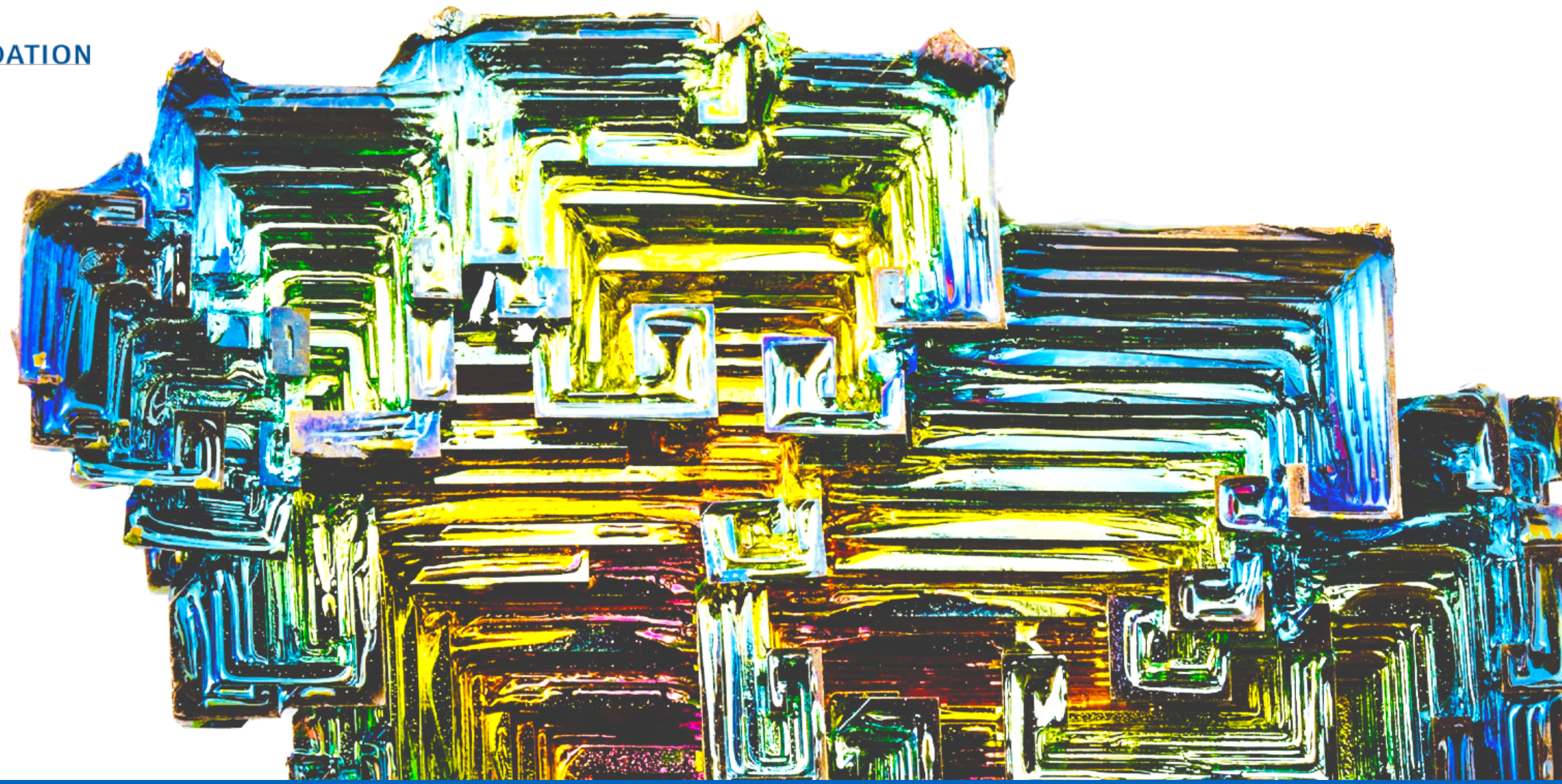
University of Zurich[UZH]

Titus Neupert

Lyon, December 21, 2018

FNSNF Swiss National Science Foundation

European Research Council



Higher order topological insulators

Content

1. HOTI basic concepts
2. Mirror symmetry route to HOTIs

   Science Advances 4, eaat0346, (2018)

3. Inversion symmetry route to HOTIs

   Nature Physics 14, 918 (2018)

4. Realization in electrical circuits

   Nature Physics 14, 925 (2018)

Collaboration

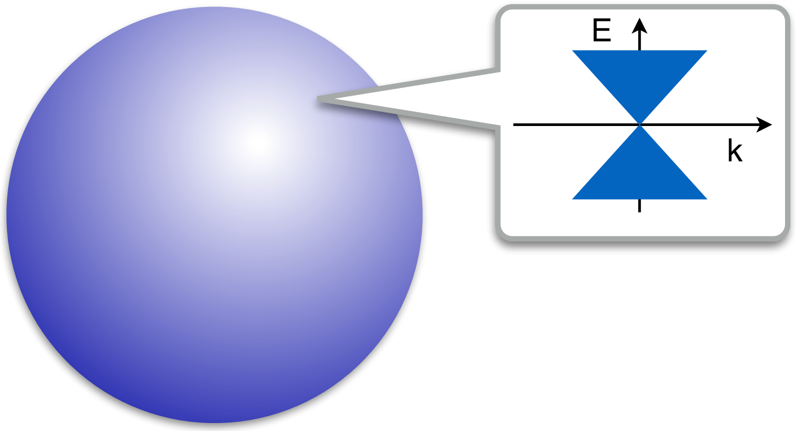
Theory

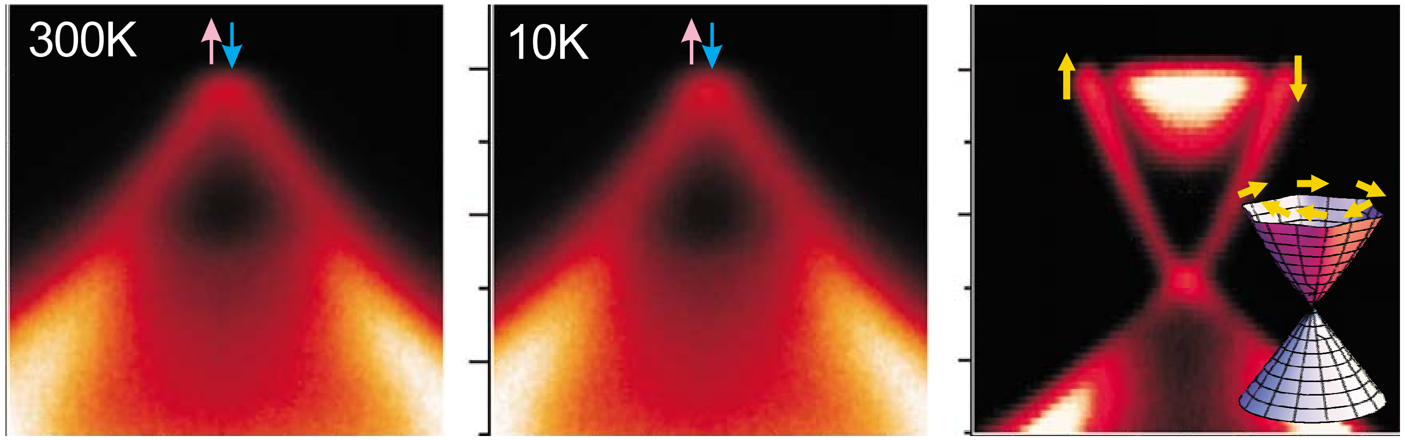
Frank Schindler
B. Andrei Bernevig
Maia Vergniory
Ashley Cook
Fiona Burnell
Zhijun Wang
Benjamin Wieder
Ronny Thomale
Martin Greiter
Yizhi You
Trithep Devakul
Giovanni Sangiovanni
Domenico Di Sante
Hugo Cui
Cheryne Jonay
…

Experiment

Sophie Gueron
Helene Bouchiat
Ali Yazdani
Laurens Molenkamp
Stuart Parkin
Matthias Bode
…

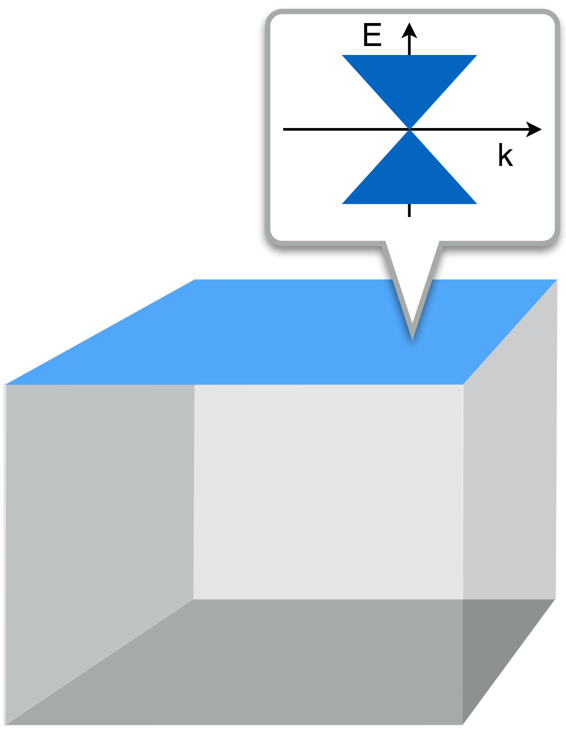
3D TI





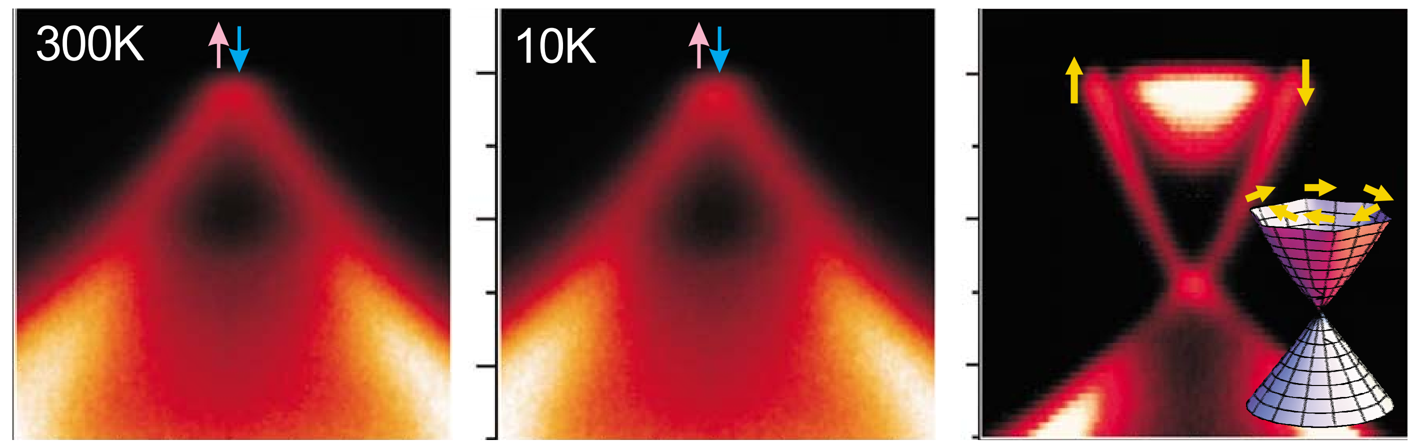
Crystals have no smooth surface!

Topological Insulators



Bulk-boundary correspondence:

gapless Dirac cones, gapped bulk band structure



Topological invariant:

$$\theta = -\epsilon_{abc} \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} \mathrm{tr} \left[\mathcal{A}_a \partial_b \mathcal{A}_c + \mathrm{i} \frac{2}{3} \mathcal{A}_a \mathcal{A}_b \mathcal{A}_c \right]$$

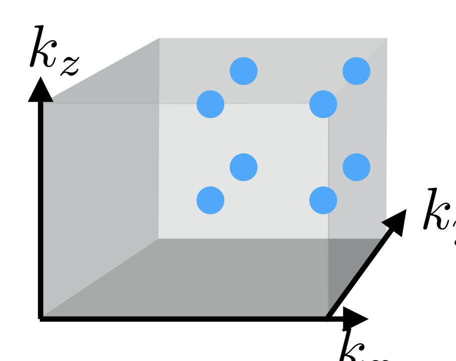
$$\mathcal{A}_{a;n,n'} = -\mathrm{i} \langle u_n | \partial_a | u_{n'} \rangle$$

$\theta = 0, \pi$ with time-reversal symmetry

Topological invariant with inversion:

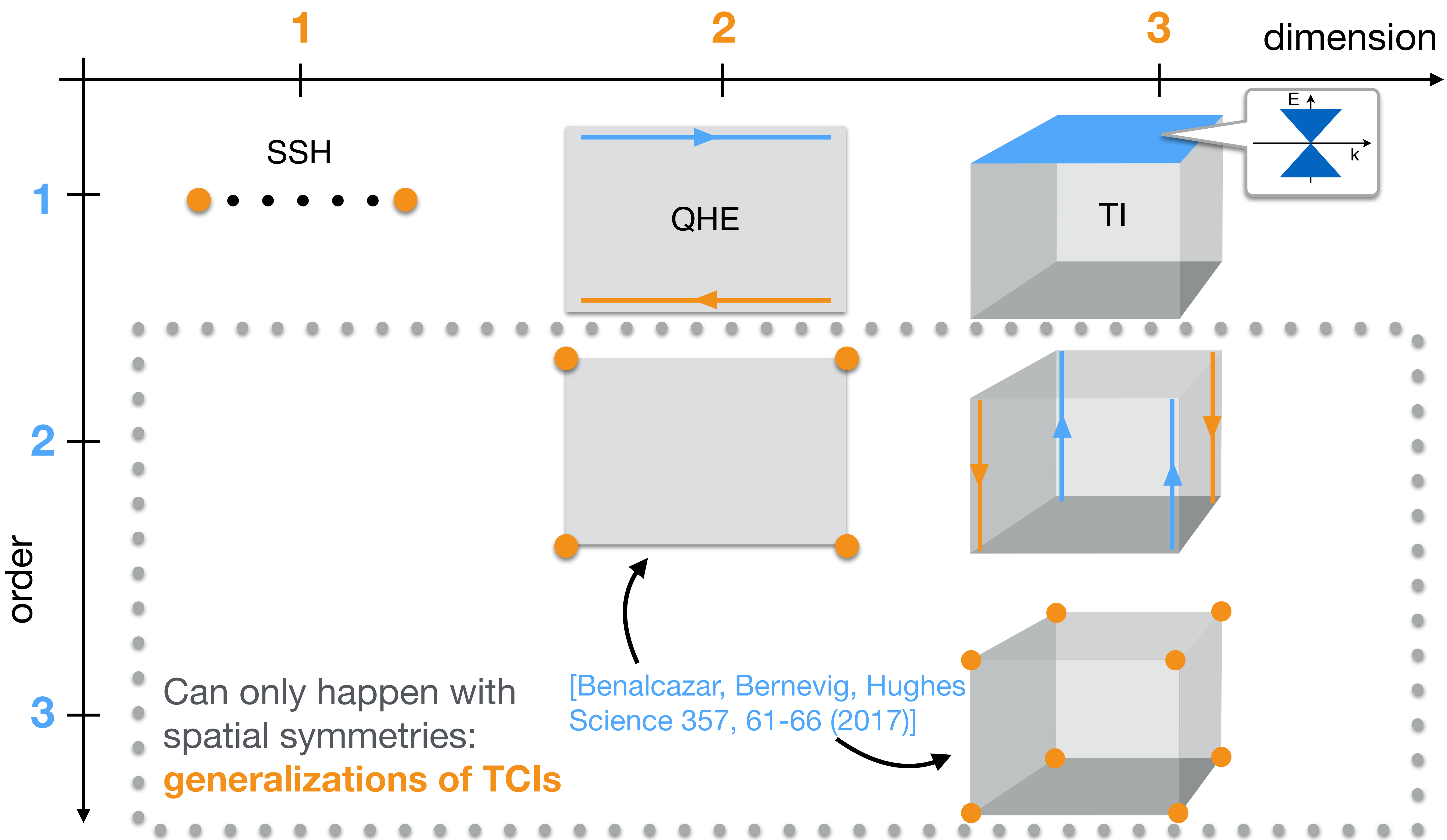
$$\prod_{k \in \mathrm{TRIM}} \xi_k = (-1)^\nu$$

product over inversion eigenvalues at time-reversal invariant momenta



Higher-order topological insulators

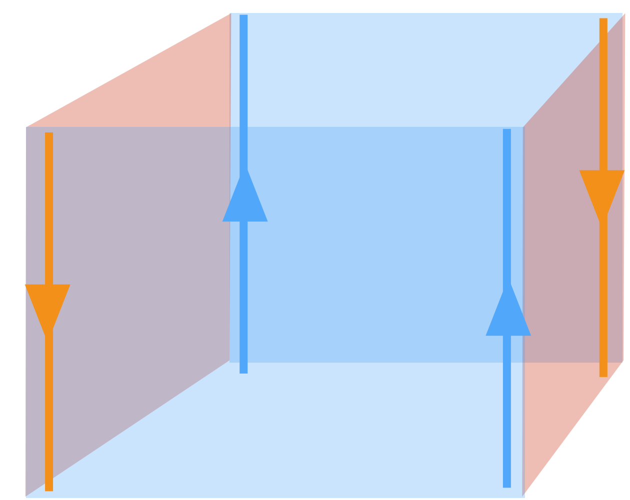
(d-m)-dimensional boundary components of a d-dimensional system are **gapless for m = N**, and are generically **gapped for m < N**



Construction of a 2nd order 3D TI

Protecting symmetry: **C_4T** (breaks T, C_4 individually)

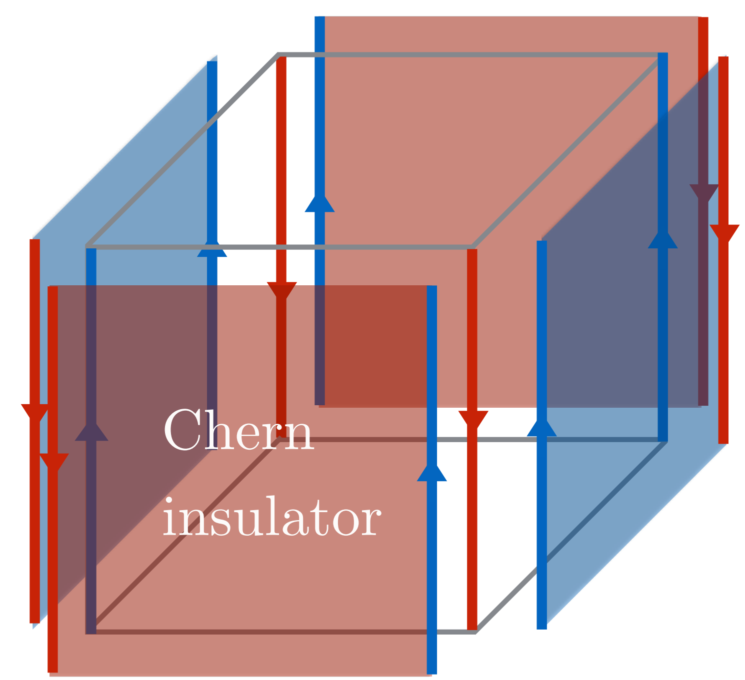
surface construction from 3D TI:
decorate surfaces alternatingly with outward and inward pointing magnetization, gives chiral 1D channels at hinges



Adding C_4T respecting IQHE layers on surface can **change number of hinge modes by multiples of 2**

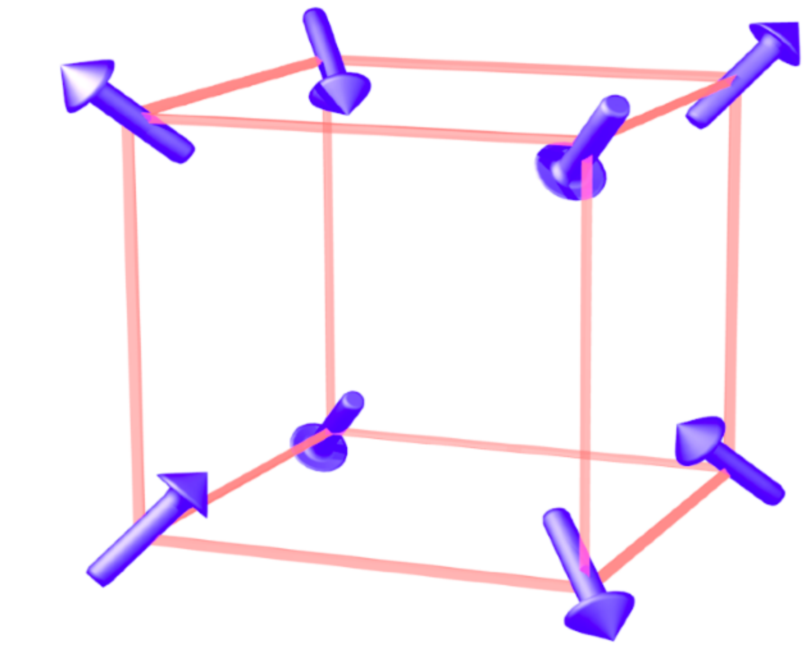
Odd number of hinge modes **stable** against any C_4T respecting surface manipulation

Bulk $\mathbb{Z}_2$ topological property



Construction of a 2nd order 3D TI

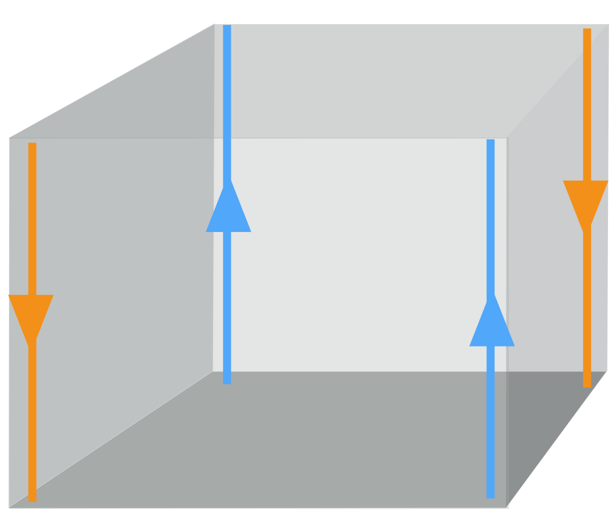
Protecting symmetry: **C_4T**
(breaks T, C_4 individually)



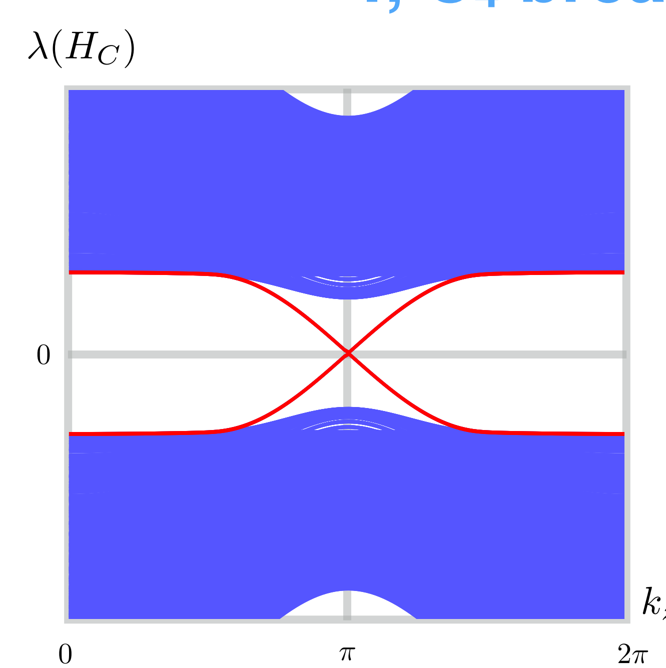
Bulk construction
TI band structure plus (sufficiently weak) triple-q (π,π,π) magnetic order

Toy model with only C_4T in *z*-direction

$$H_4(\vec{k}) = \underbrace{\left(M + \sum_i \cos k_i\right) \tau_z\sigma_0 + \Delta_1 \sum_i \sin k_i \, \tau_y\sigma_i}_{\text{3D TI}} + \underbrace{\Delta_2(\cos k_x - \cos k_y)\,\tau_x\sigma_0}_{\text{T, } C_4 \text{ breaking term}}$$



Spectrum of column geometry



Topological invariant of a 2nd order 3D TI

Same quantization with C_4T as with T alone:

$\boxed{\theta = 0, \pi \quad \text{is topological invariant}}$

$$Z_{\rm top} = e^{\mathrm{i}\frac{\theta}{8\pi^2}\int \mathrm{d}^4x\,\boldsymbol{E}\cdot\boldsymbol{B}}$$

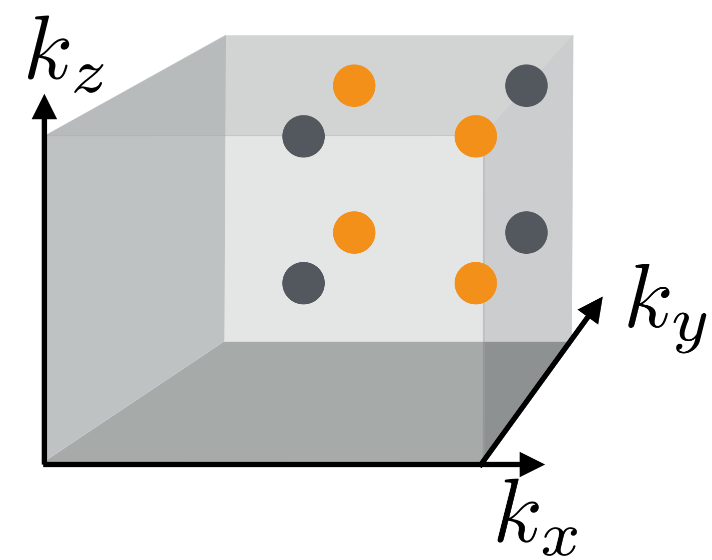
Different from existing indices, because $\boxed{(C_4T)^4 = -1}$

Case of additional **inversion times TRS**

Band inversion formula for topological index à la Fu Kane for C_4T invariant momenta

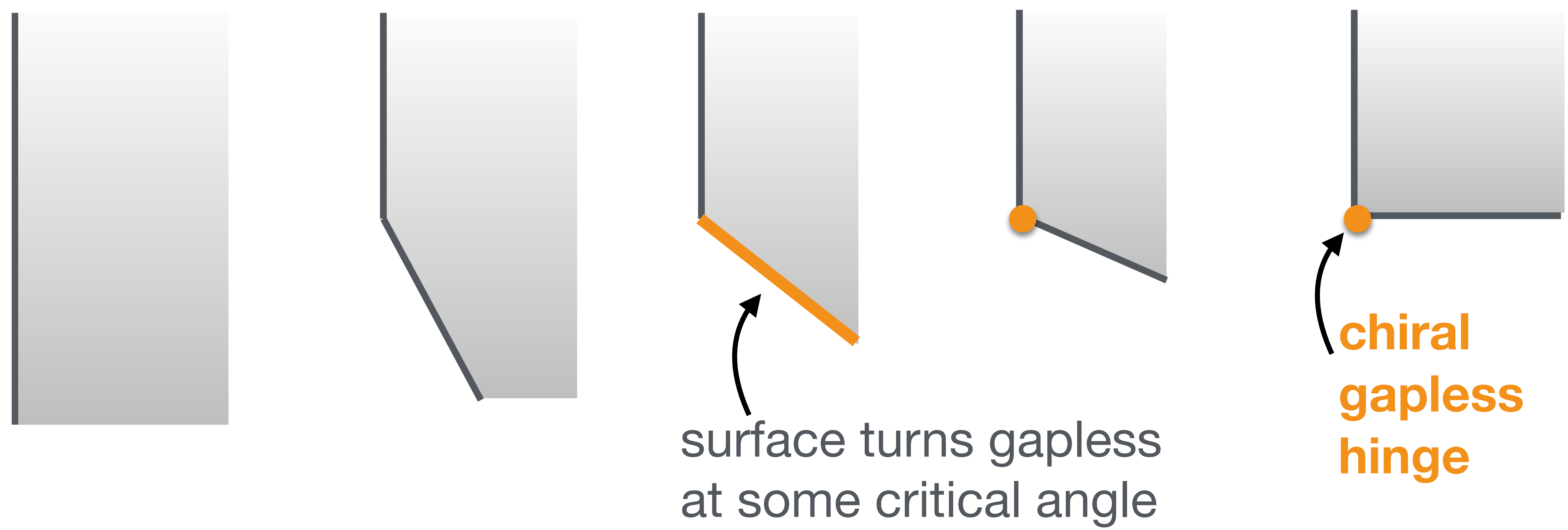
$$\boxed{(-1)^\nu = \prod_{\vec{k}\in\mathcal{I}_{\hat{C}_4^z\hat{T}}} \xi_{\vec{k}}}$$

$$\mathcal{I}_{\hat{C}_4^z\hat{T}} = \{(0,0,0),(\pi,\pi,0),(0,0,\pi),(\pi,\pi,\pi)\}$$



Gapless surfaces?

consider adiabatically inserting a hinge

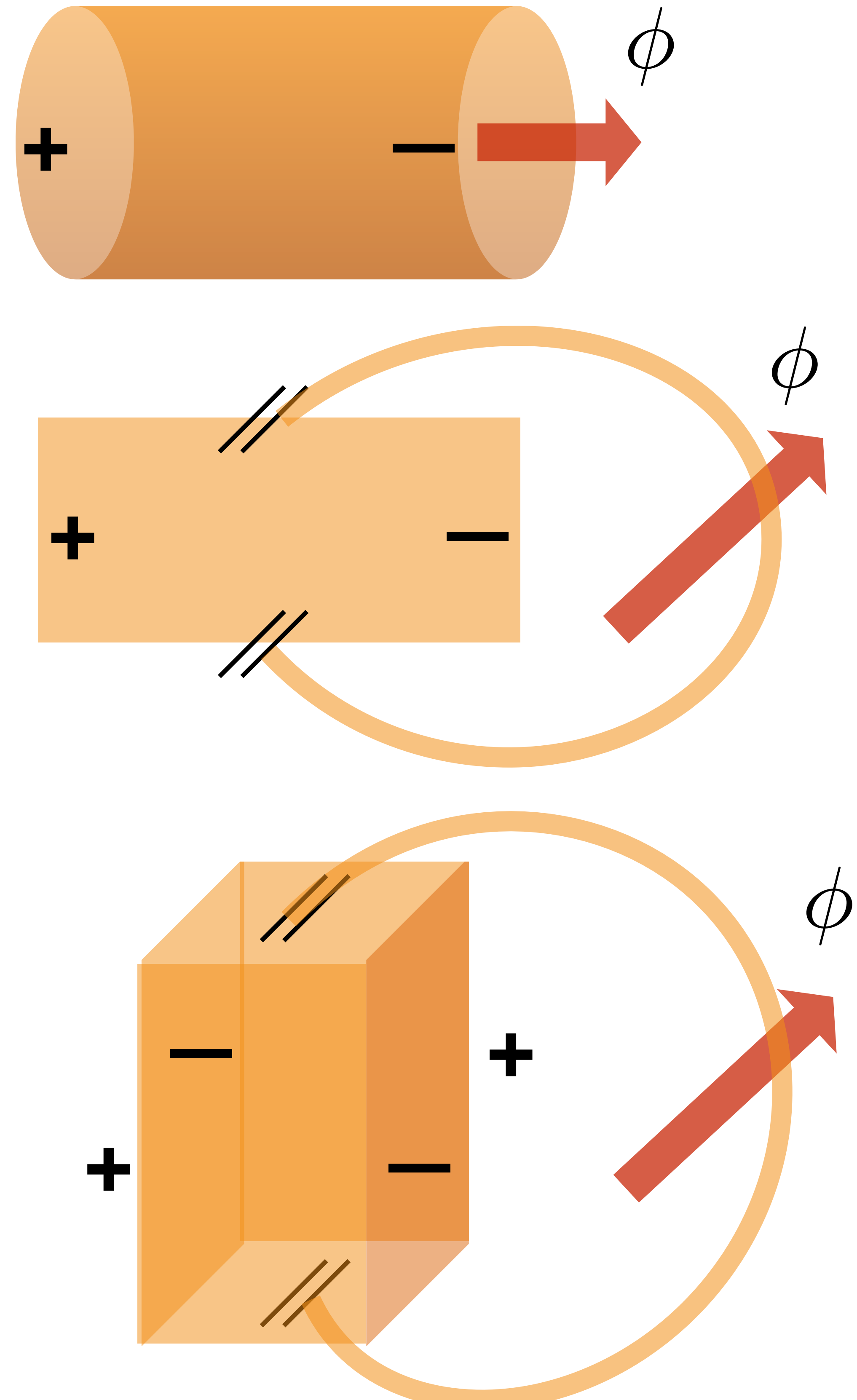


$$H_4(\vec{k}) = \left(M + \sum_i \cos k_i\right) \tau_z\sigma_0 + \Delta_1 \sum_i \sin k_i\, \tau_y\sigma_i + \Delta_2(\cos k_x - \cos k_y + \boxed{r} \sin k_x \sin k_y)\, \tau_x\sigma_0$$

Critical angle **nonuniversal**, not fixed to particular crystallographic direction. Different from gapless surfaces of TCIs.

Electromagnetic response

Flux insertion in quantum Hall system creates quantized **dipole**



Flux insertion in chiral higher-order TI creates quantized **quadrupole**

2nd order 3D topological superconductor

$$H_4(\vec{k}) = \left(M + \sum_i \cos k_i \right) \tau_z\sigma_0 + \Delta_1 \sum_i \sin k_i \, \tau_y\sigma_i + \Delta_2(\cos k_x - \cos k_y)\, \tau_x\sigma_0$$

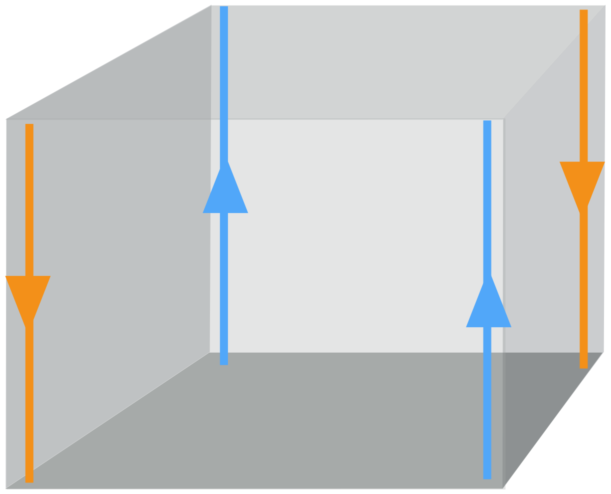
has a **particle hole symmetry** $P = \tau_y\sigma_y K$

Interpretation: Superconductor with generic dispersion and superposition of two order parameters

Δ_1 spin triplet, p-wave $d_{\vec{k},i} = \mathrm{i}\Delta_1 \sin k_i$
Balian-Werthamer state in superfluid Helium-3-B

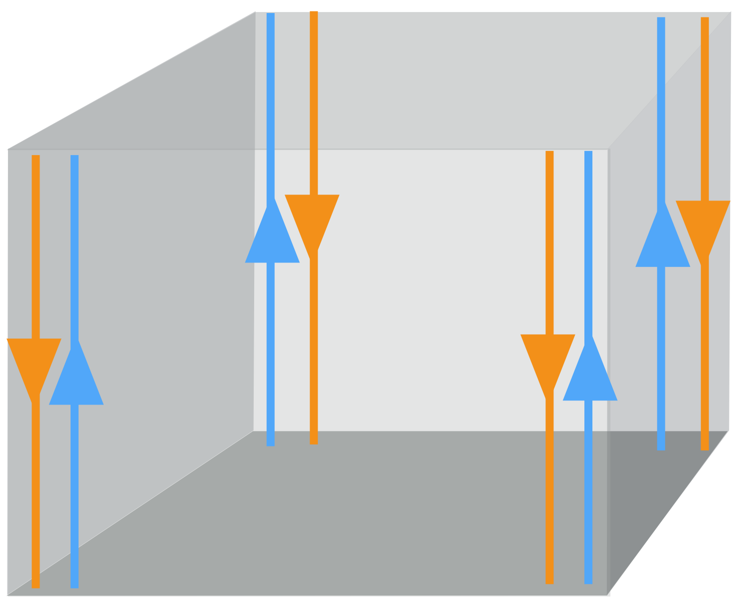
Δ_2 spin singlet $d_{x^2-y^2}$-wave

$\boxed{p + \mathrm{i}d \text{ superconductor with chiral Majorana hinge modes}}$



Time-reversal symmetric 2nd order 3D TI

One Kramers pair of modes on each hinge,
like quantum spin Hall edge



Two routes: (1) Mirror symmetry
(2) Inversion symmetry

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Mirror symmetry route: 3D HOTI

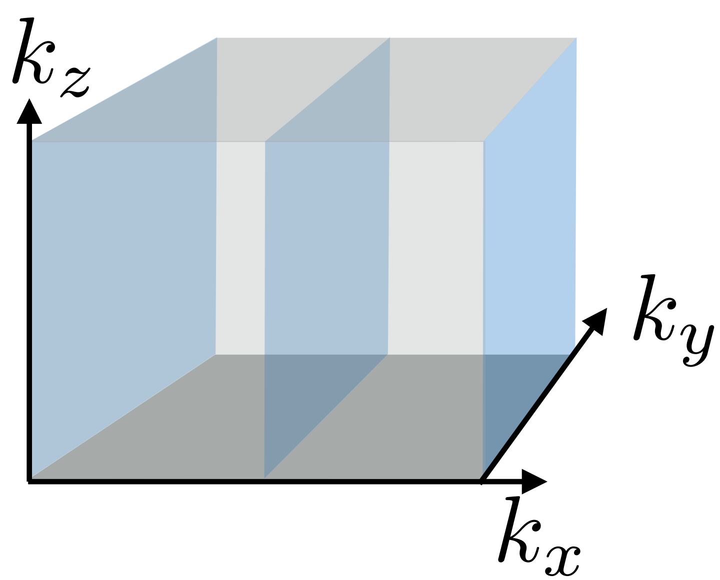
Mirror Chern numbers

Stabilize more than one Dirac cone by adding crystalline symmetries

Mirror symmetry:

eigenvalues +i and -i in spinful system

eigenstates on the mirror invariant planes in momentum space



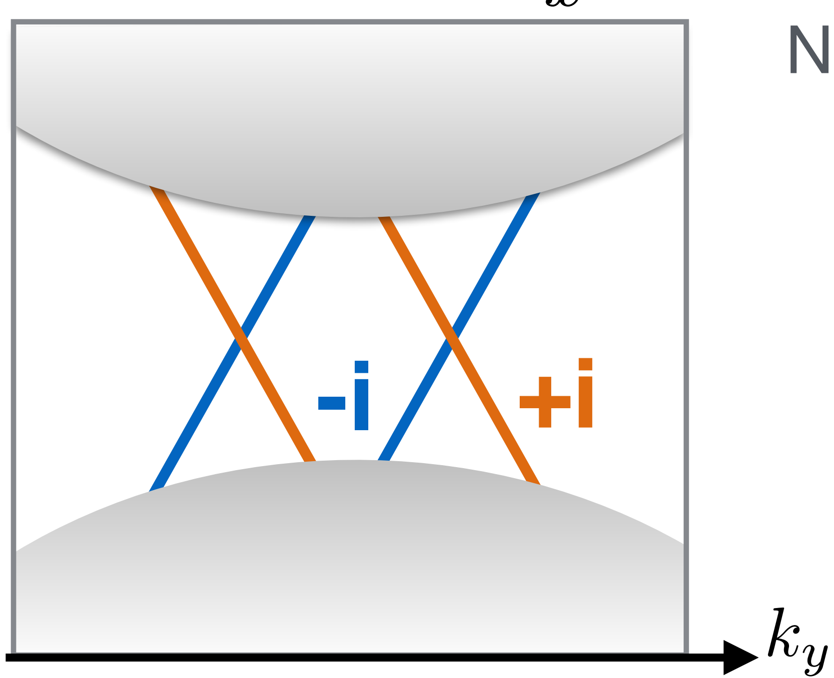
Mirror Chern number:

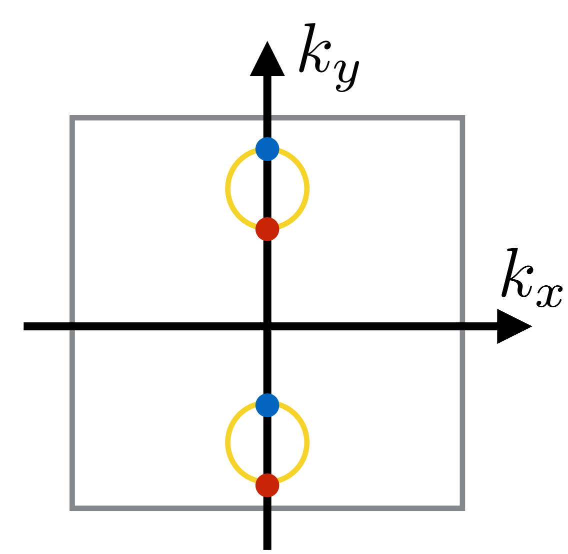
Chern number in +i/-i subspace on the plane

$$C_{\pm} = \frac{1}{2\pi} \int \mathrm{d}^2\boldsymbol{k}\, \mathrm{tr}\left[\partial_{k_y} \mathcal{A}_z^{\pm} - \partial_{k_z} \mathcal{A}_y^{\pm}\right]_{k_x = 0/\pi}$$

$\in \mathbb{Z}$ Time-reversal symmetry: $C_+ = -C_-$

Number of Dirac cones crossing line in surface BZ



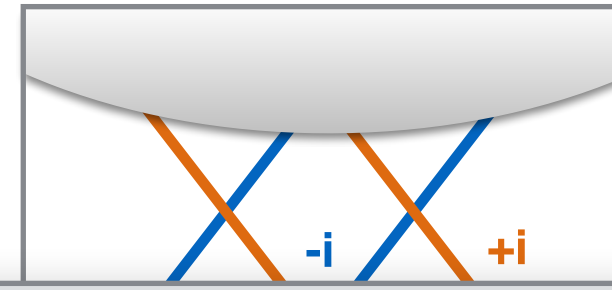


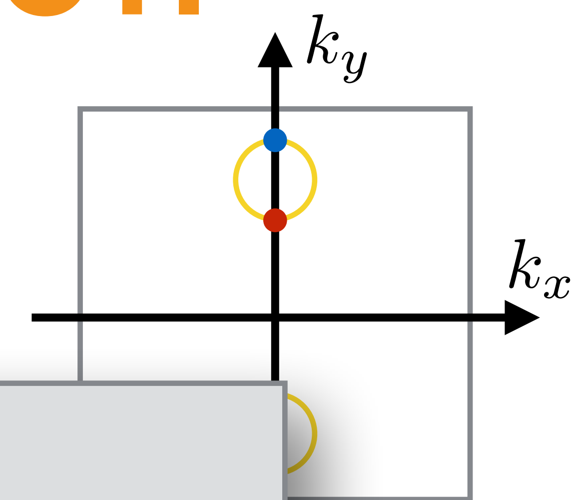
[L. Fu, Phys. Rev. Lett., 2011]

Mirror symmetry route: 3D HOTI

Bending the surface of a topological crystalline insulator

mirror Chern number = 2





$\mathbb{Z}$ (upmovers − downmovers) with mirror eigenvalue −i

= with time-reversal $\mathbb{Z}$

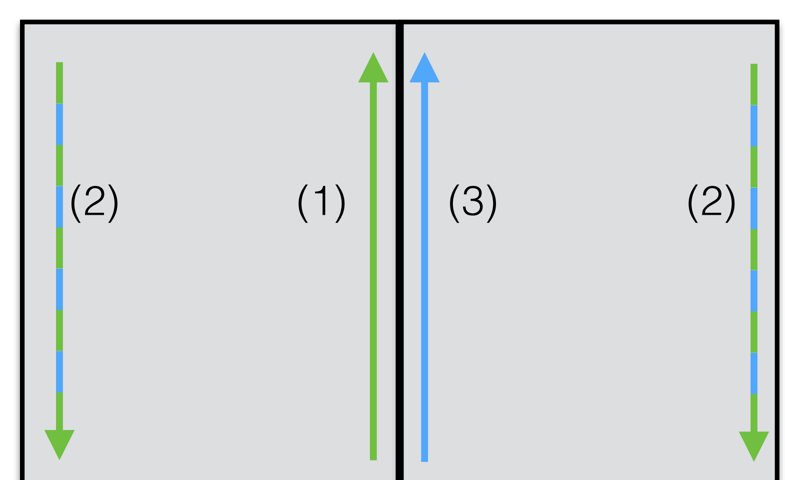
$\mathbb{Z}$ (upmovers − downmovers) with mirror eigenvalue + i

One upmover with mirror eigenvalue +i (=mirror Chern number/2)

Requires 3D bulk.

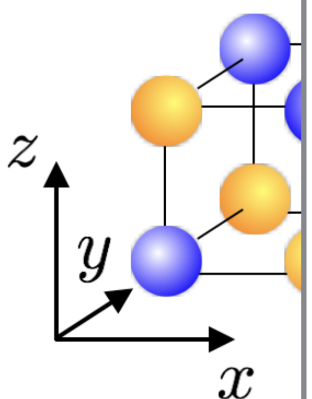
Allowed 2D surface perturbations:

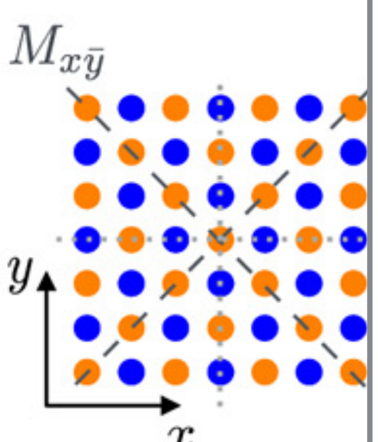
Number of upmovers of both mirror eigenvalues are equal

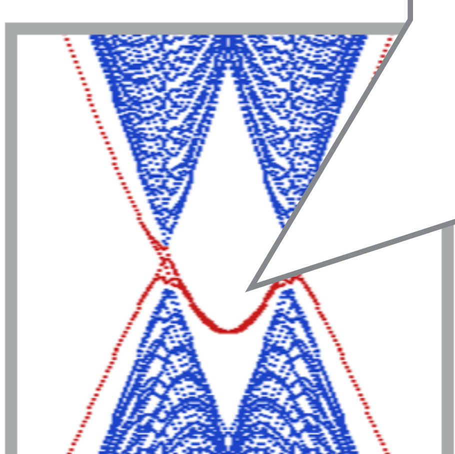


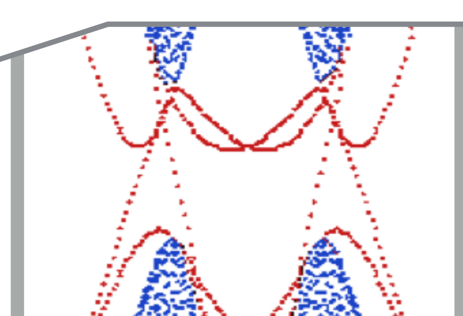
SnTe is a HOTI

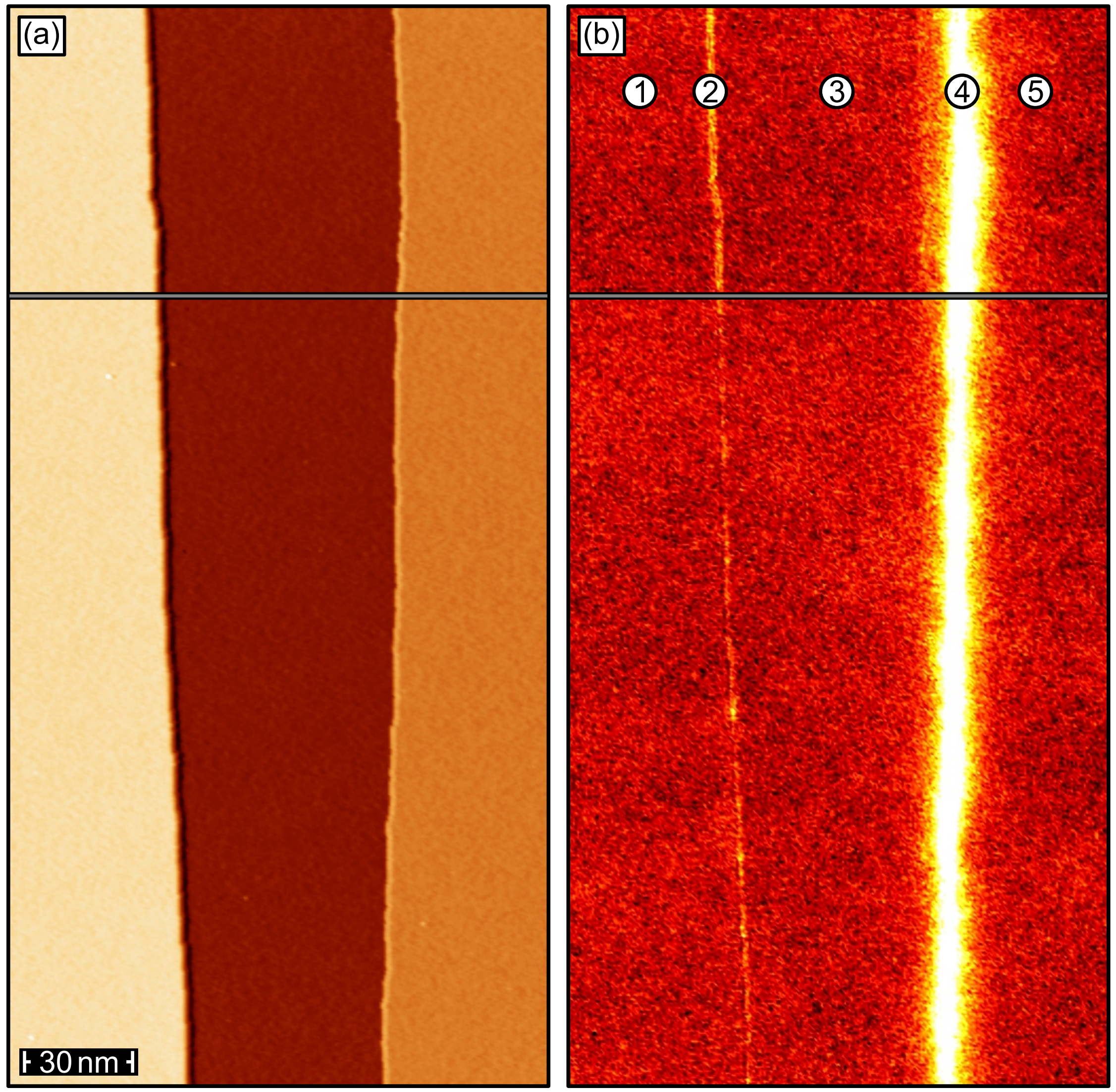
SnTe has f … the surface;
hinge state …



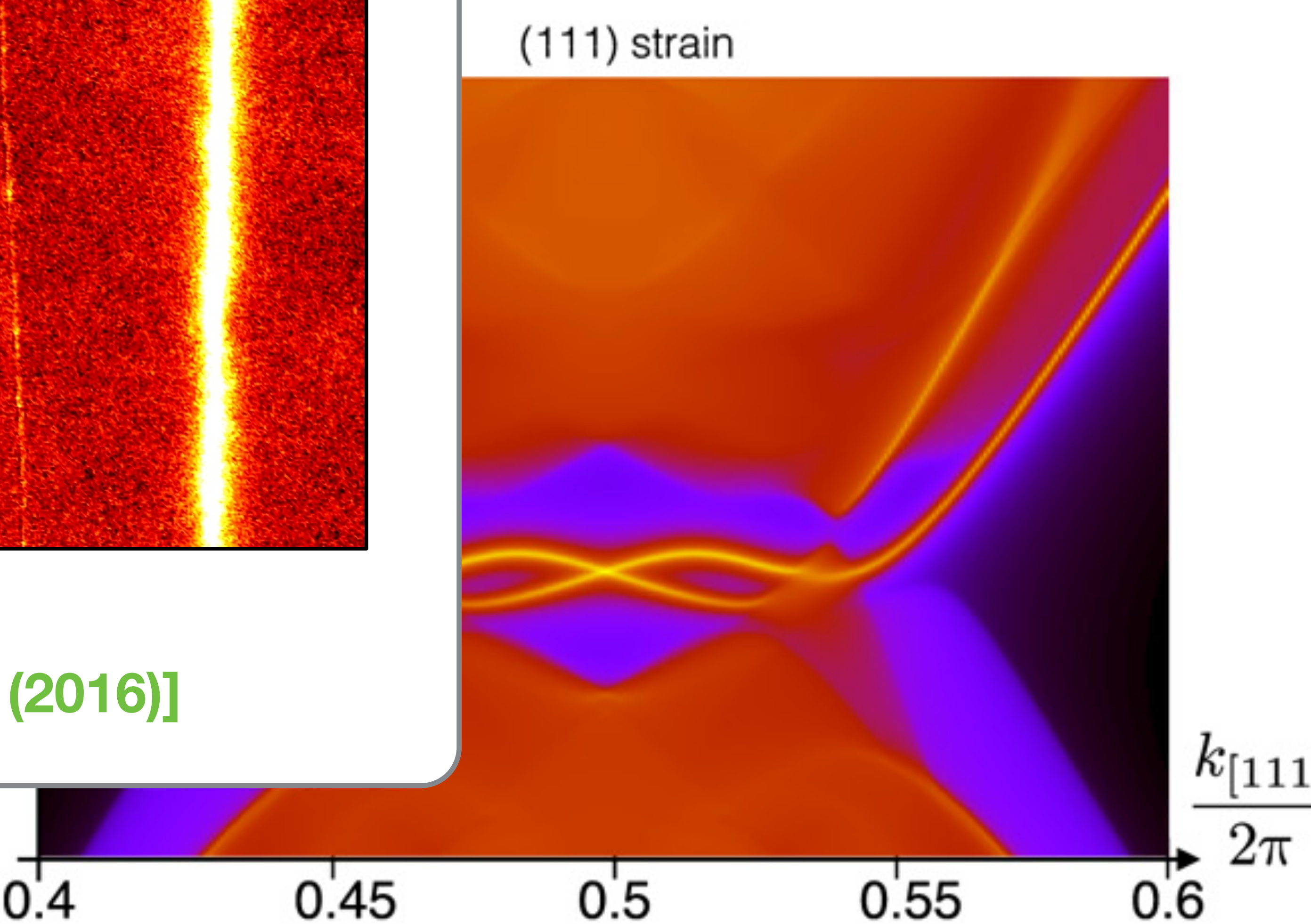








[P. Sessi et al.,
Science, 354, 1269-1273 (2016)]



[F. Schindler et al., Science Advances 2018]

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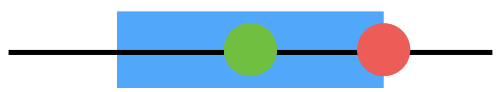
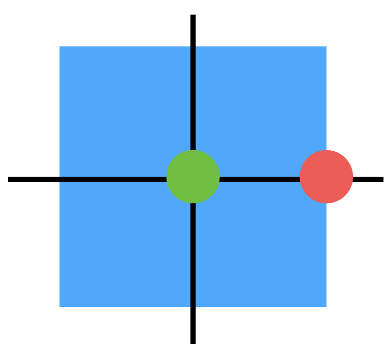
Inversion symmetry route: 3D HOTI

1D



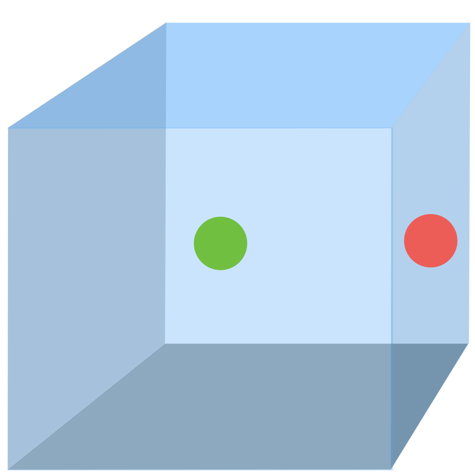
1 band inversion = trivial

2D



2 band inversions = trivial
1 band inversion = TI (Fu Kane)

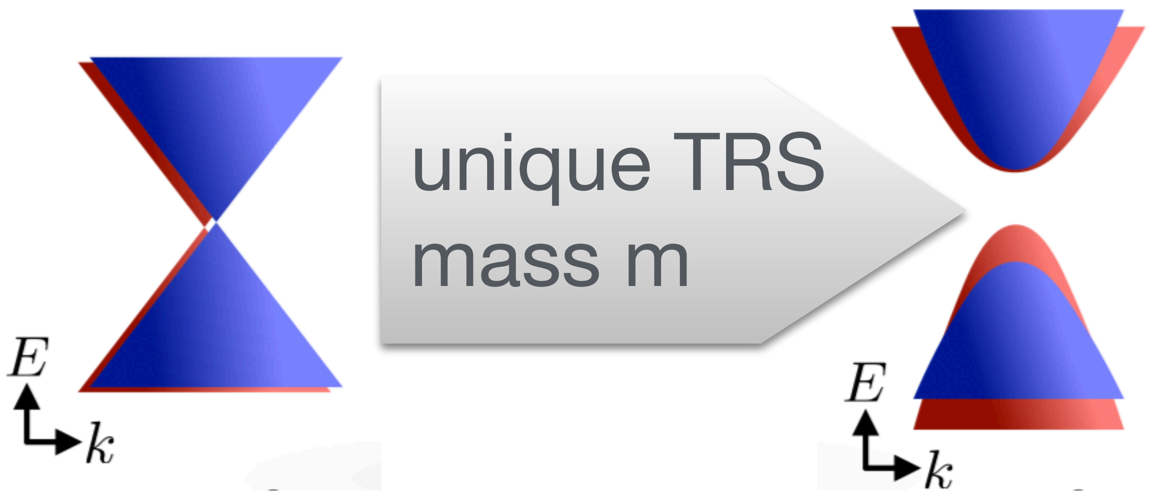
3D



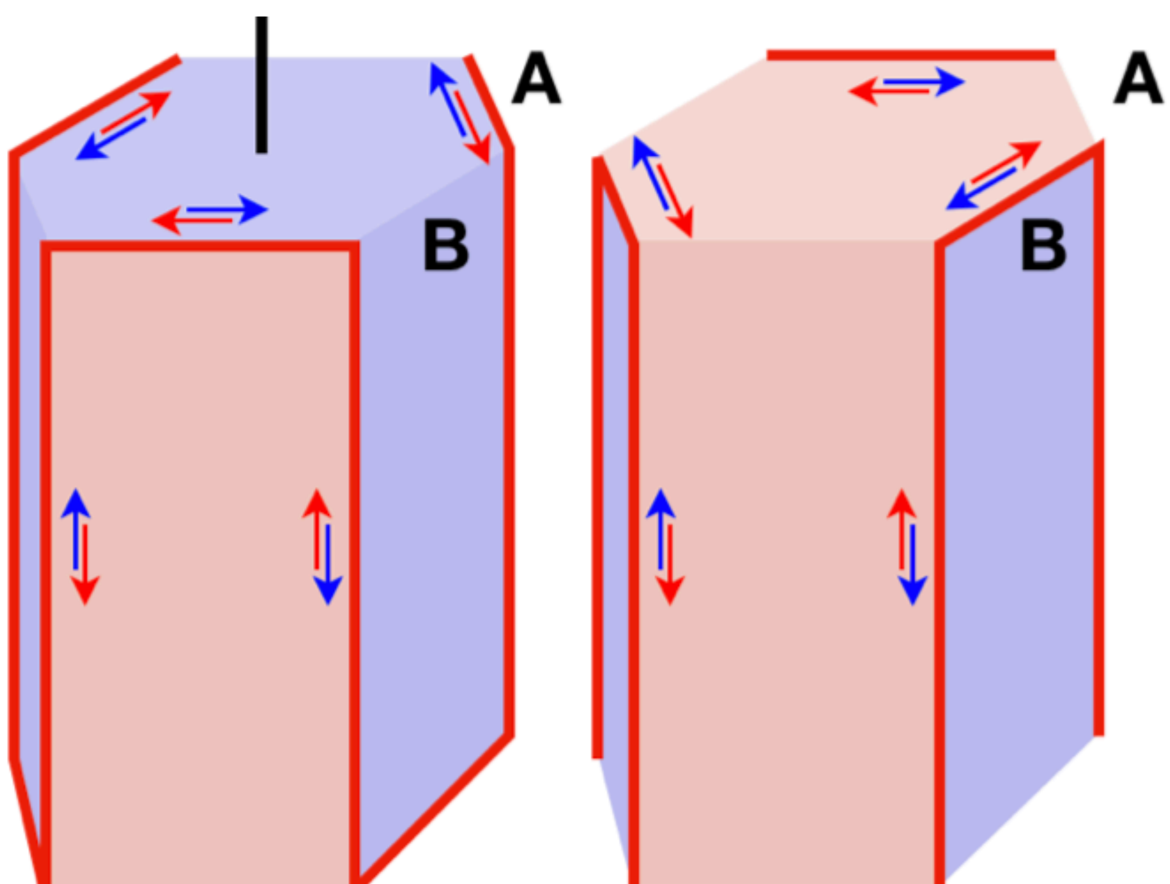
4 band inversions = trivial
1/3 band inversions = TI (Fu Kane)
2 band inversions = HOTI (e.g., Bi)

Bulk-boundary correspondence for “double band inversion”

Decoupled subspaces:
Two surface Dirac cones



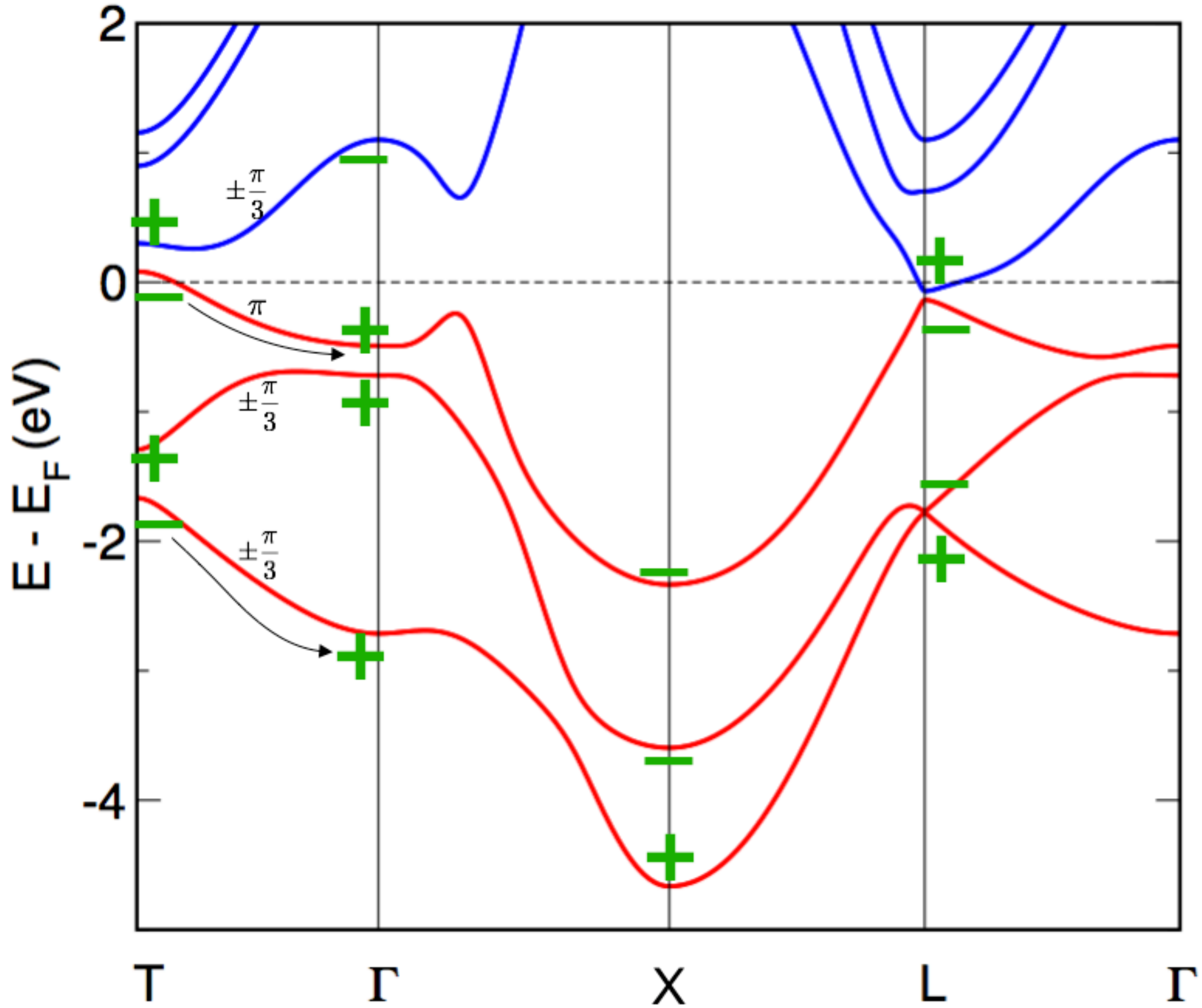
I: m → − m



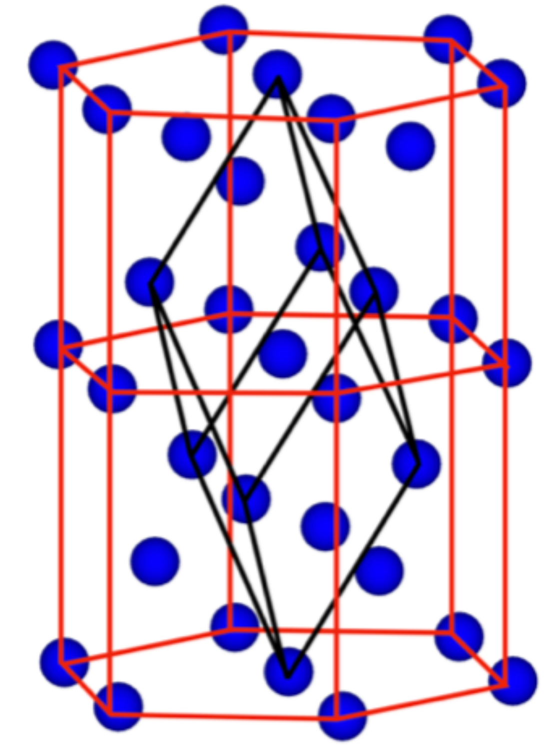
Bismuth is a HOTI

Double band inversion

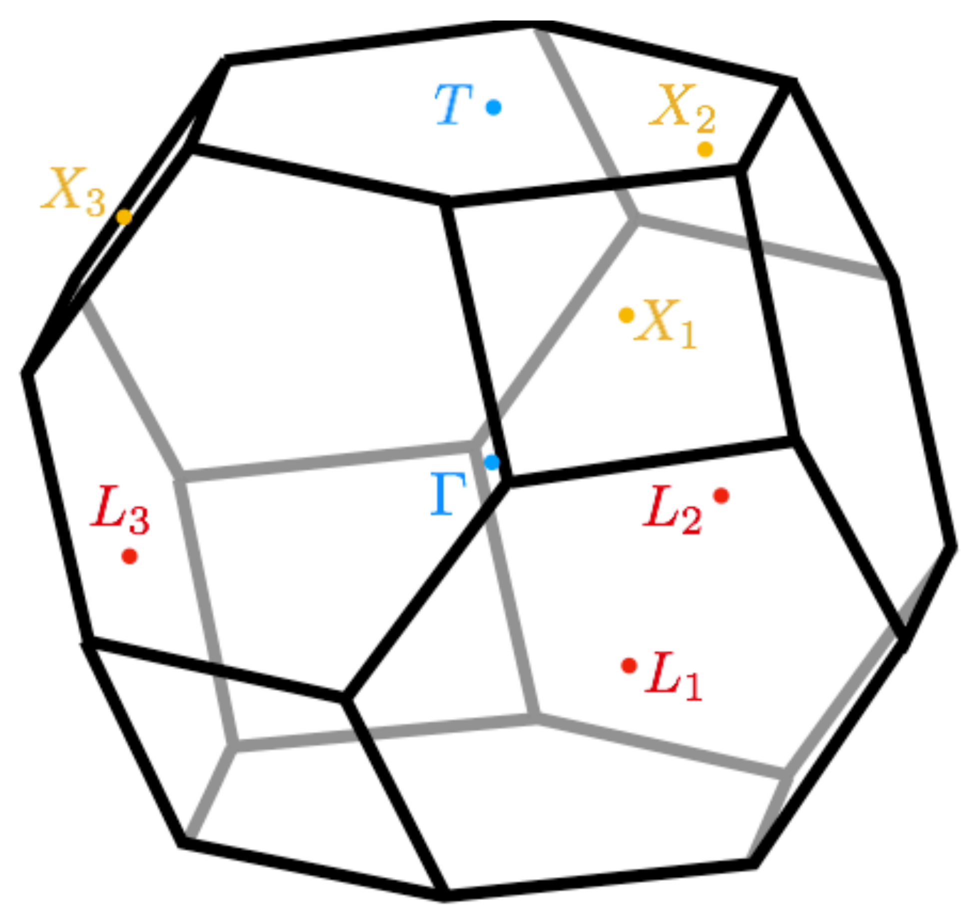
trivial weak indices, albeit
stack of 2D QSHE



SG 166:
C_3, I, M_x



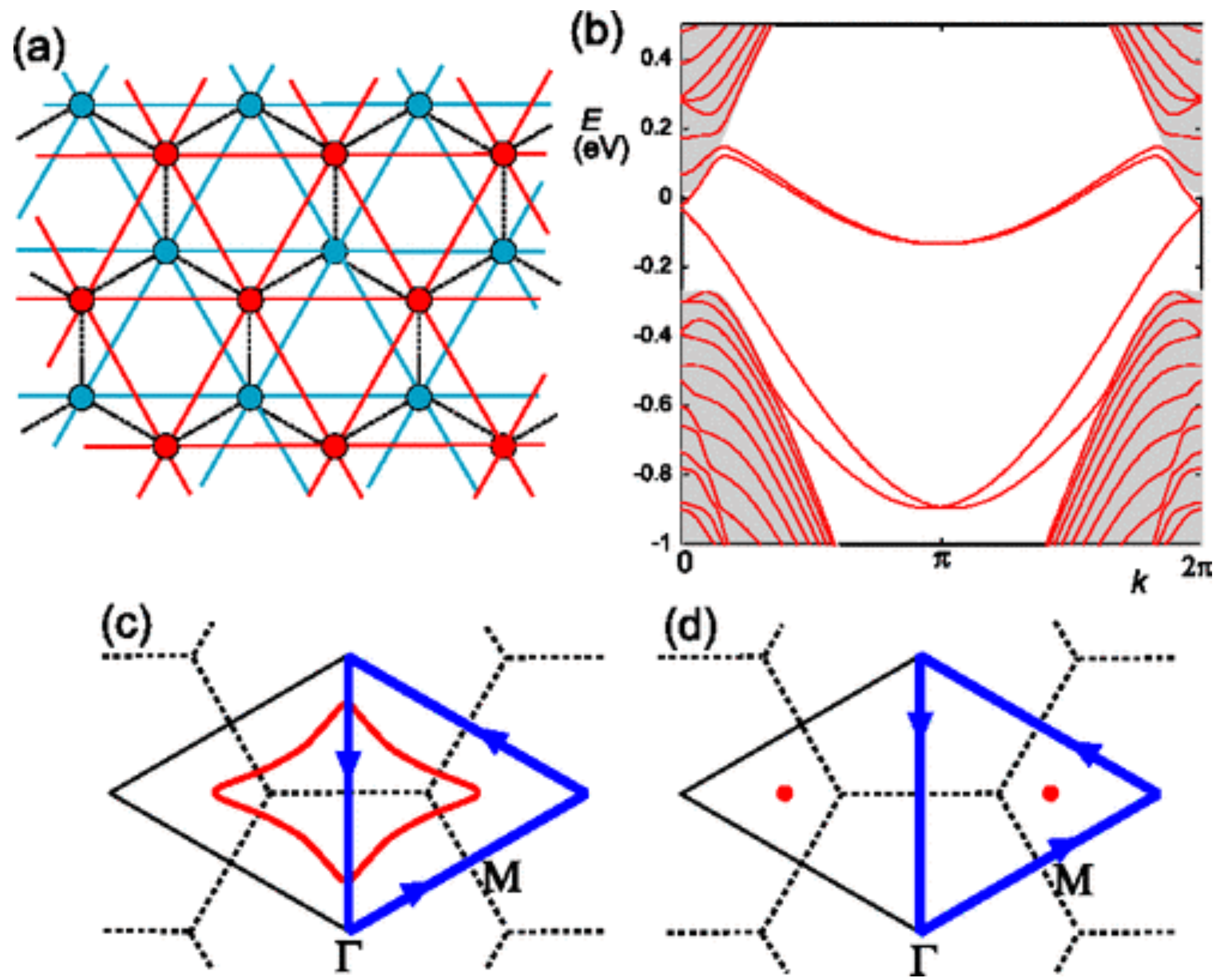
structure: weakly coupled layers of buckled honeycomb (bilayers)



BISMUTH HISTORY 101

Quantum Spin Hall Effect and Enhanced Magnetic Response by Spin-Orbit Coupling

Shuichi Murakami
Phys. Rev. Lett. **97**, 236805 – Published 6 December 2006



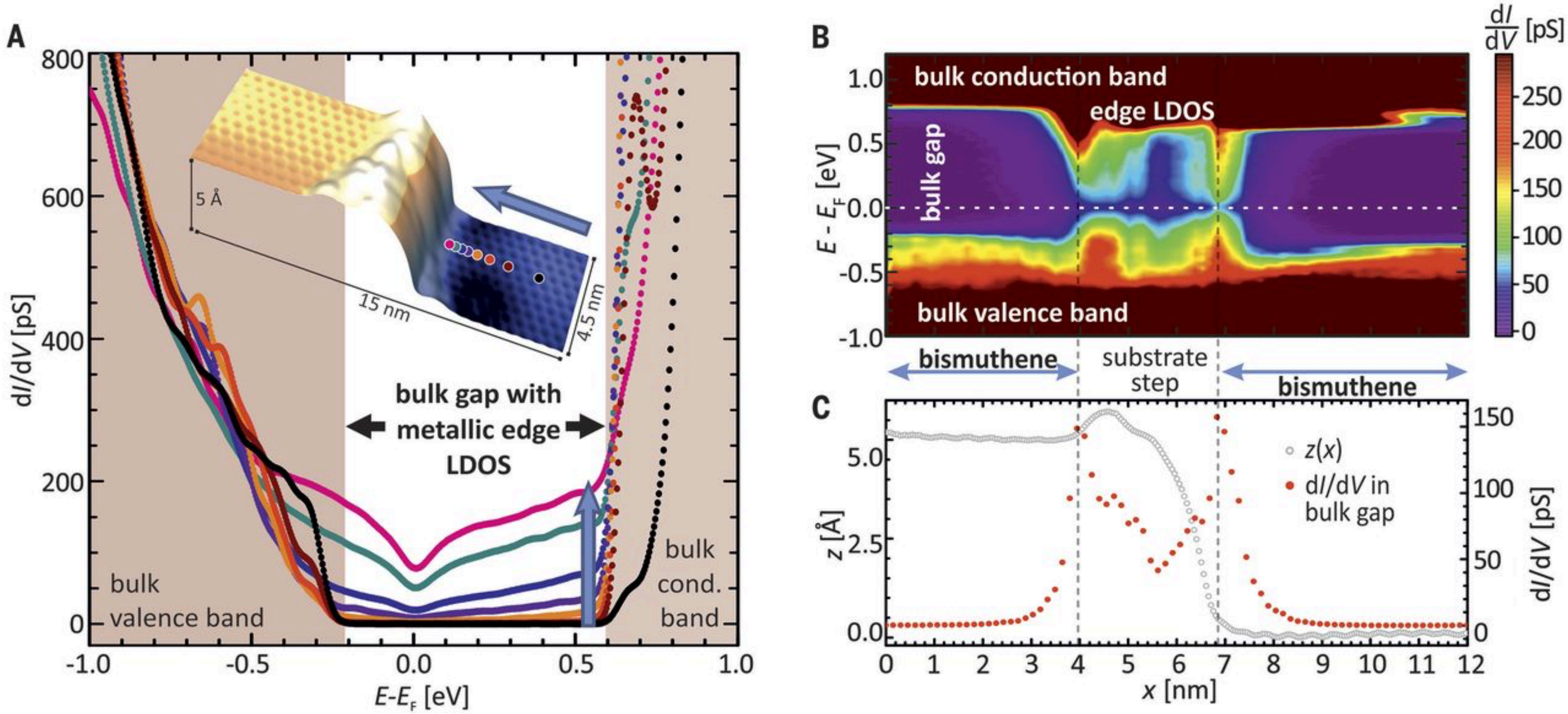
Bi bilayer is 2D TI

Bismuthene on a SiC substrate: A candidate for a high-temperature quantum spin Hall material

F. Reis[1,*], G. Li[2,3,*], L. Dudy[1], M. Bauernfeind[1], S. Glass[1], W. Hanke[3], R. Thomale[3], J. Schäfer[1,†], R. Claessen[1]

+ See all authors and affiliations

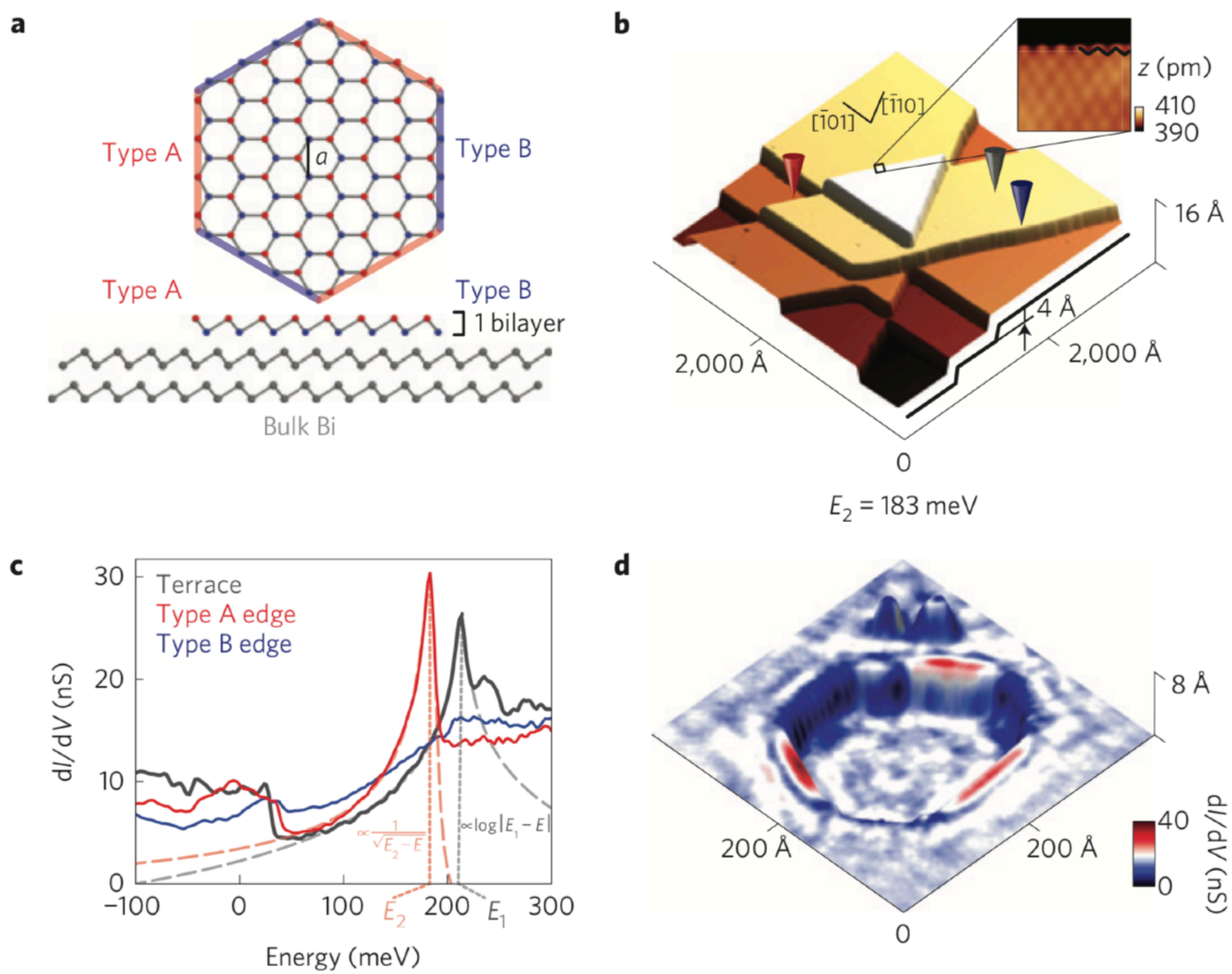
Science 21 Jul 2017:
Vol. 357, Issue 6348, pp. 287-290
DOI: 10.1126/science.aai8142

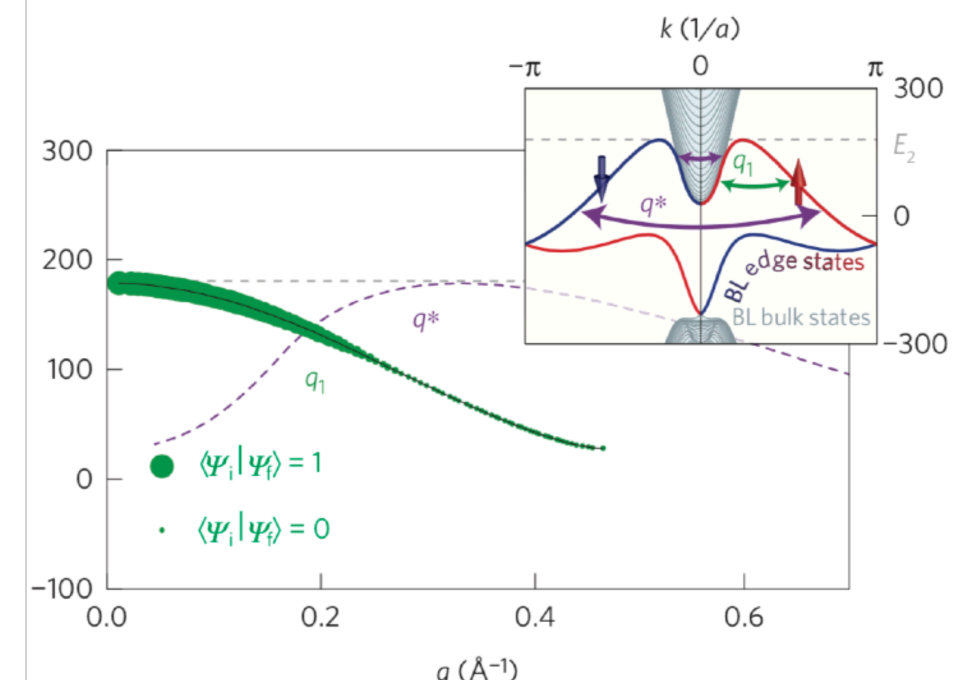


One-dimensional topological edge states of bismuth bilayers

Ilya K. Drozdov, A. Alexandradinata, Sangjun Jeon, Stevan Nadj-Perge, Huiwen Ji, R. J. Cava, B. Andrei Bernevig & Ali Yazdani

Nature Physics **10**, 664–669 (2014) | Download Citation



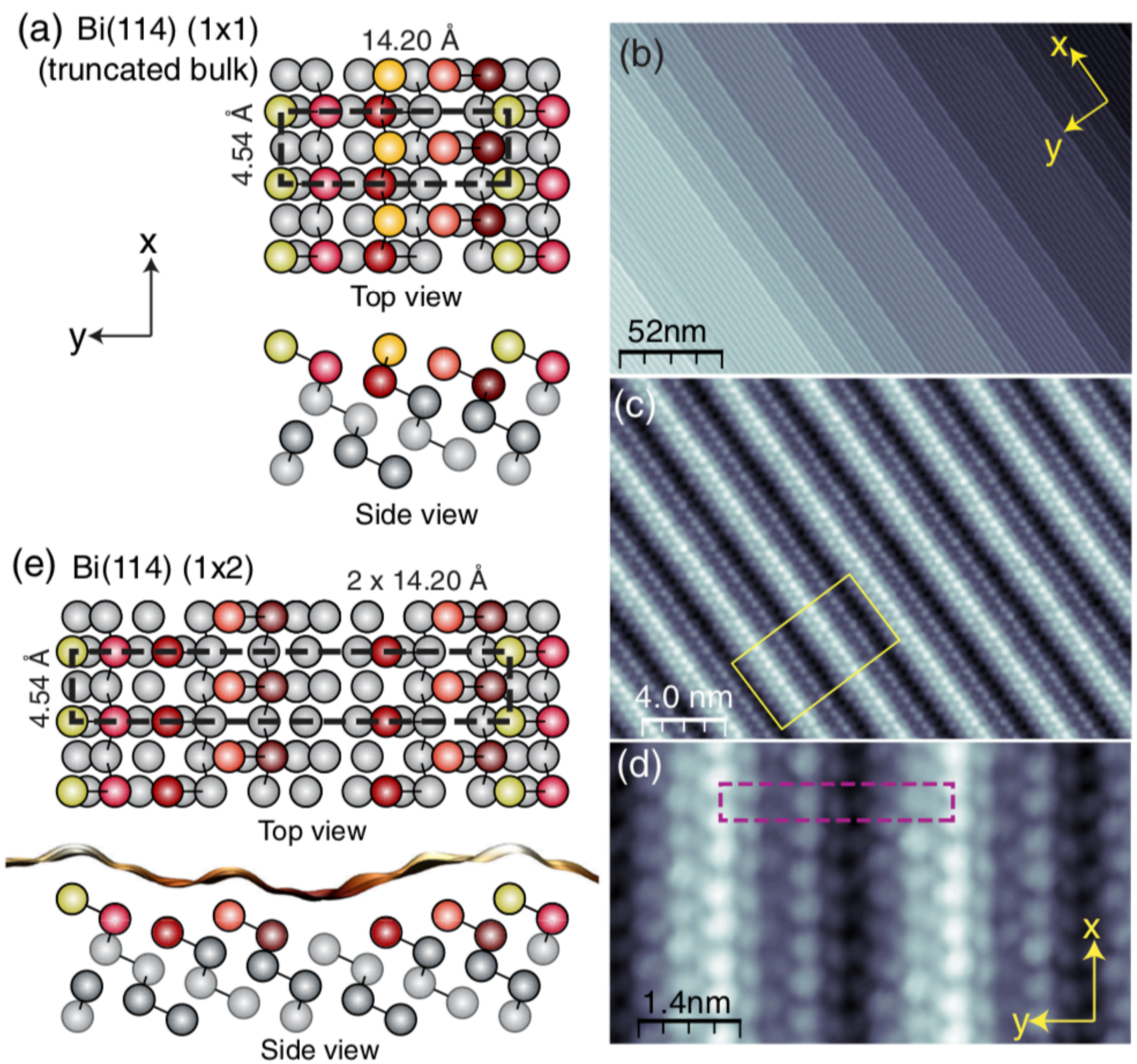


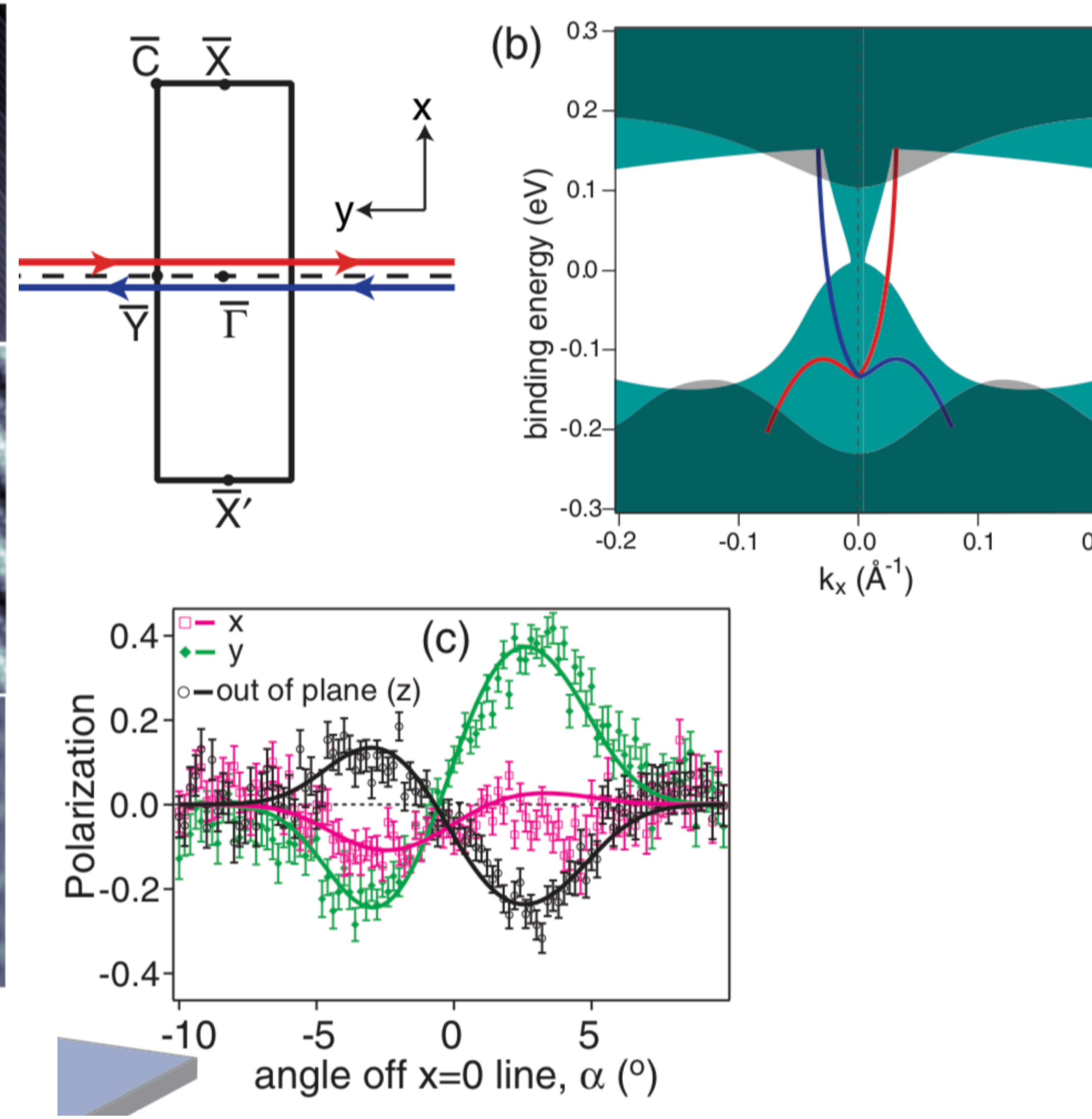
Featured in Physics Editors' Suggestion Access by l

Nondegenerate Metallic States on Bi(114): A One-Dimensional Topological Metal

J. W. Wells, J. H. Dil, F. Meier, J. Lobo-Checa, V. N. Petrov, J. Osterwalder, M. M. Ugeda, I. Fernandez-Torrente, J. I. Pascual, E. D. L. Rienks, M. F. Jensen, and Ph. Hofmann

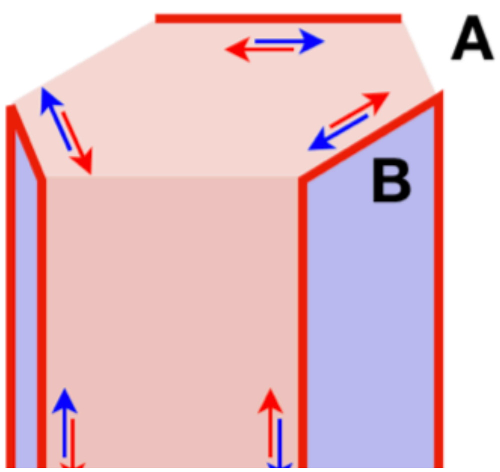
Phys. Rev. Lett. **102**, 096802 – Published 2 March 2009

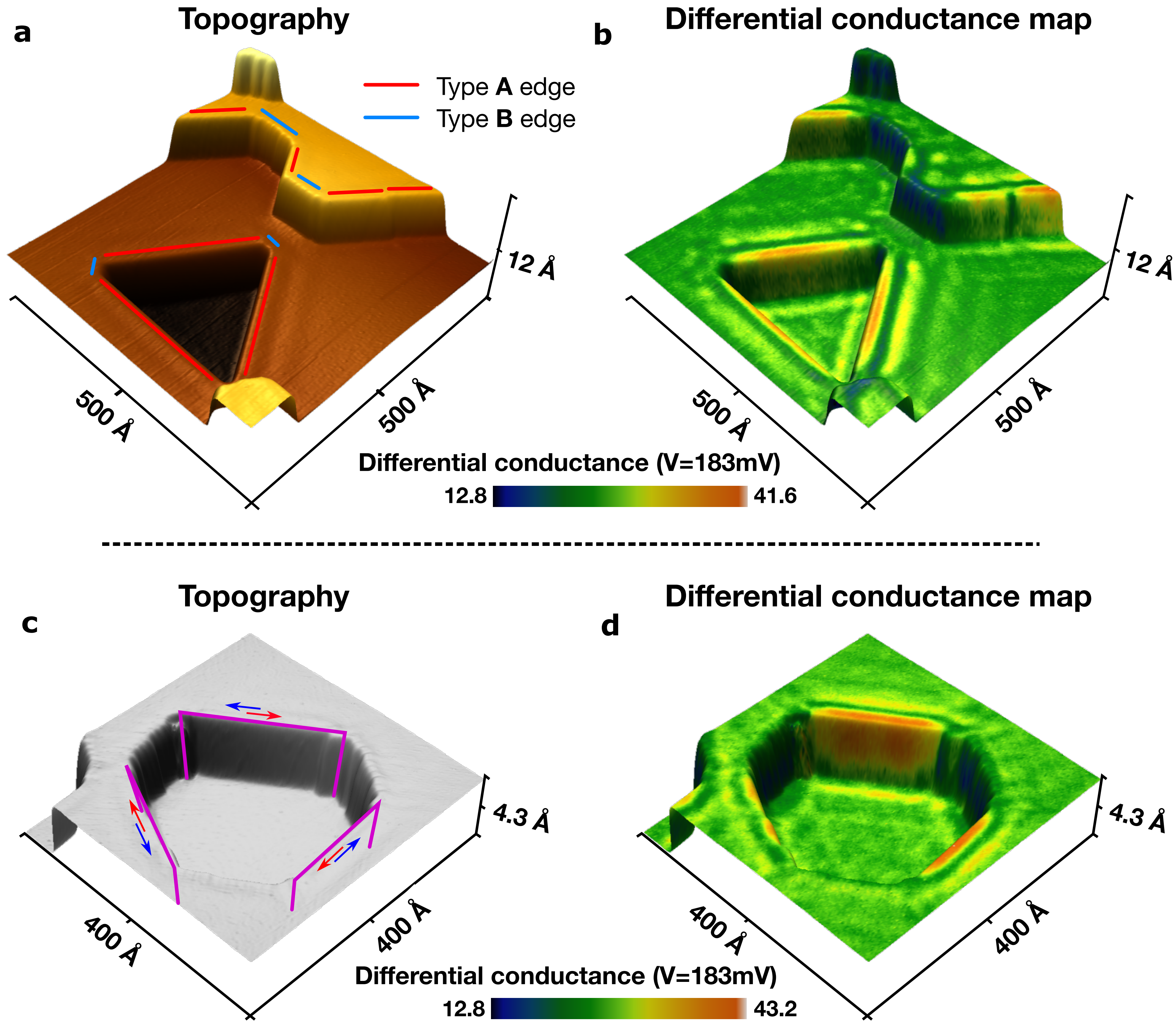




BISMUTH HISTORY 101

HOTI Evidence 1) STM



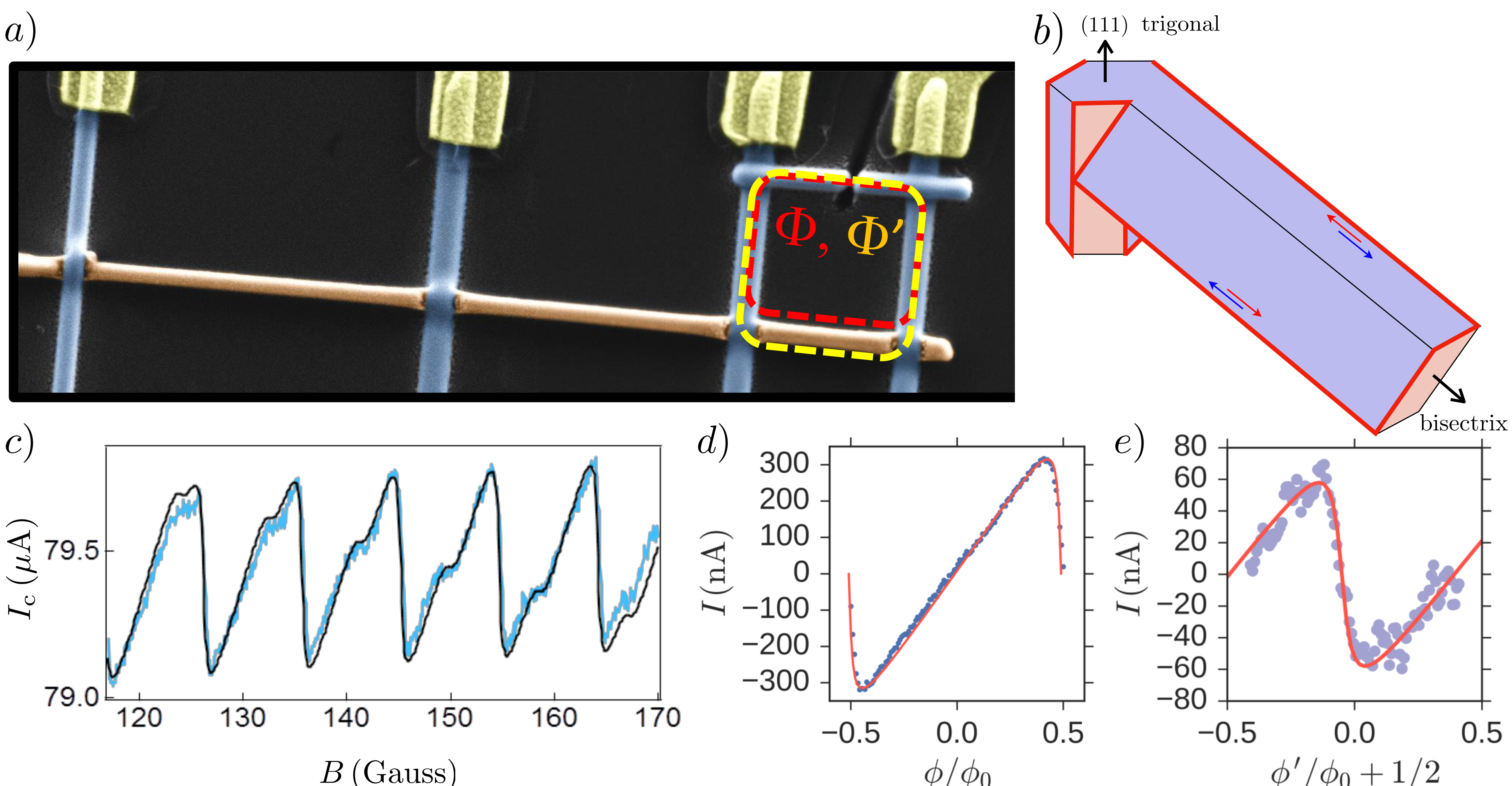


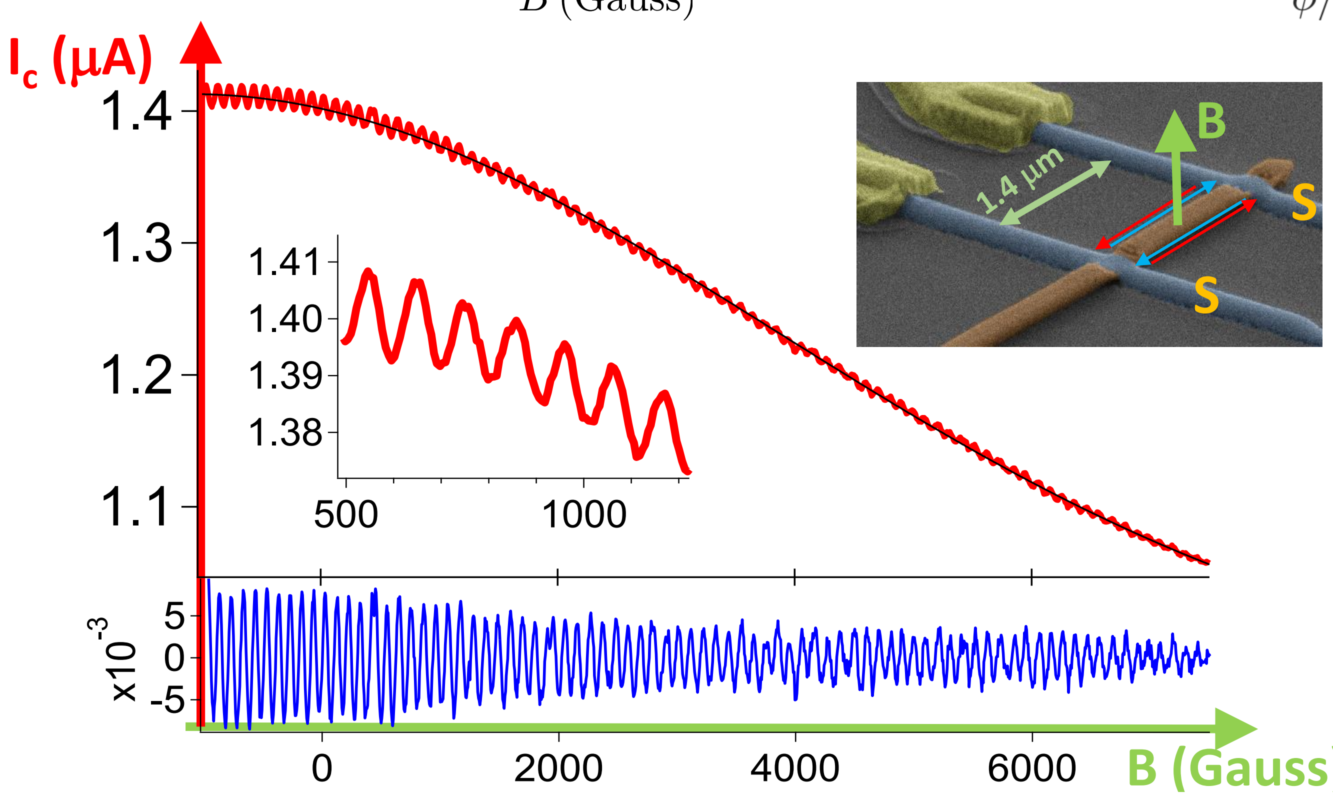
questions:

i) thickness dependence

ii) hybridization with lower hinge mode

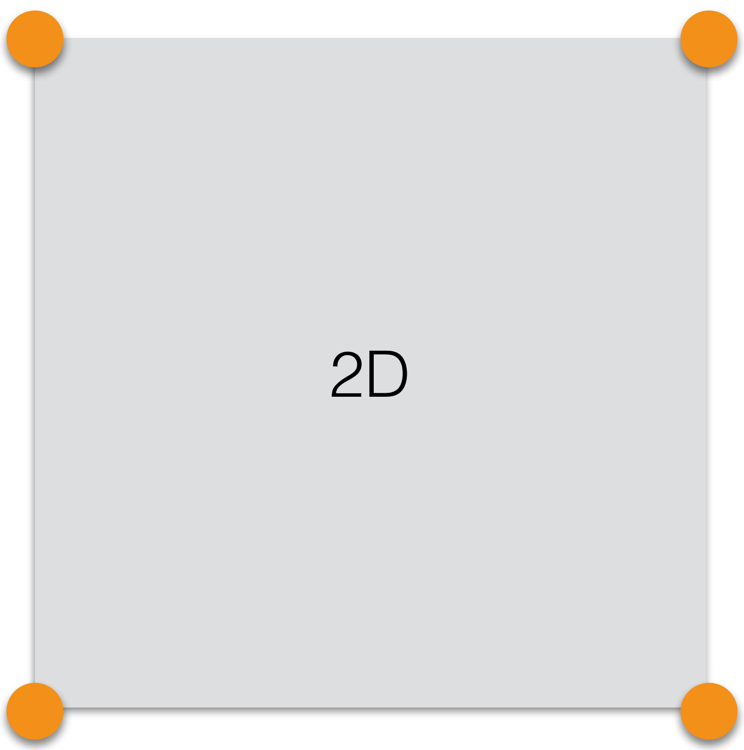
HOTI Evidence 2) Josephson interferometry





interference between hinge modes only (no SQUID)

Content



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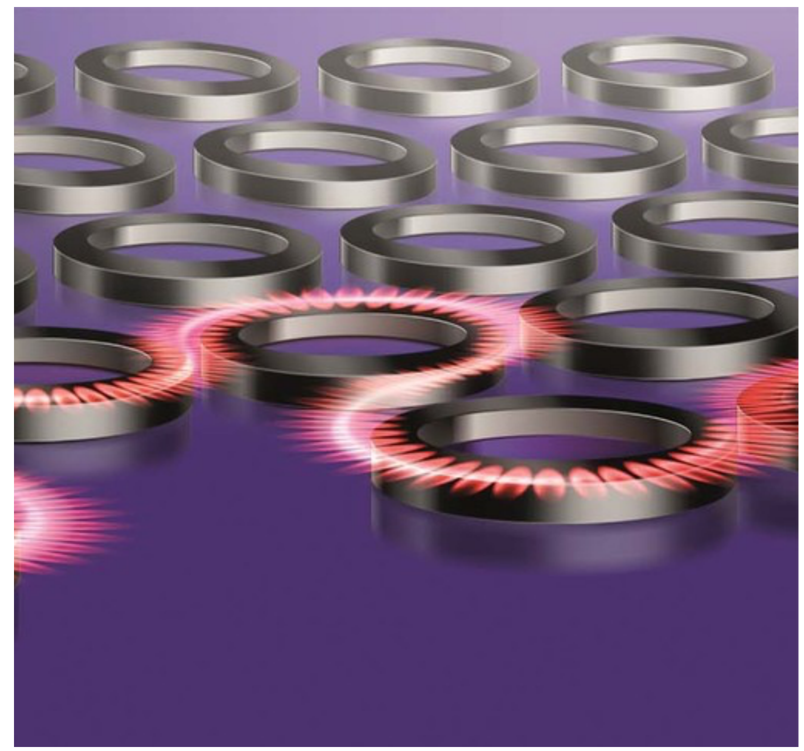
[Benalcazar, Bernevig, Hughes
Science 357, 61-66 (2017)]

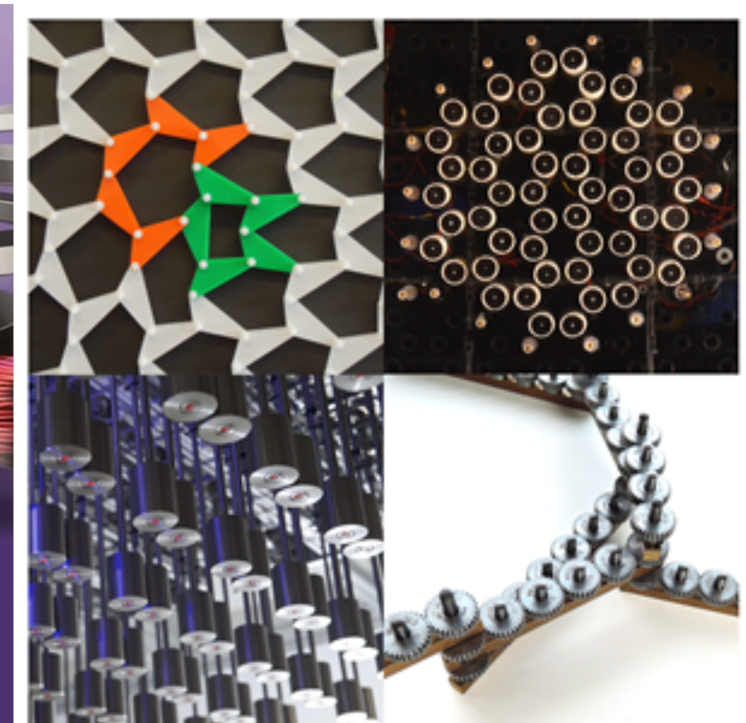
Realization of topological systems in classical systems

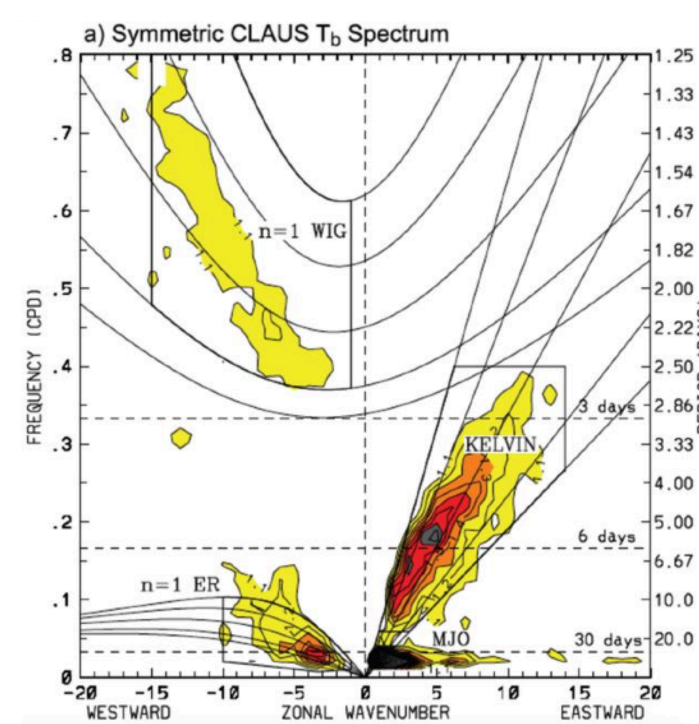
Berry phase is not inherently quantum mechanical

- mechanical systems
- acoustic systems
- photonic systems
- electrical circuits
- … the earth … has Chern number 2

[Delplace et al., Science (2017)]







Theoretical framework

Network of inductors and capacitors

There is no Fermi sea: band topology not manifest in ground state

Design topology of response function (impedance) instead of Hamiltonian

Kirchhoff's law

$$I_a(\omega) = \sum_{b=1,2,\cdots} J_{ab}(\omega)\, V_b(\omega)$$

capacities inverse inductances

$$J_{ab}(\omega) = \mathrm{i}\omega\, C_{ab} - \frac{\mathrm{i}}{\omega} W_{ab}$$

circuit Laplacian

Impedance

$$Z_{ab}(\omega) = G_{aa}(\omega) + G_{bb}(\omega) - G_{ab}(\omega) - G_{ba}(\omega)$$

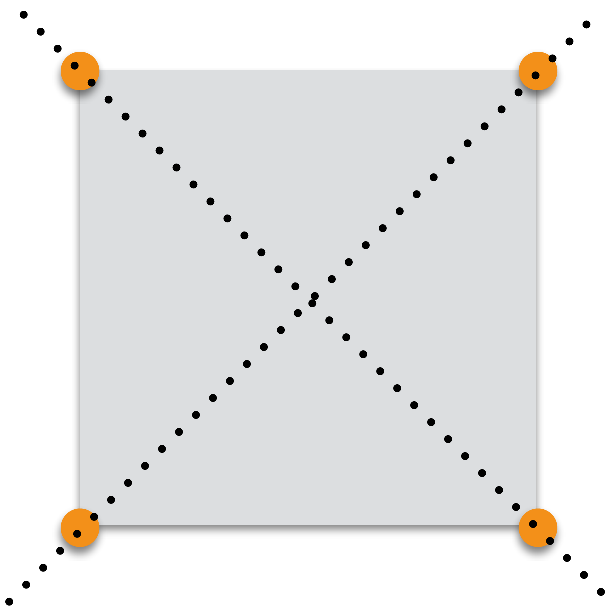
$$G(\omega) = J^{-1}(\omega)$$

Zeros in J dominate response Z

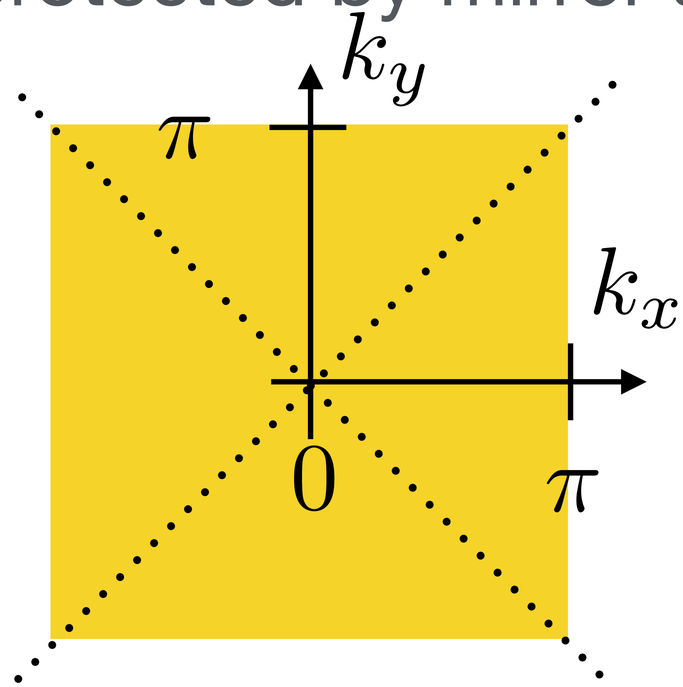
[Benalcazar, Bernevig, Hughes Science 357, 61-66 (2017)]

HOTI model

2D corner states protected by mirror and chiral symmetry



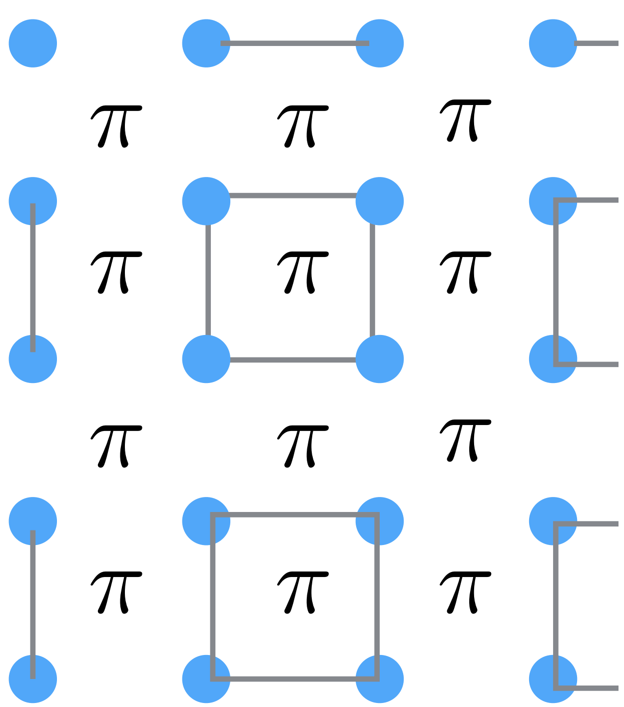
real space



Brillouin zone

$$
\begin{aligned}
M_{x\bar{y}} R(k_x, k_y) M_{x\bar{y}}^{-1} &= -R(k_y, k_x) \\
\mathcal{C} R(\boldsymbol{k}) \mathcal{C}^{-1} &= -R(\boldsymbol{k})
\end{aligned}
$$

$$
R(k, k) = \begin{pmatrix} 0 & q_+(k) & 0 & 0 \\ q_+(k)^\dagger & 0 & 0 & 0 \\ 0 & 0 & 0 & q_-(k) \\ 0 & 0 & q_-(k)^\dagger & 0 \end{pmatrix}
$$

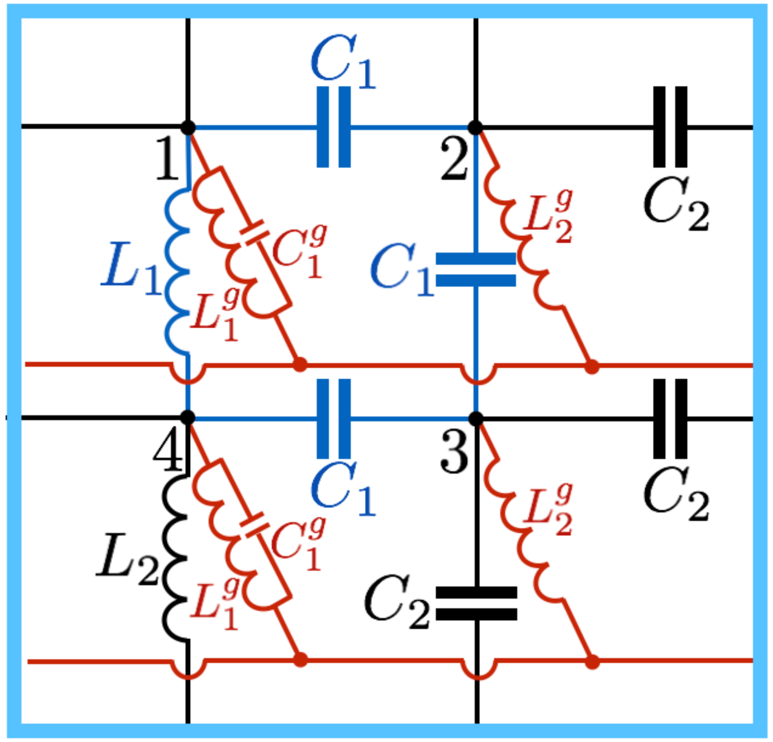


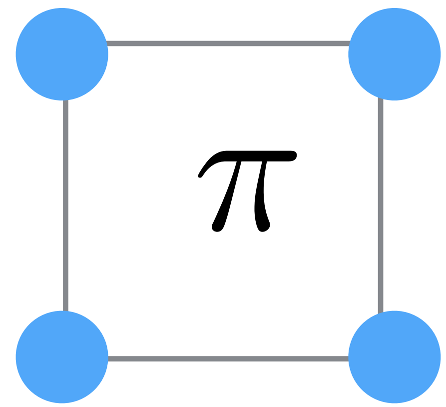
$$
\begin{aligned}
R(\boldsymbol{k}) = &(1 + \lambda \cos\, k_x) \sigma_1 \tau_0 \\
&+ (1 + \lambda \cos\, k_y) \sigma_2 \tau_2 \\
&- \lambda \sin\, k_x\, \sigma_2 \tau_3 \\
&+ \lambda \sin\, k_y\, \sigma_2 \tau_1,
\end{aligned}
$$

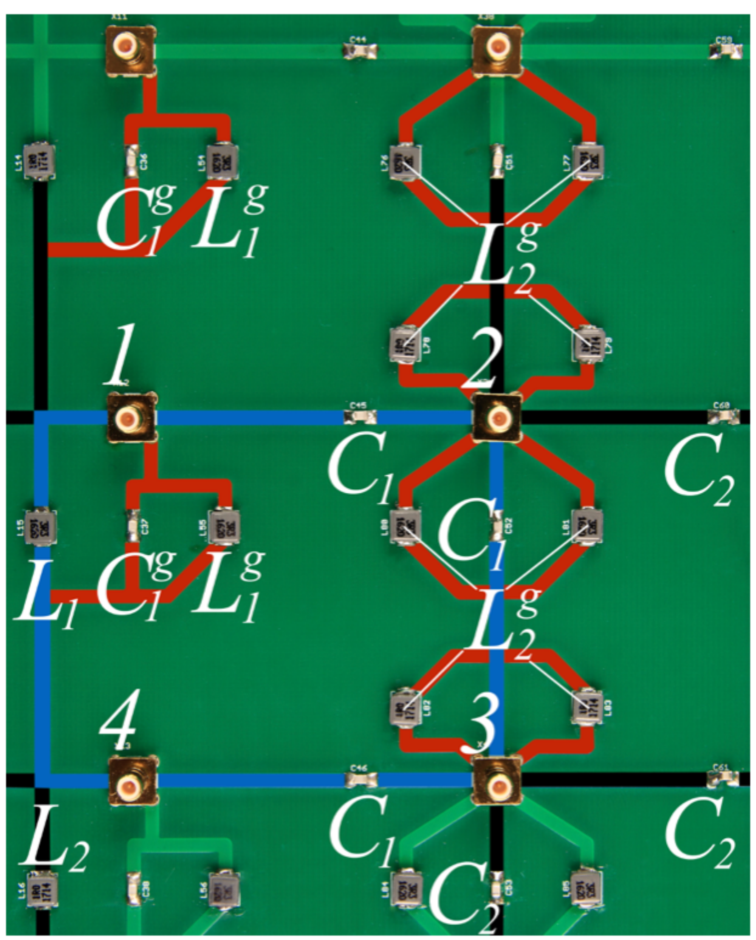
Topological invariant: mirror graded winding number

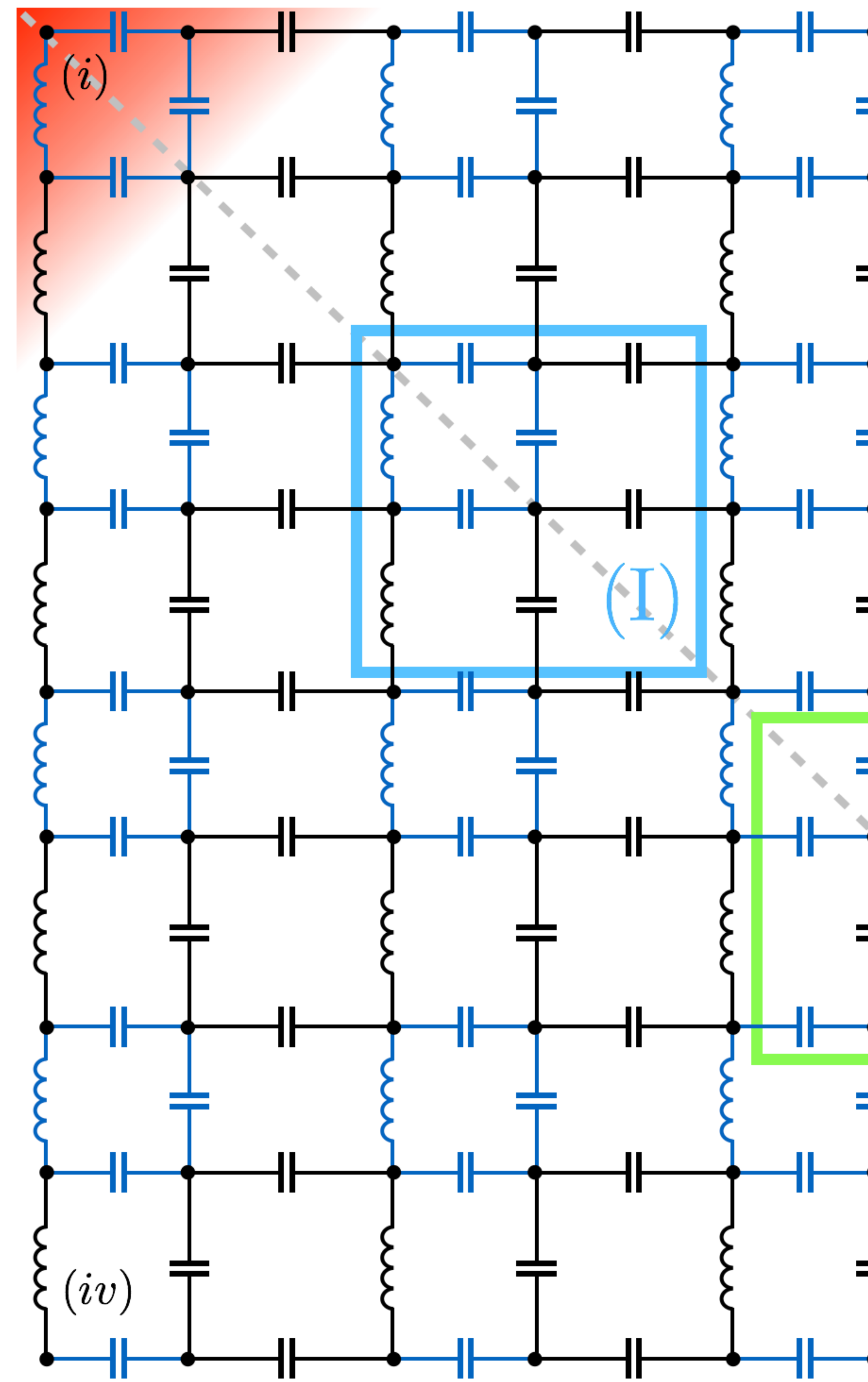
$$
\nu_\pm := \frac{\mathrm{i}}{2\pi} \int_0^{2\pi} \mathrm{d}k \, \mathrm{tr}\, \tilde{q}_\pm^\dagger(k) \partial_k \tilde{q}_\pm(k) \qquad \nu := \frac{\nu_+ - \nu_-}{2} \in \mathbb{Z}
$$

Electric circuit realization



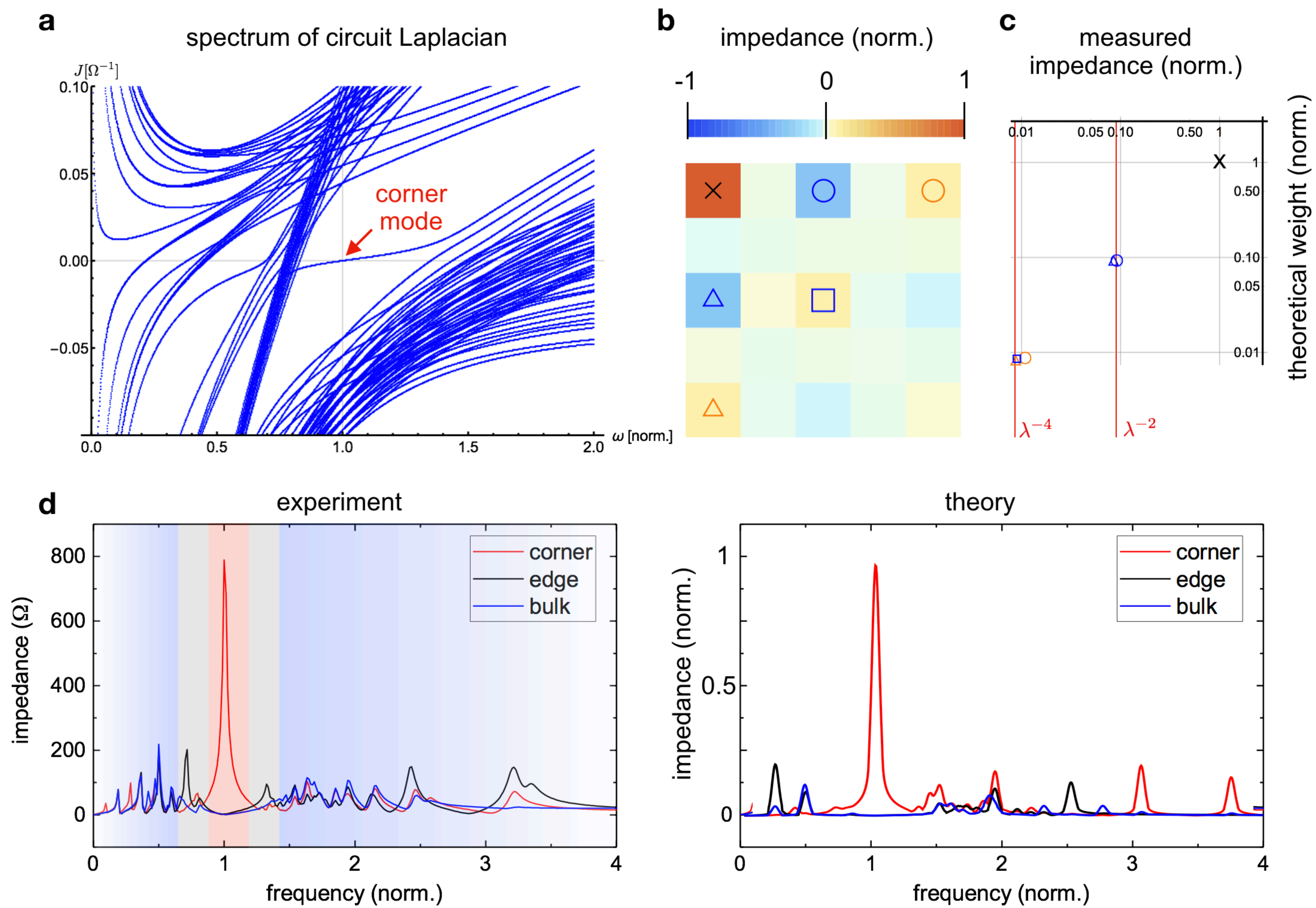






Electric circuit realization

measured corner state:
exponential localization



Other synthetic HOTIs

Microwave resonators

[Peterson et al. Nature (2018)]

Photonic waveguides

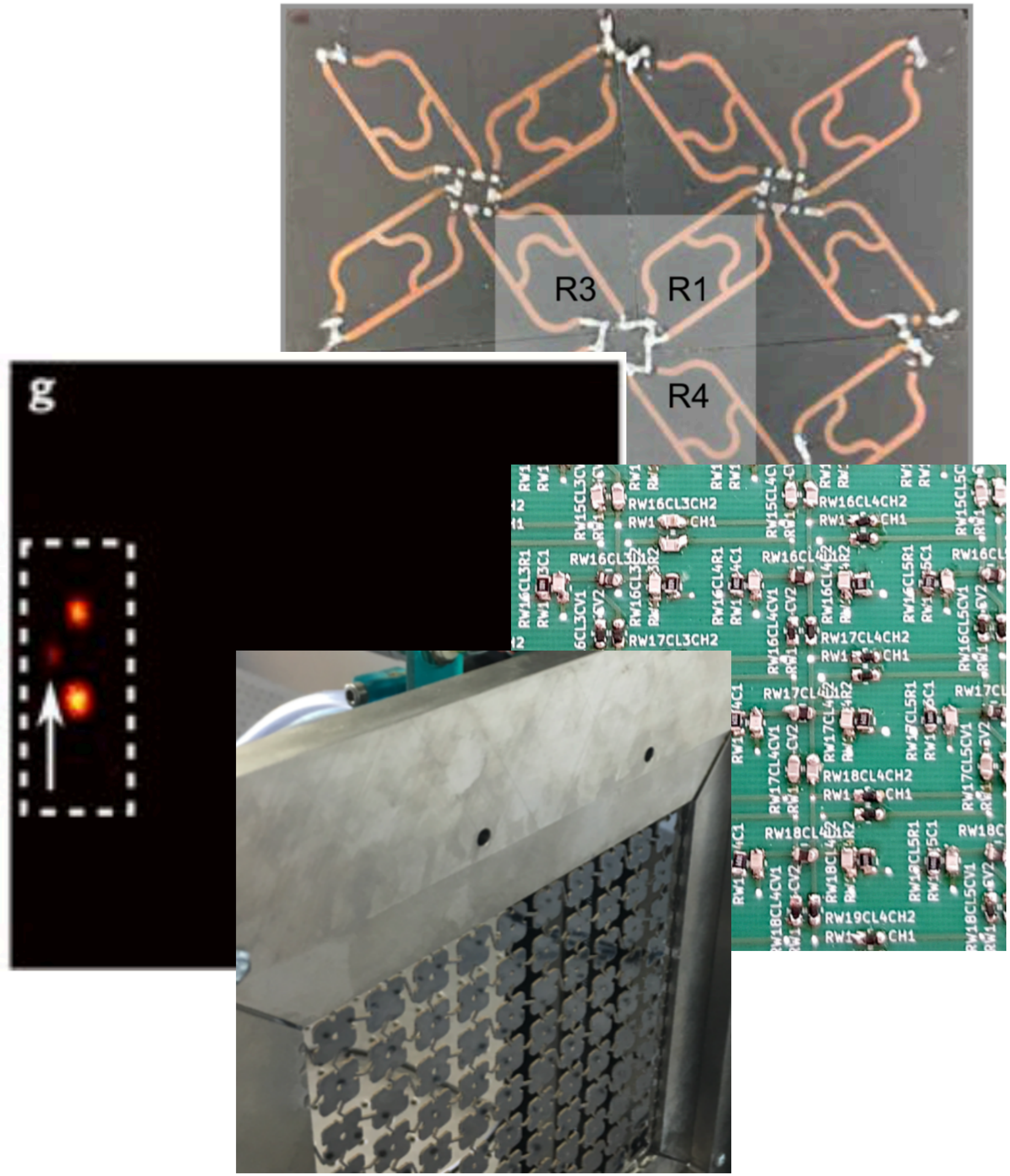
[Noh et al. Nature Photonics (2018)]

Mechanical system

[Serra-Garcia et al. Nature (2018)]

Electric circuit

[Serra-Garcia et al. arxiv (2018)]



Summary: **Higher-order topology**

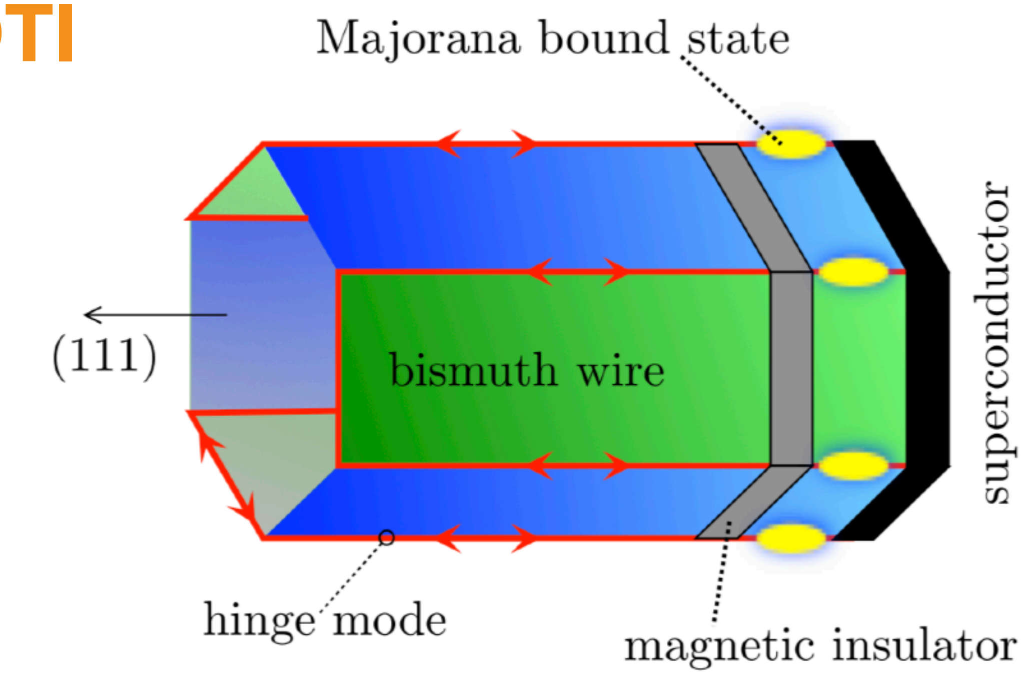
HOTIS generalize the **bulk-boundary correspondence** of topological matter

need **spatial symmetry**

SnTe is HOTI

Bismuth is e-h compensated **HOTI**

Topological quantum computing: hexon of **Majoranas**



[C. Beenakker, Condensed matter online journal club]
[Hsu et al., PRL (2018)]

Electric circuits realization of **HOTI**