Rencontre ANR WKBHJ 26-27 Novembre 2015, ENS-Lyon

Thursday November 26, 2015. Amphi A

14:00–14:30. Opening & discussion.

14:30–15:20. Albert Fathi (ENS-Lyon): *On the set of singularities of a solution of the Hamilton-Jacobi equation.*

We consider a continuous viscosity solution $U : M \times [0, +\infty[\to \mathbb{R}$ of the evolutionary equation

$$\partial_t U(x,t) + H(x, \partial_x U(x,t)) = 0,$$

where M is a compact manifold, and $H : T^*M \to \mathbb{R}$ is a Tonelli Hamiltonian.

We use topological tools to understand the connected components of the set S of points in $M \times]0, +\infty[$ where U is not differentiable. Our result implies the following fact: if D be a compact smooth domain with boundary ∂D contained in $M \times]0, +\infty[$., and U is differentiable at each point of ∂D, then U is also differentiable at each point of D.

15:30–16:20. Samuel Petite (Université de Picardie Jules Verne): *Calibrated configurations for Frenkel-Kontorova type models in almost-periodic environments.*

Abstract: The Frenkel-Kontorova model describes how an infinite chain of atoms minimizes the total energy of the system when the energy takes into account the interaction of nearest neighbors as well as the interaction with an exterior environment. An almost-periodic environment leads to consider a family of interaction energies which is stationary with respect to a minimal topological dynamical system. With E. Garibaldi and P. Thieullen, we introduce, in this context, the notion of calibrated configuration (stronger than the standard minimizing condition) and, for continuous superlinear interaction energies, we show the existence of these configurations for some environment of the dynamical system. Furthermore, in one dimension, we give sufficient conditions on the family of interaction energies to ensure, for any environment, the existence of calibrated configurations when the underlying dynamics is uniquely ergodic.

16:30-17:00. Coffee Break

17:00-17:50 Jean-Michel Roquejoffre (Université Paul Sabatier): *Uniqueness in a class of Hamilton-Jacobi equations with constraints.*

The model under study is a time-dependent Hamilton-Jacobi equation, which incorporates a tuning function whose role is to keep the maximal value of the unknown at the constant value 0. The main result is that the full problem has a unique classical solution. The motivation is the singular limit of a selection-mutation model in population dynamics, which exhibits concentration on the zero level set of the solution of the Hamilton-Jacobi equation. The uniqueness result implies strong convergence and error estimates for the selection-mutation model. Joint work with S. Mirrahimi

18:00-19:00. Discussion

19:30 Dinner at Restaurant Gamboni

Friday November 27, 2015. Salle 435

9:30–10:20. Valentine Roos (ENS Paris): *Viscosity and variational solutions of the evolutive Hamilton-Jacobi equation.*
Abstract: Two different notions of weak solutions were introduced for the evolutive Hamilton-Jacobi equation, which coincide when the Hamiltonian is convex in the fiber. The aim of this talk is to compare them for non-convex Hamiltonians. Using the entropy characterization of viscosity solutions, it is possible to determine if the weak solutions coincide during a short time, for simple initial data in dimension 1 and 2.

10:30-11:00. Coffee Break

11:00–11:50. Alessio Figalli (University of Texas at Austin): *An overview of obstacle problems*
Abstract: Obstacle problems are an important class of free-boundary problems.t The general setting is the following: Given an unknown pair (u, Ω) where u solves a "nice" equation inside Ω and has some behavior across $\partial\Omega$, understand the regularity of both u and $\partial\Omega$. The aim of this talk is to give an overview of some classical and more recent results in the area, and to present some open problems. In particular, in the end of the talk, I plan to mention a recent direction which consists in considering obstacle problems where the obstacle is not given but it is only known to solve a Hamilton-Jacobi equation.

12:00. Final discussion followed by Lunch