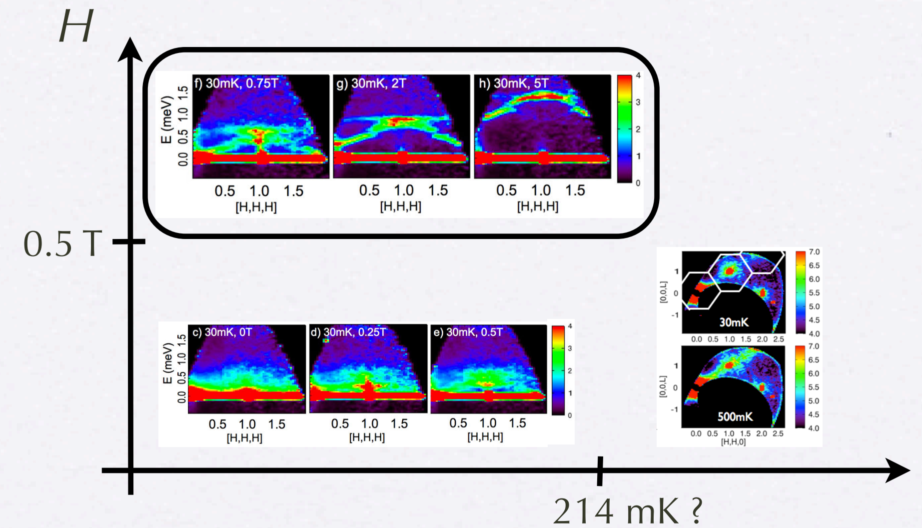
experiment



spin wave theory

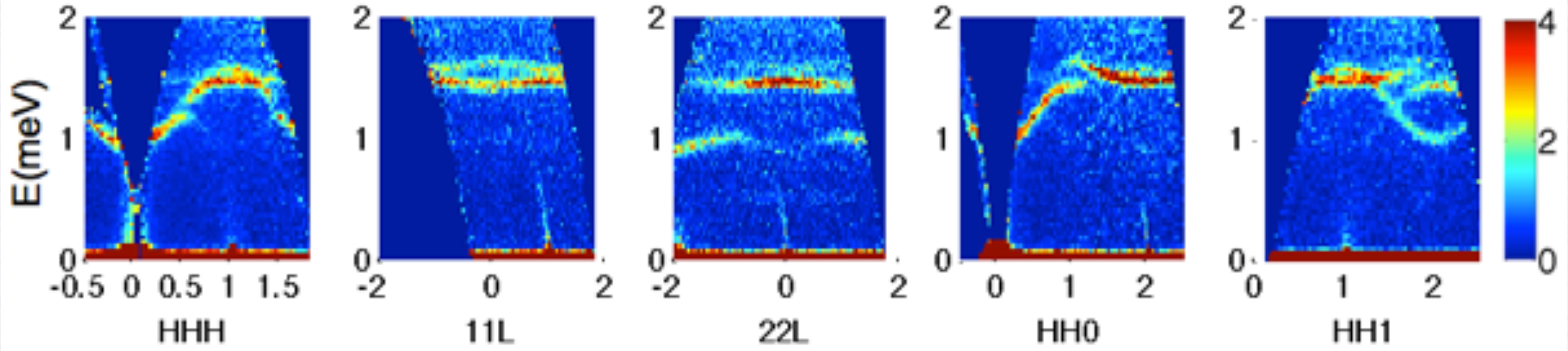
Ross, Savary, Gaulin, Balents PRX 2011

$Yb_2Ti_2O_7$



high field H = 5T

experiment

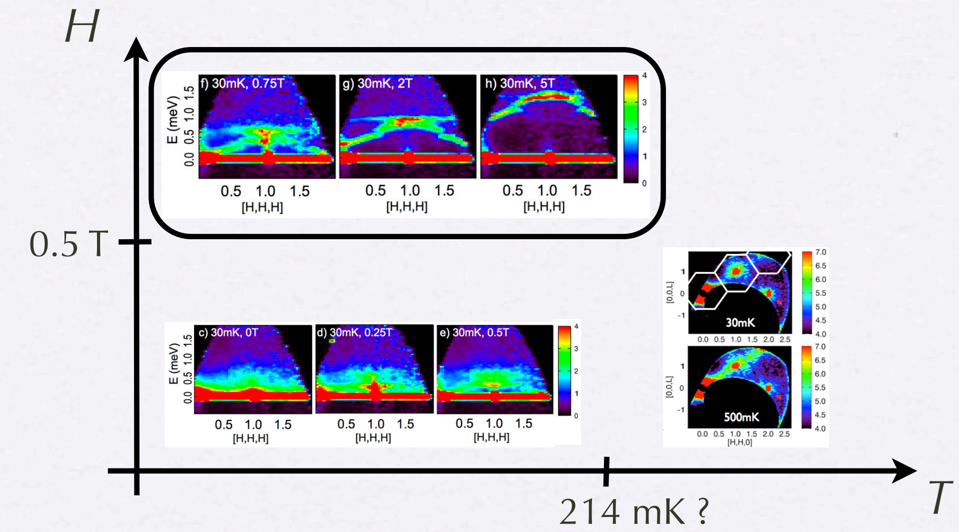


spin wave theory

1. classical high-field ground state
2. Holstein-Primakoff bosons in the spirit of large s
3. calculation of the inelastic structure factor

Ross, Savary, Gaulin, Balents PRX 2011

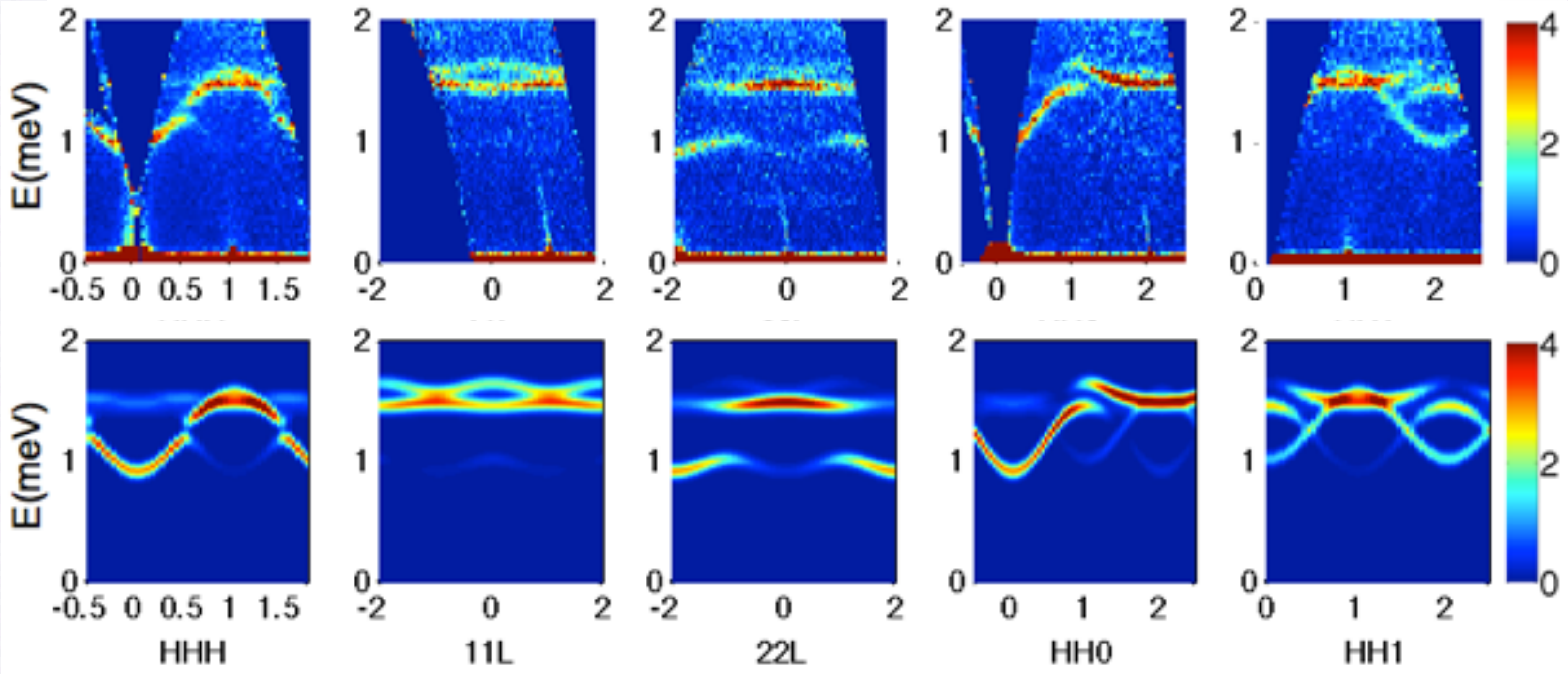
$Yb_2Ti_2O_7$



high field H = 5T

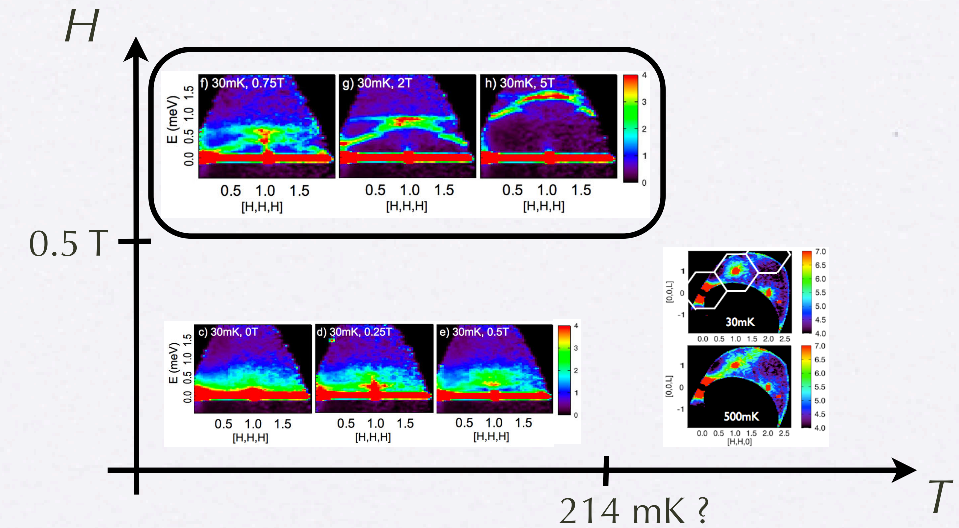
experiment

spin wave theory



Ross, Savary, Gaulin, Balents PRX 2011

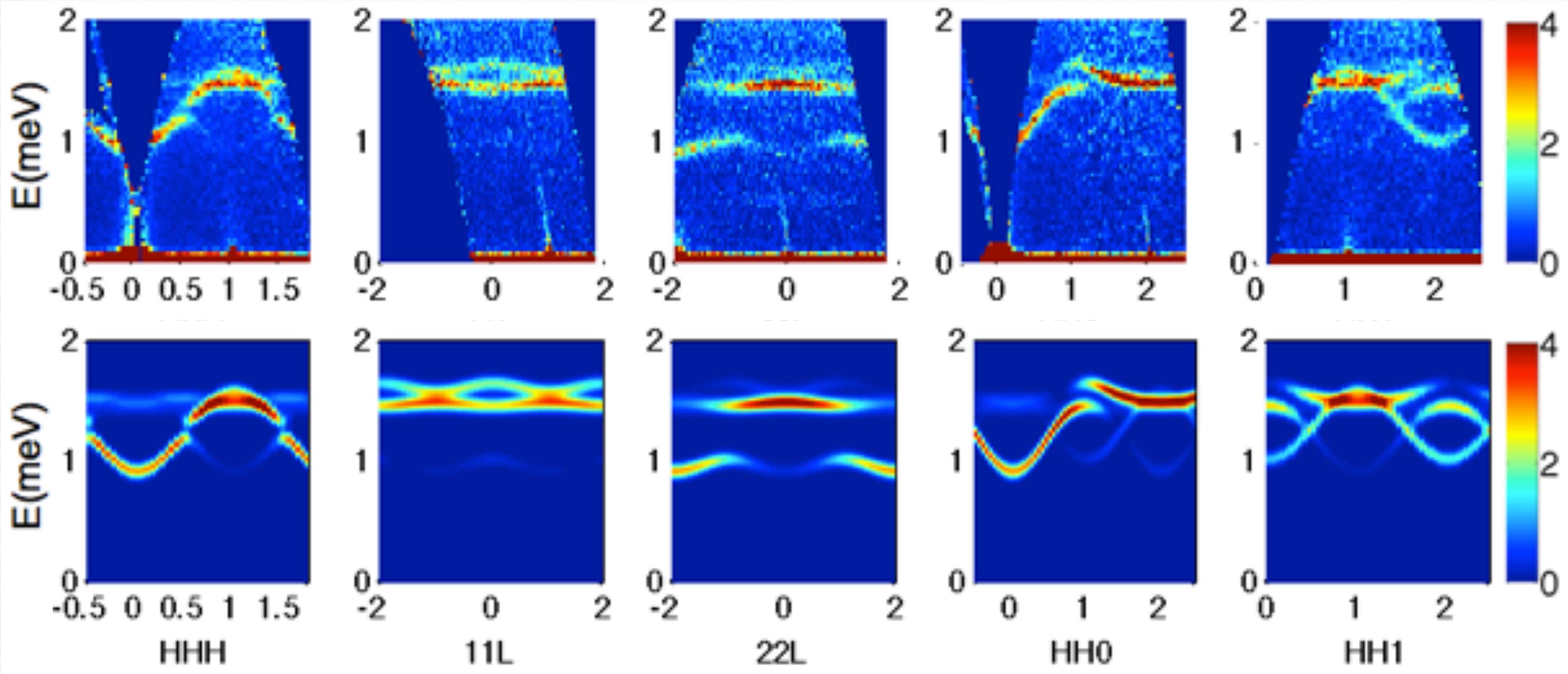
$Yb_2Ti_2O_7$



high field H = 5T

experiment

spin wave theory

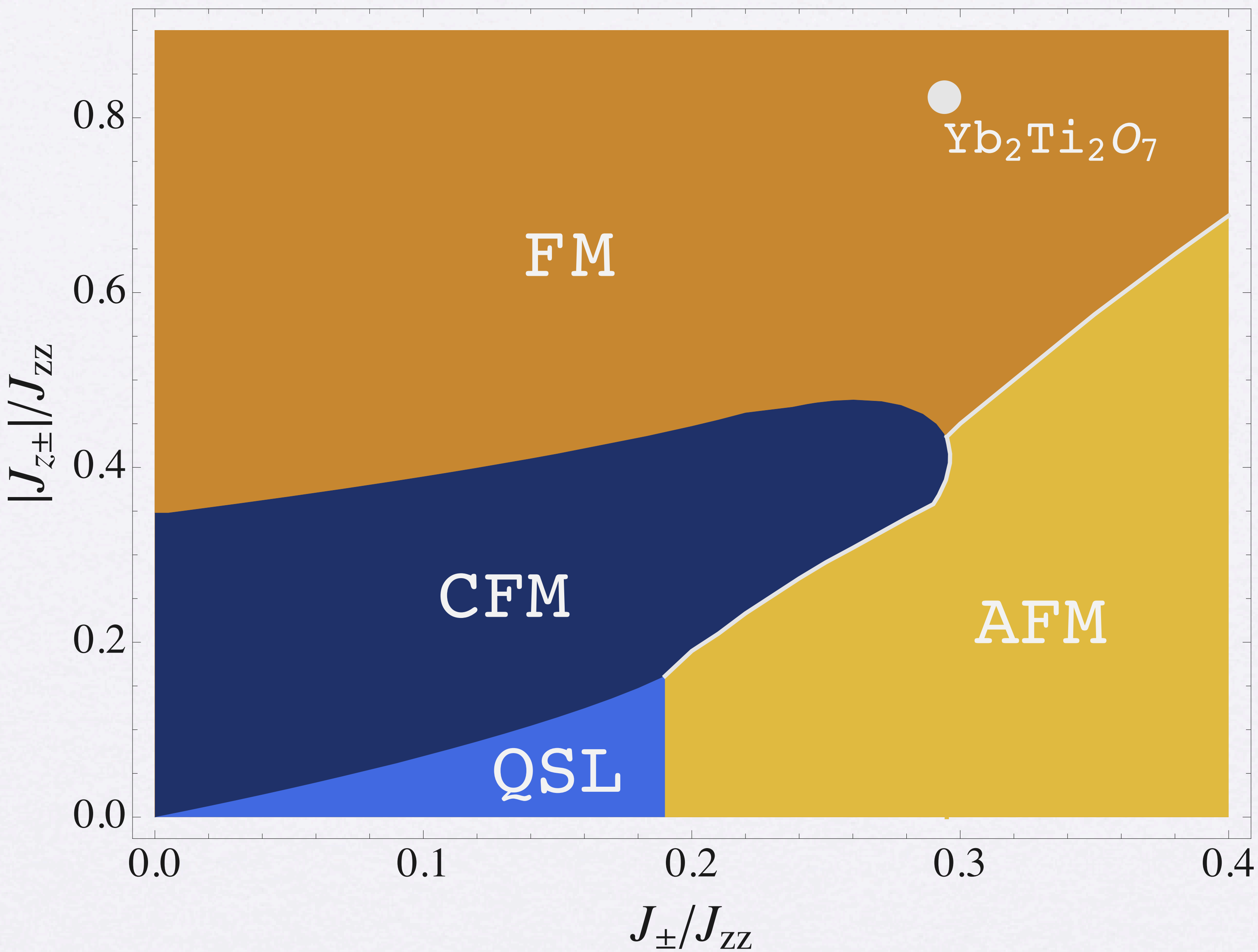


$$J_{zz} = 0.17, \quad J_{\pm} = 0.05, \quad J_{z\pm} = -0.14, \quad J_{\pm\pm} = 0.05 \quad \text{meV}$$

Ross, Savary, Gaulin, Balents PRX 2011

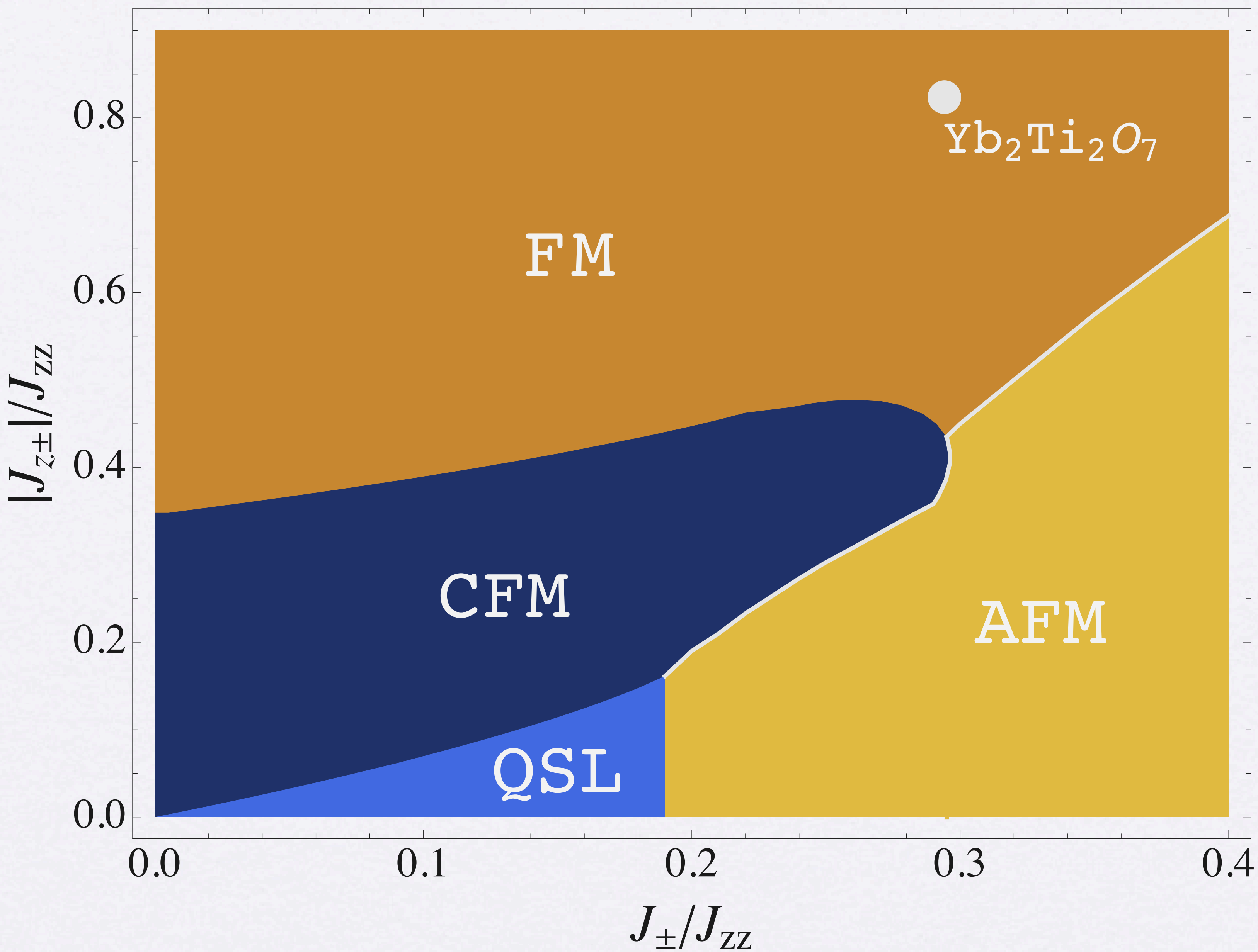
$Yb_2Ti_2O_7$

$$J_{zz} = 0.17, \quad J_{\pm} = 0.05, \quad J_{z\pm} = -0.14, \quad J_{\pm\pm} = 0.05 \quad \text{meV}$$



$Yb_2Ti_2O_7$

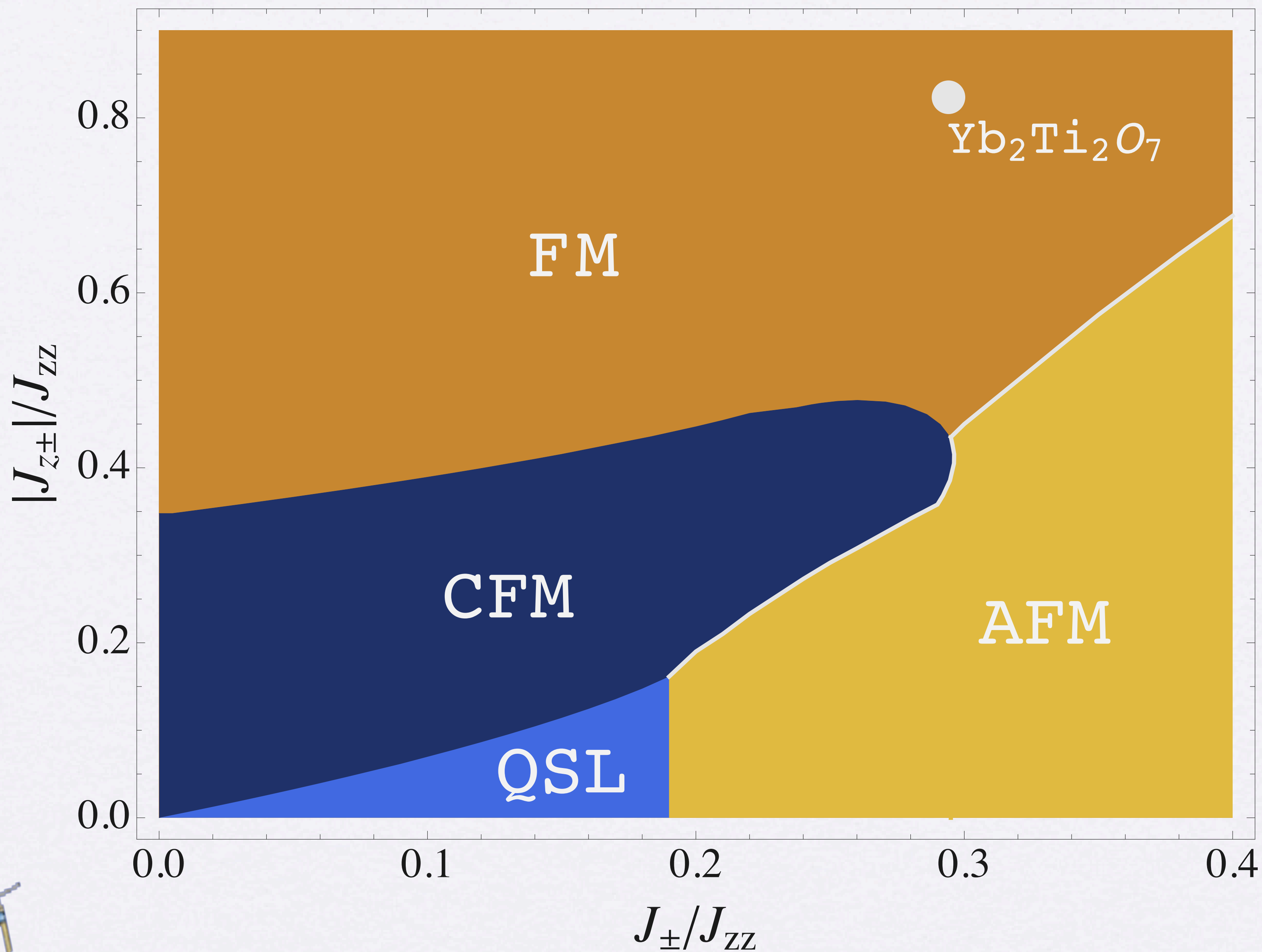
$$J_{zz} = 0.17, \quad J_{\pm} = 0.05, \quad J_{z\pm} = -0.14, \quad J_{\pm\pm} = 0.05 \quad \text{meV}$$



some uncertainties

$Yb_2Ti_2O_7$

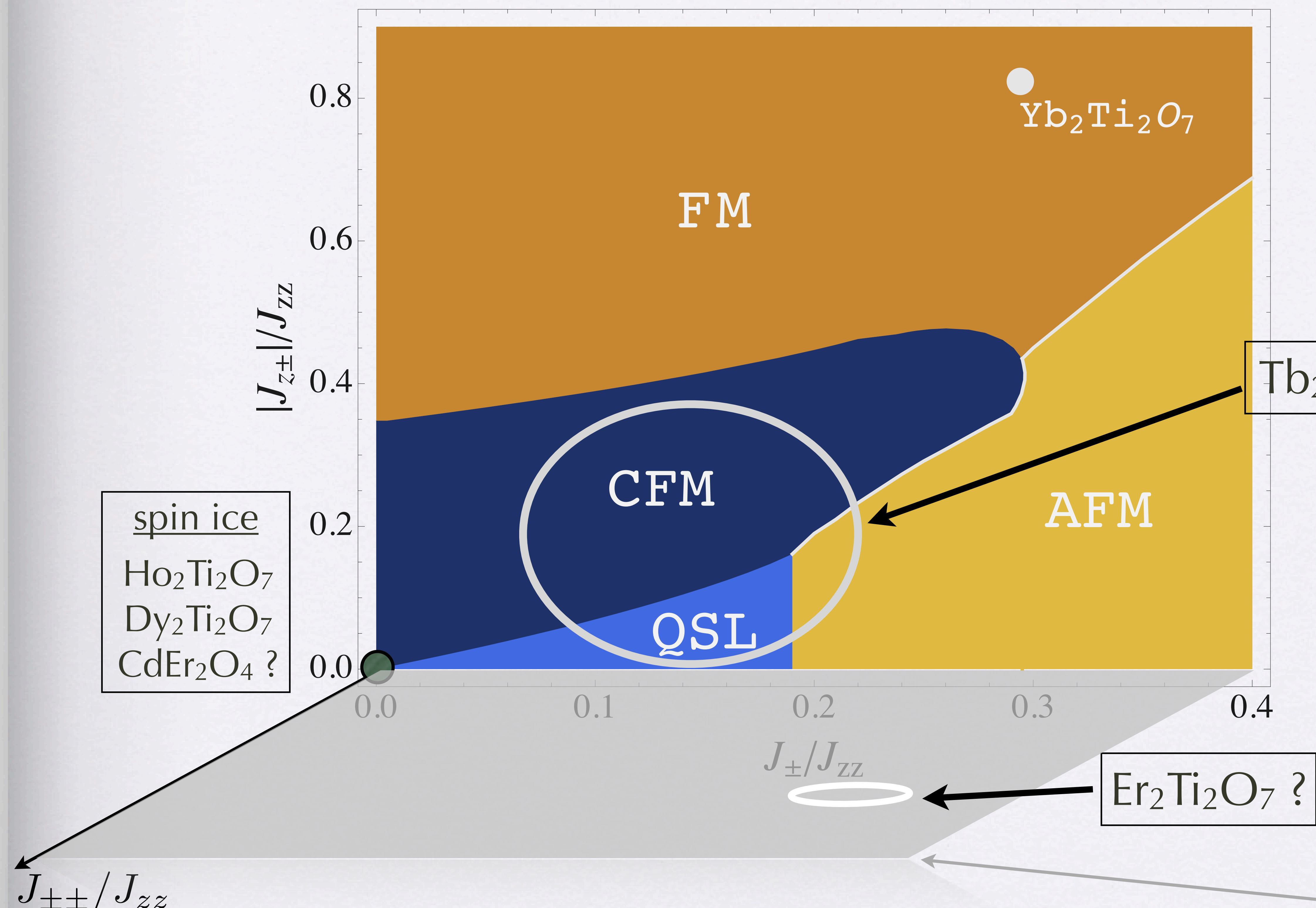
$$J_{zz} = 0.17, \quad J_{\pm} = 0.05, \quad J_{z\pm} = -0.14, \quad J_{\pm\pm} = 0.05 \quad \text{meV}$$



some
uncertainties

$Tb_2Ti_2O_7$? $Er_2Ti_2O_7$? $CdEr_2O_4$? ...

Materials



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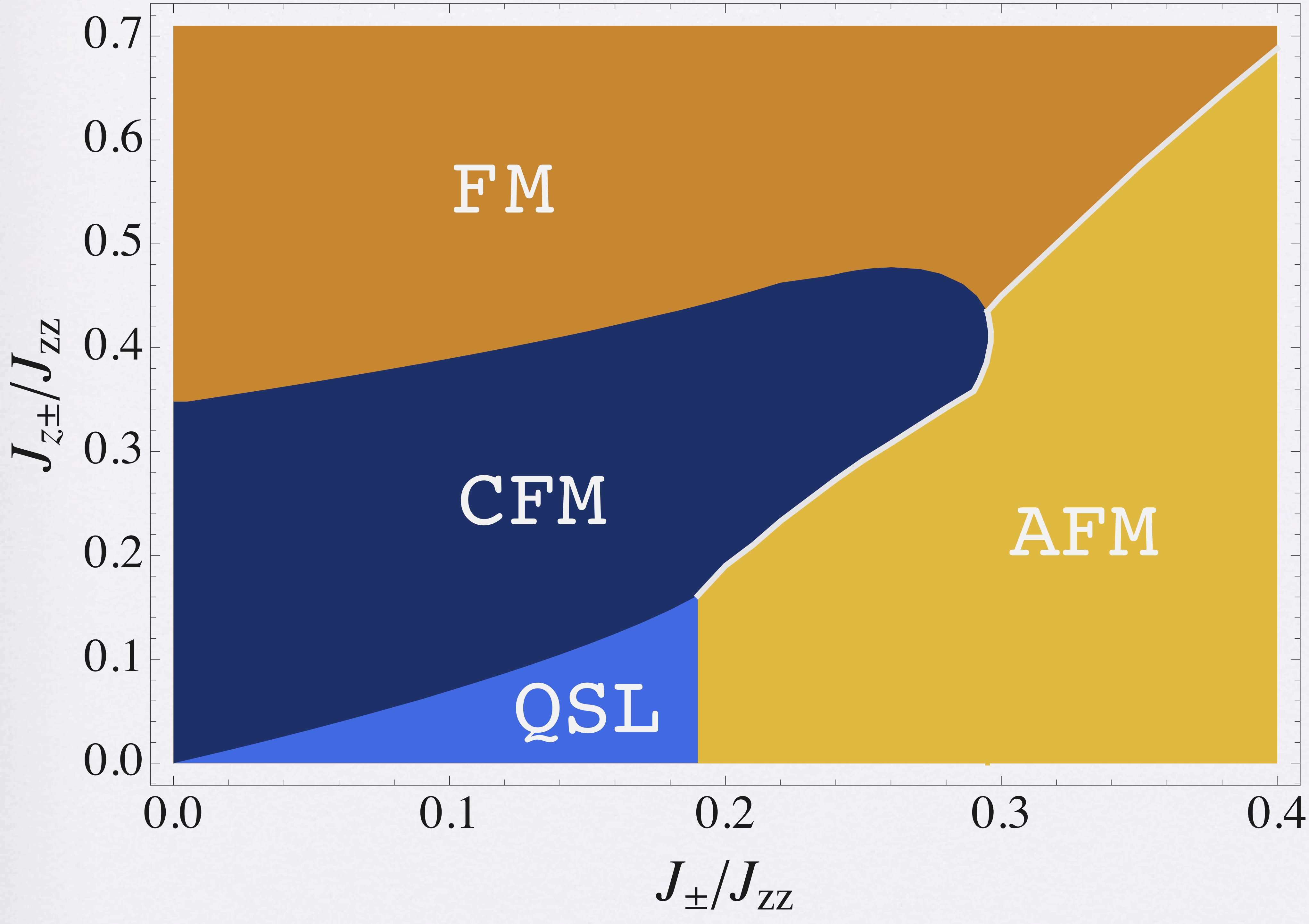
Conclusions and perspectives

- Model and phase diagram which should apply to a **wide spectrum of materials**
- Realization of the **U(1) QSL** in a phase diagram for **real materials**
- Existence of a **new phase of matter: the Coulomb FM**
- Need **numerics**
- Need exchange constants of **more materials**
- Need more **low temperature specific heat** data
- Effects of **disorder**
- Effects of **temperature**
- **Longer range** interactions...


Thank you for your attention

extra slides for questions

Phase diagram



Order parameters

$\langle \Phi \rangle$	$\langle S^z \rangle$	phase
$\neq 0$	$= 0$	AFM
$\neq 0$	$\neq 0$	FM
$= 0$	$= 0$	QSL
$= 0$	$\neq 0$	CFM

Geometry

$$\begin{cases} \hat{\mathbf{e}}_0 = (1,1,1)/\sqrt{3} \\ \hat{\mathbf{e}}_1 = (1,-1,-1)/\sqrt{3} \\ \hat{\mathbf{e}}_2 = (-1,1,-1)/\sqrt{3} \\ \hat{\mathbf{e}}_3 = (-1,-1,1)/\sqrt{3}, \end{cases} \qquad \begin{cases} \hat{\mathbf{a}}_0 = (-2,1,1)/\sqrt{6} \\ \hat{\mathbf{a}}_1 = (-2,-1,-1)/\sqrt{6} \\ \hat{\mathbf{a}}_2 = (2,1,-1)/\sqrt{6} \\ \hat{\mathbf{a}}_3 = (2,-1,1)/\sqrt{6} \end{cases}$$

$$\hat{\mathbf{b}}_i = \hat{\mathbf{e}}_i \times \hat{\mathbf{a}}_i$$

$$\gamma = \begin{pmatrix} 0 & 1 & w & w^2 \\ 1 & 0 & w^2 & w \\ w & w^2 & 0 & 1 \\ w^2 & w & 1 & 0 \end{pmatrix}$$

$$w = e^{2\pi i/3}$$

$$\zeta = -\gamma^*$$

Curie-Weiss temperature

$$\Theta_{\text{CW}} = \frac{1}{2k_B(2g_{xy}^2 + g_z^2)} \left[\, g_z^2 J_{zz} - 4g_{xy}^2 (J_{\pm} + 2J_{\pm\pm}) - 8\sqrt{2}\, g_{xy}\, g_z J_{z\pm} \,\right]$$

parameters

$$J_{zz} = -\frac{1}{3}(2J_1 - J_2 + 2(J_3 + 2J_4))$$

$$J_{\pm} = \frac{1}{6}(2J_1 - J_2 - J_3 - 2J_4)$$

$$J_{z\pm} = \frac{1}{3\sqrt{2}}(J_1 + J_2 + J_3 - J_4)$$

$$J_{\pm\pm} = \frac{1}{6}(J_1 + J_2 - 2J_3 + 2J_4)$$

$$J_1 = \frac{1}{3}\left(-J_{zz} + 4J_{\pm} + 2\sqrt{2}J_{z\pm} + 2J_{\pm\pm}\right)$$
$$J_3 = \frac{1}{3}\left(-J_{zz} - 2J_{\pm} + 2\sqrt{2}J_{z\pm} - 4J_{\pm\pm}\right)$$
$$J_2 = \frac{1}{3}\left(J_{zz} - 4J_{\pm} + 4\sqrt{2}J_{z\pm} + 4J_{\pm\pm}\right)$$
$$J_4 = \frac{1}{3\sqrt{2}}\left(-\sqrt{2}J_{zz} - 2\sqrt{2}J_{\pm} + 2J_{z\pm} + 2\sqrt{2}J_{\pm\pm}\right)$$

$$\mathbf{J}_{01} = \begin{pmatrix} J_2 & J_4 & J_4 \\ -J_4 & J_1 & J_3 \\ -J_4 & J_3 & J_1 \end{pmatrix}$$

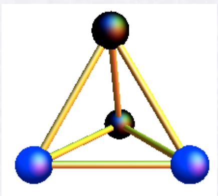
Perturbation Theory

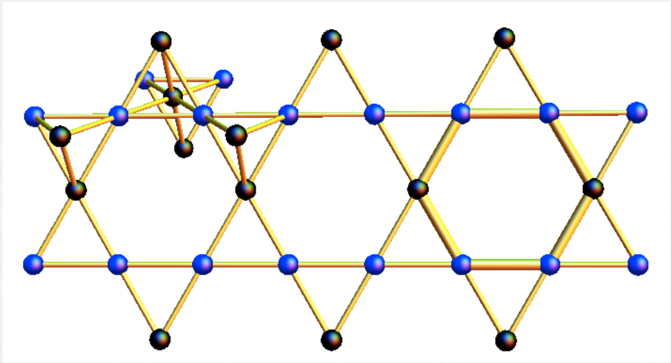
$$H^{\text{eff}}_{\text{ring}} = -K \sum_{\{i,j,k,l,m,n\}=\text{hexagon}} \left(\mathsf{S}^+_i \mathsf{S}^-_j \mathsf{S}^+_k \mathsf{S}^-_l \mathsf{S}^+_m \mathsf{S}^-_n + \text{h.c.}\right)$$

$$K = \frac{12 J^3_{\pm}}{J^2_{zz}}$$

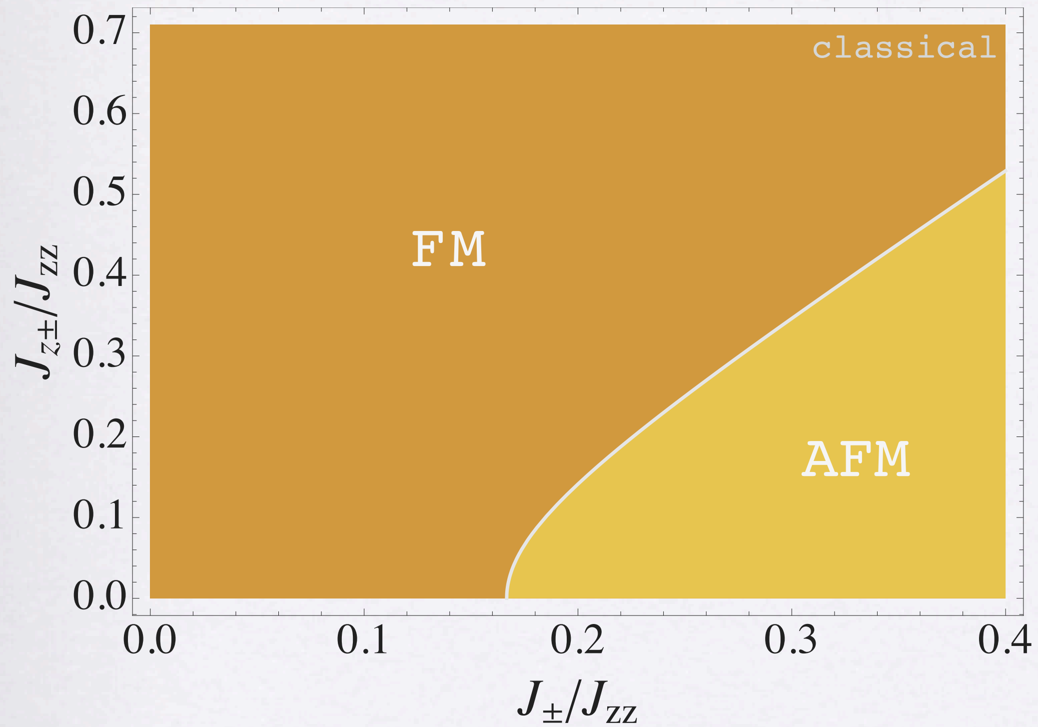
$$H^{\text{eff}}_{\text{3rd Ising}} = -J_{(3)} \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} \mathsf{S}^z_i \mathsf{S}^z_j$$

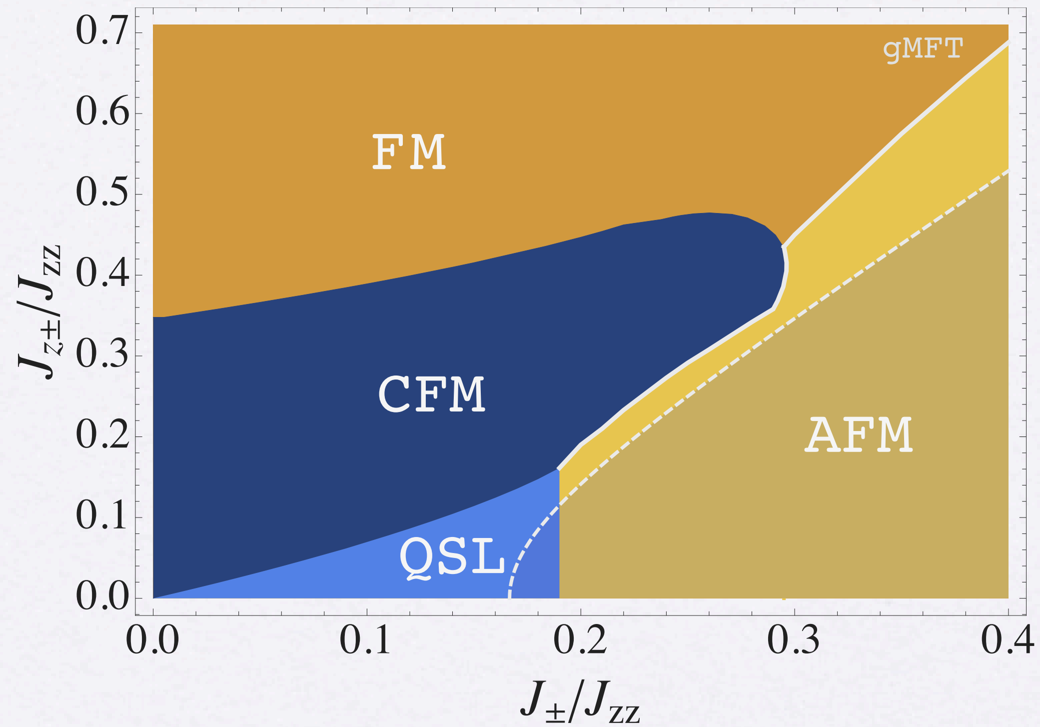
$$J_{(3)} = \frac{3 J^2_{z\pm}}{J_{zz}}$$



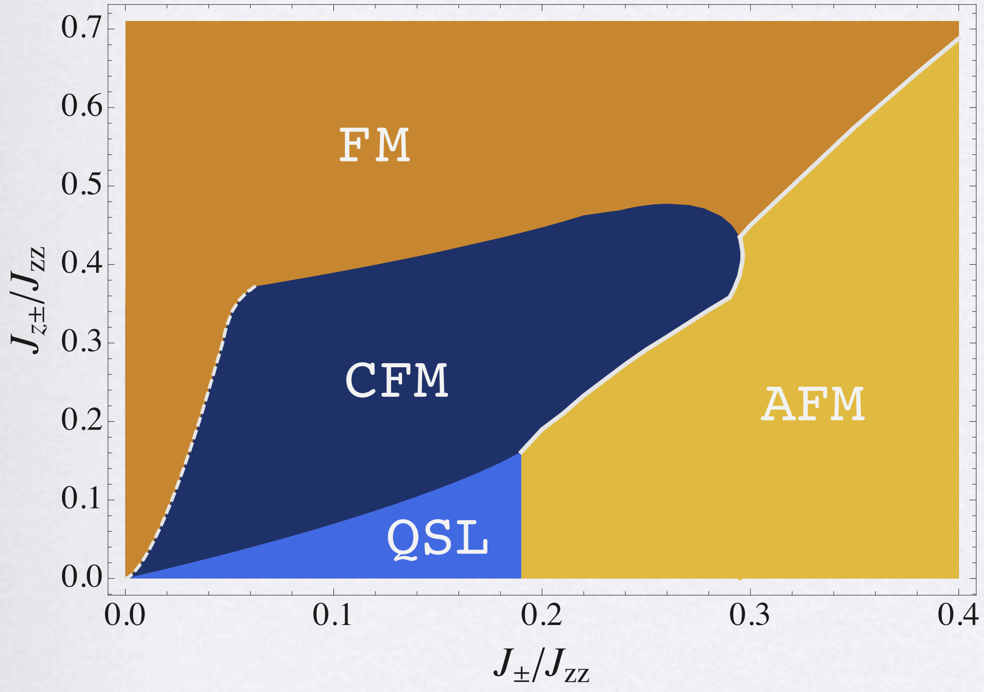


Other diagrams

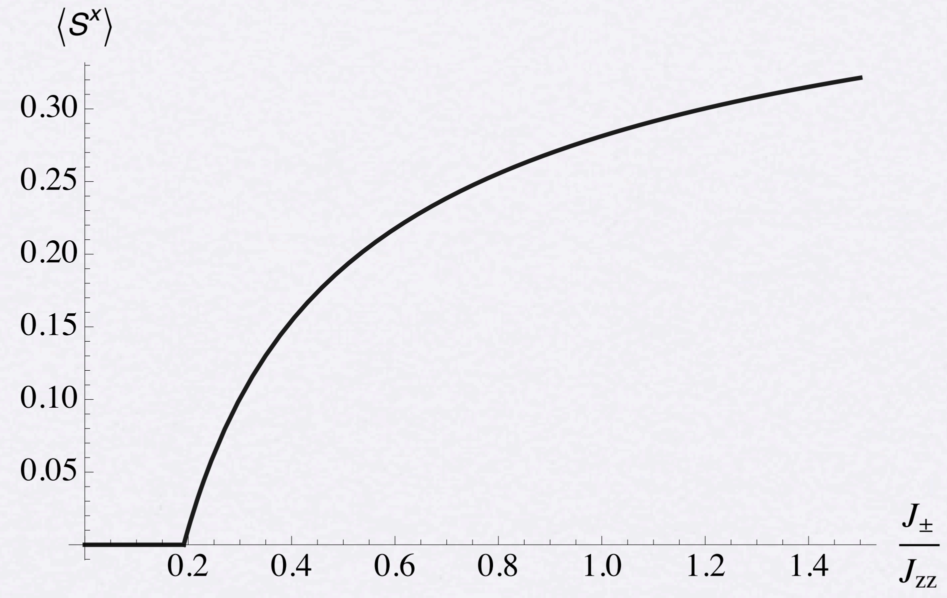




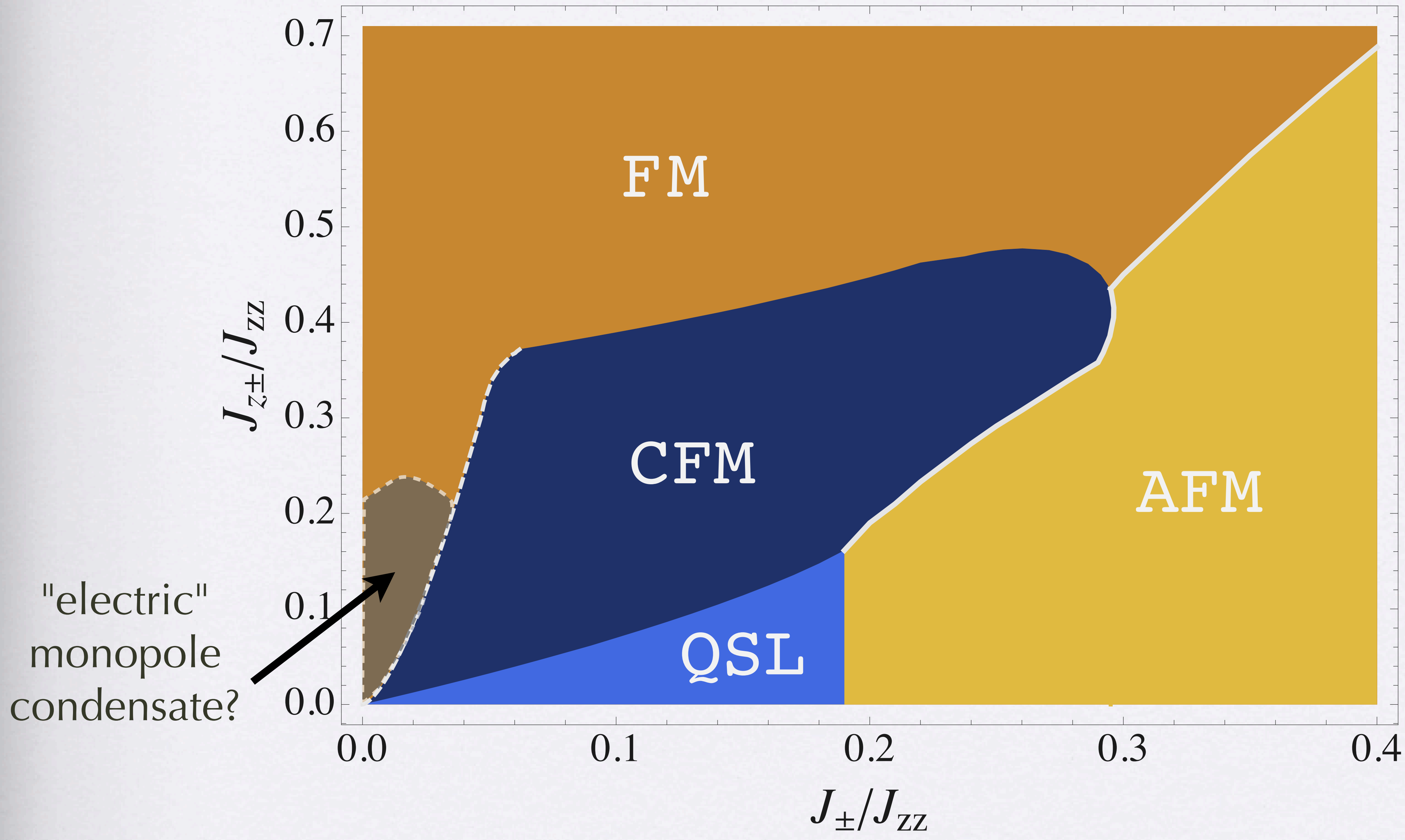
corrected gMFT diagram



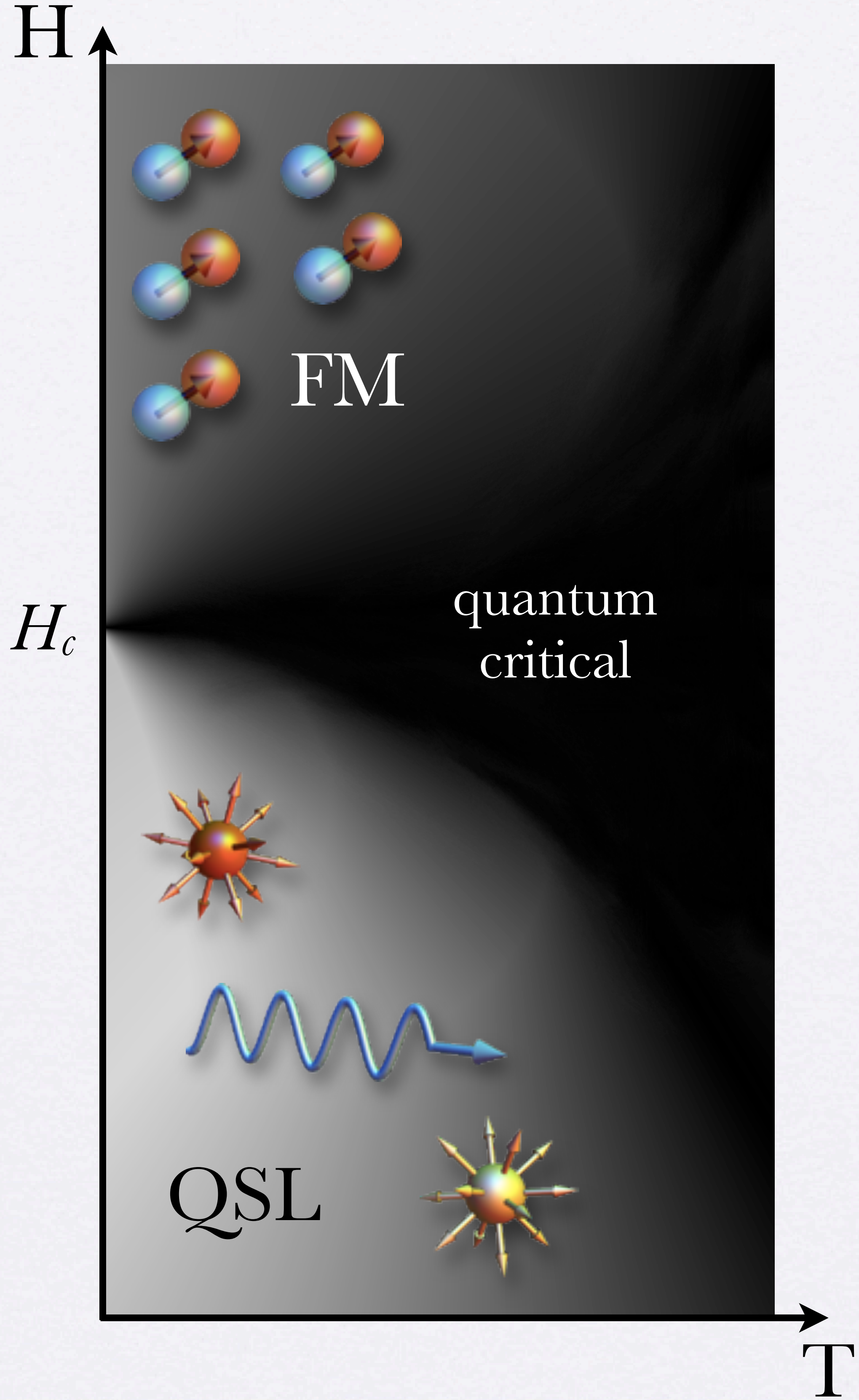
staggered magnetization for $J_{z\pm} = 0$



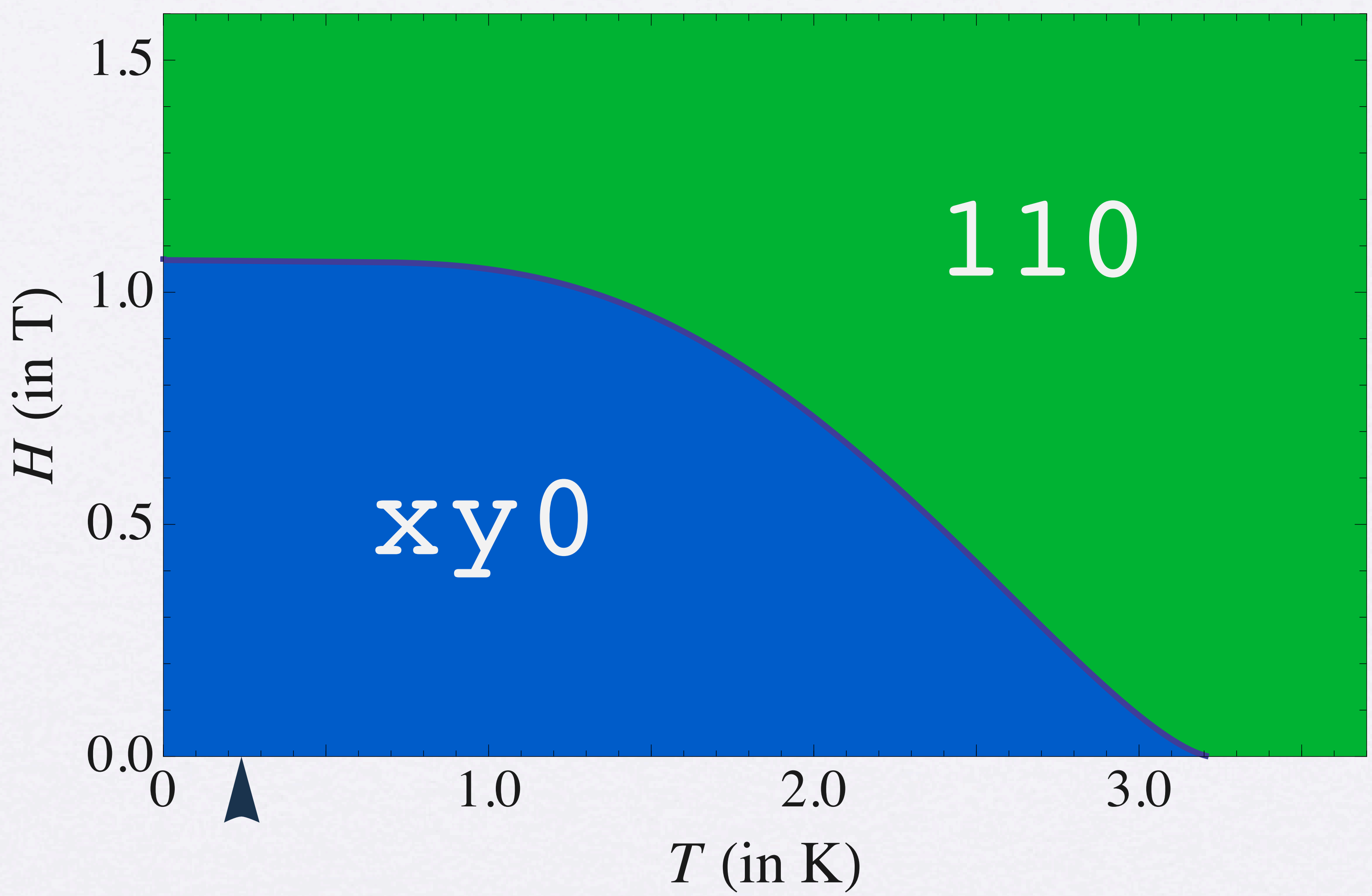
"Electric" monopole condensate?



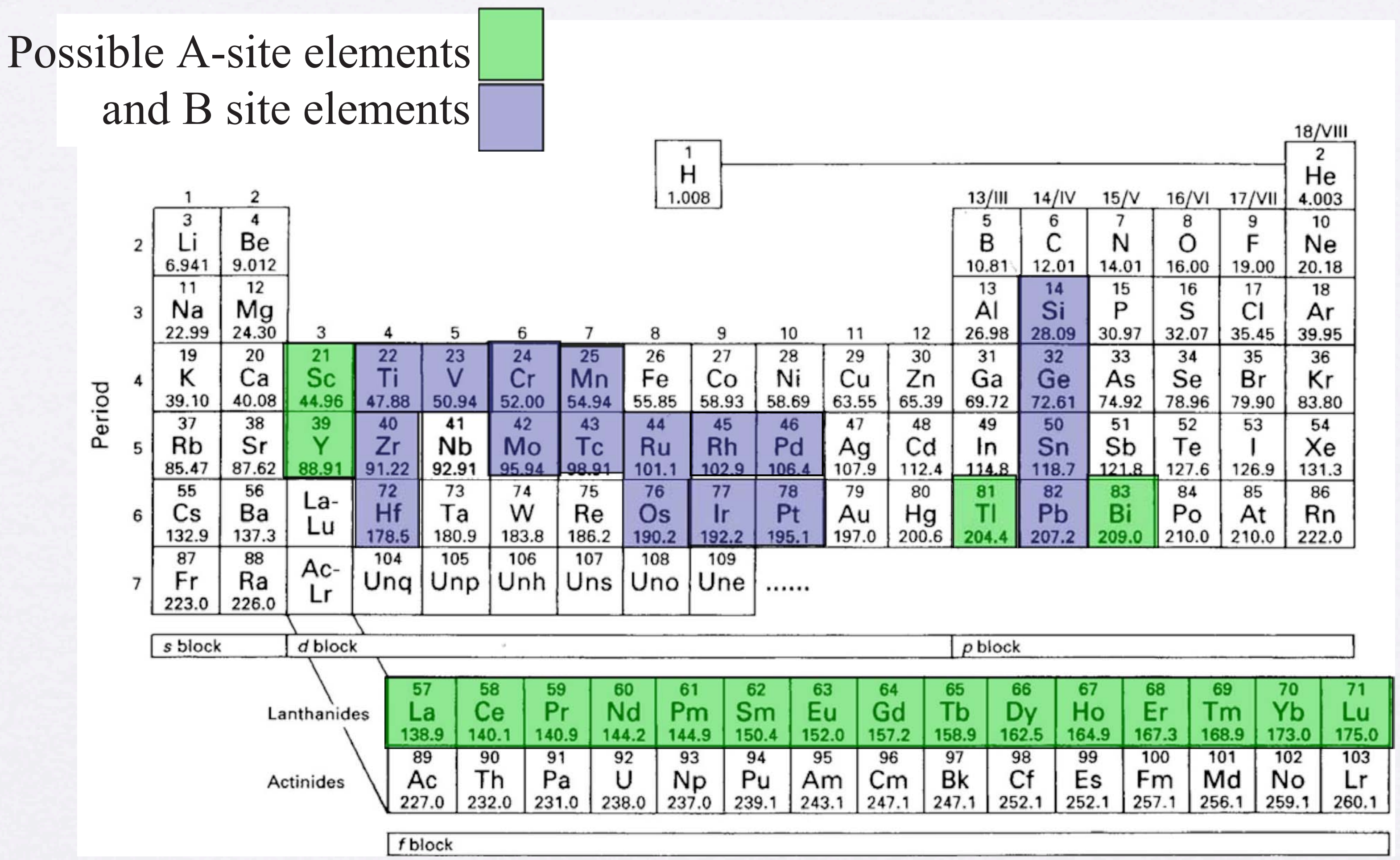
Field



Classical *T-H* MFT diagram for $Yb_2Ti_2O_7$

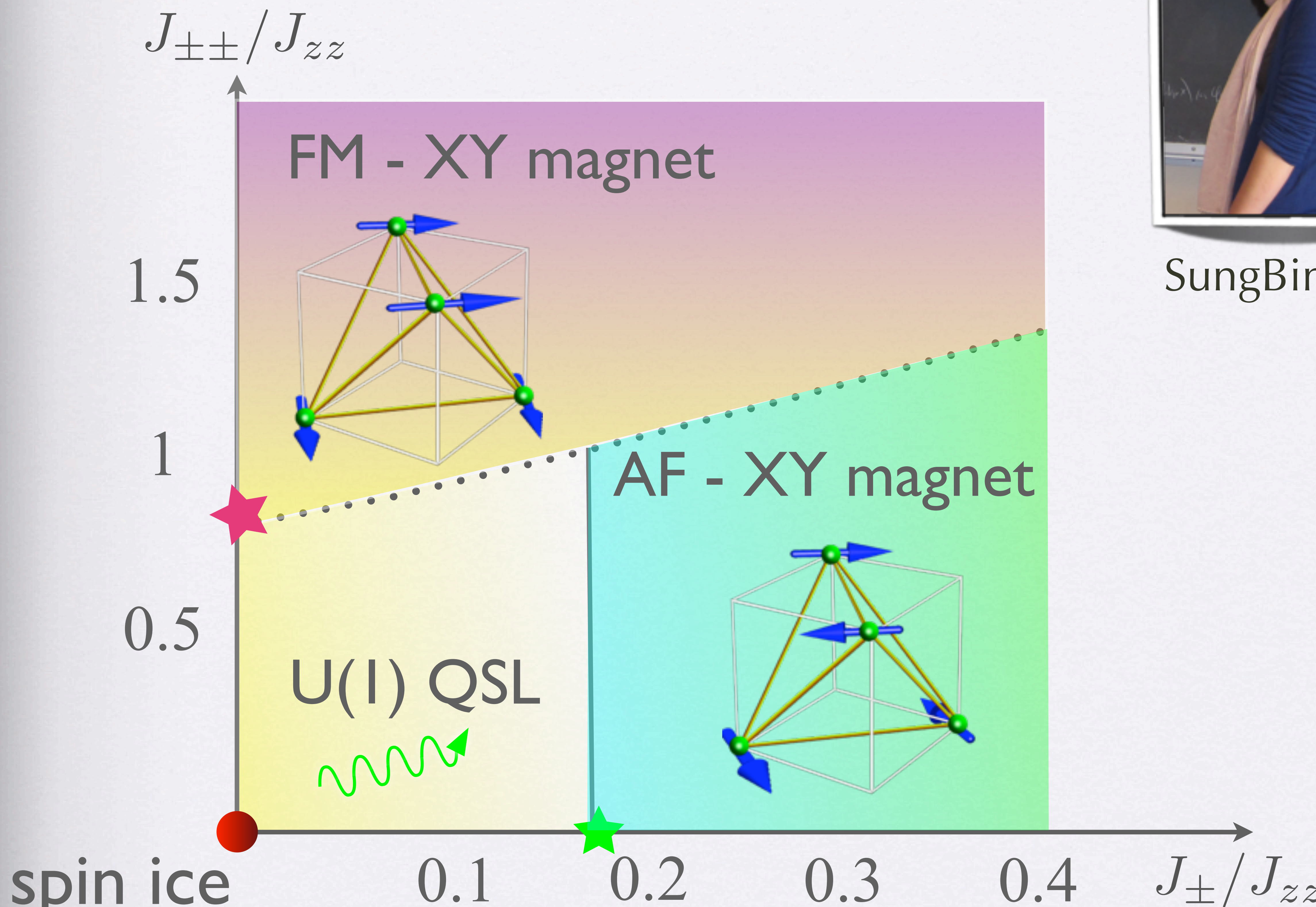


Rare-earth pyrochlores



Gardner, Gingras, Greedan, RMP 2010

$J_{\pm\pm}$



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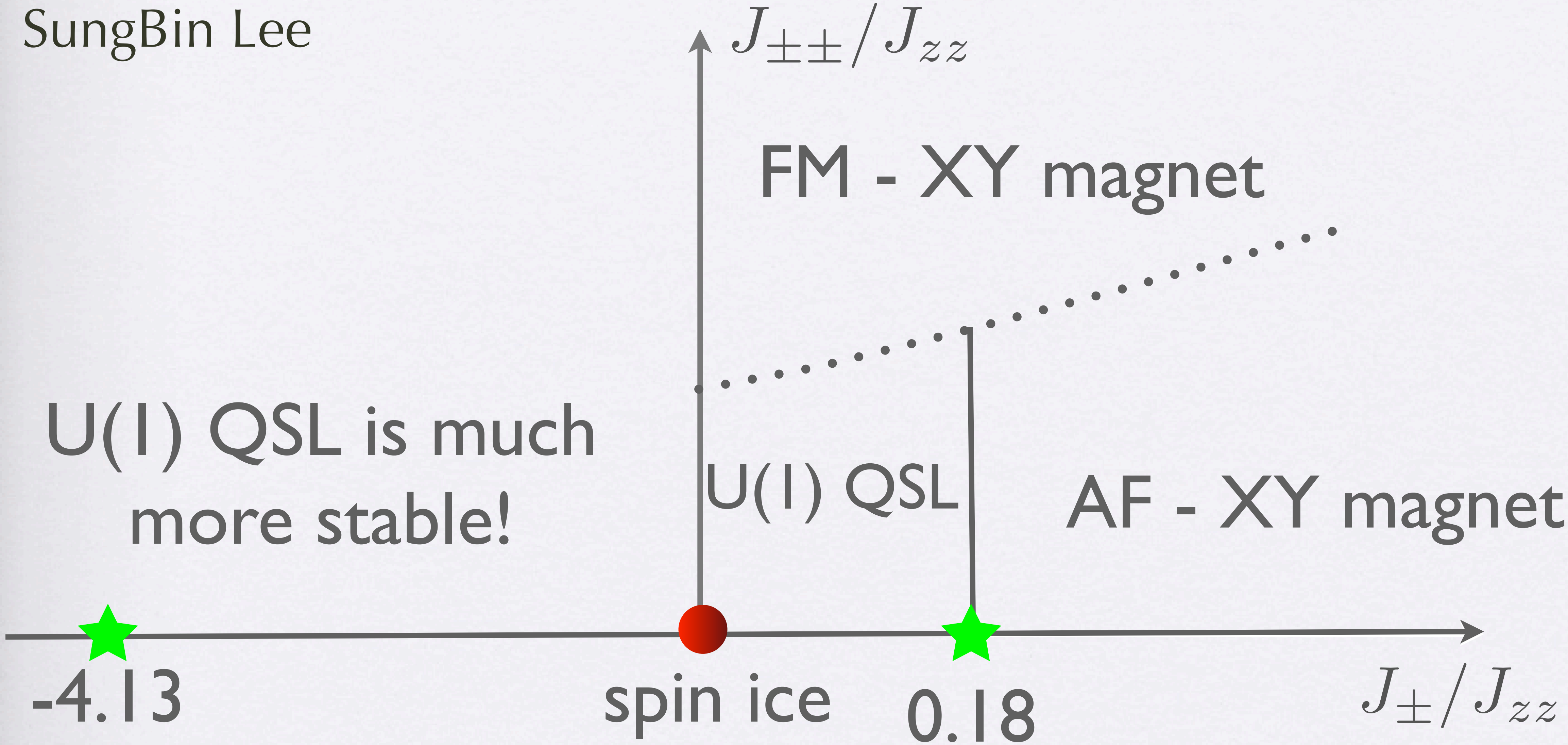
$J_\pm < 0$



From U(1) QSL to ???

quartic spinon hopping $\Phi_r^\dagger \Phi_r^\dagger \Phi_{r+e_\mu} \Phi_{r+e_u}$

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(Q) Which one is energetically favored ?

	$\langle \Phi_r \rangle$	$\langle \Phi_r \Phi_{r'} \rangle$	$\langle \Phi_{r_A}^\dagger \Phi_{r_B} \rangle$	characteristics
XY magnet	$\neq 0$	$\neq 0$	$\neq 0$	ordering on XY
Z_2	0	$\neq 0$	0	no ordering gapped excitation
U(1)-XY*	0	0	$\neq 0$	ordering on XY gapless photon
Z_2-XY*	0	$\neq 0$	$\neq 0$	ordering on XY gapped excitation