exceptional geometry and string compactifications

Henning Samtleben

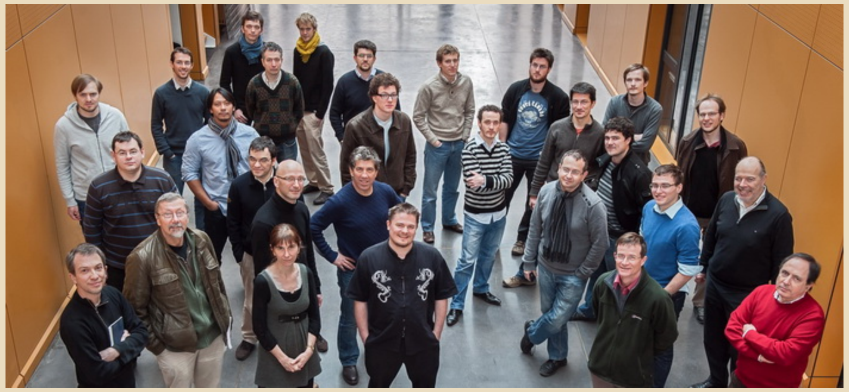
ENS de Lyon meets **SISSA**
Lyon 12/2017

Lab PHYS
ENS de LYON

Theoretical physics group (équipe 4) at Laboratoire de physique, ENS de Lyon

head of group: Jean-Michel Maillet

- condensed matter theory
- statistical physics
- mathematical physics



Theoretical physics group (équipe 4)

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- condensed matter theory
- statistical physics
- mathematical physics

faculty

Angel Alastuey
Jeremie Bouttier
David Carpentier
Pascal Degiovanni
François Delduc
Pierre Delplace
Andrey Fedorenko
Krzysztof Gawedzki
Peter Holdsworth
Karol Kozlowski
Etera Livine
Marc Magro
Jean Michel Maillet
Giuliano Niccoli
Edmond Orignac
Tommaso Roscilde
Henning Samtleben
Lucile Savary

postdocs

Marco Marciani
Takashi Kameyama

PhD students

Clément Cabart
Christophe Goeller
Savish Goomanee
Callum Gray
Yannick Herfray
Sylvain Lacroix
Thibaud Louvet
Raphaël Menu
Baptiste Pezelier
Valentin Raban
Benjamin Roussel
Jérôme Thibaut
Lavi Kumar Upreti

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condensed matter theory

- topological matter: topological insulators, topological interacting phases, Dirac phases, hydrodynamics, topological superconductivity, dynamical systems
- strong correlations in boson and fermion systems: Tomonaga-Luttinger liquids, Mott transition
- relativistic phases in condensed matter: graphene, Weyl/Dirac semimetals, quantum transport, effects of disorder
- quantum magnetism: frustrated systems, spin ladders, magnetic monopole quasi-particles, Coulomb and topological phase transitions
- Bose-Einstein condensation: long-range effects
- mesoscopic physics: quantum nanoelectronics, electron quantum optics, decoherence, quantum technologies
- non-equilibrium quantum many-body systems: quantum quenches, correlation spreading, many-body localization
- quantum correlations in many-body systems: entanglement and beyond

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statistical physics

- disordered systems: functional renormalization group, random field systems, elastic manifolds, depinning
- macroscopic fluctuation theory
- critical Casimir forces, magnetic, fluid and quantum systems
- emergent electrodynamics: lattice gauge theories for spin systems
- solvable lattice models and their connections with enumerative/algebraic combinatorics
- quantum plasmas: path integrals, recombination, ionic criticality

+ related activity in (all) other groups

Theoretical physics group (équipe 4)
at Laboratoire de physique, ENS de Lyon

mathematical physics

- integrable systems: quantum separation of variables and correlation functions, quantum critical models, integrable probability
- asymptotic analysis: multiple integrals, Riemann–Hilbert problems
- conformal field theory: nonequilibrium CFT
- $AdS_5 \times S^5$ string theory: integrable deformations of (string) sigma models
- supersymmetric field theories: 6D SCFT, M5 branes, higher gauge theories
- supergravity: supersymmetry on curved space, duality symmetries
- quantum gravity: TQFTs, discrete path integrals, holography & entanglement, random maps and 2DQG

exceptional geometry and string compactifications

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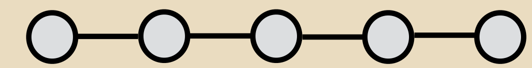
Lab PHYS
ENS de LYON

based on work with
O. Hohm (MIT), C. Pope (Texas A&M),
A. Baguet, M. Magro (ENSL)

outline

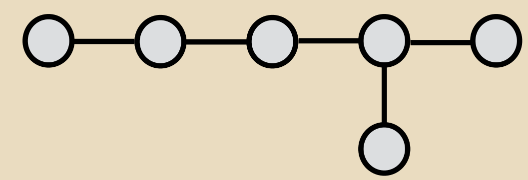
gravity and extended geometry

A) Kaluza-Klein theory & Riemannian geometry



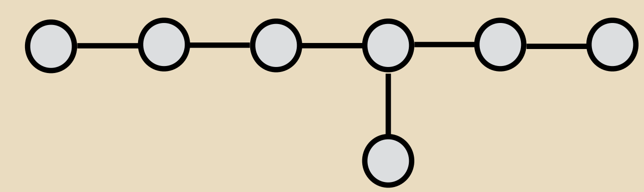
1919 : extra dimensions in Einstein's general relativity: $D = 4 + 1$

D) String theory & generalized geometry



1970's, then 2000 – : $D = 10 + \mathbf{10}$

E) M theory & exceptional geometry



1980's, then 2010 – : $D = 11 + \mathbf{??}$

applications

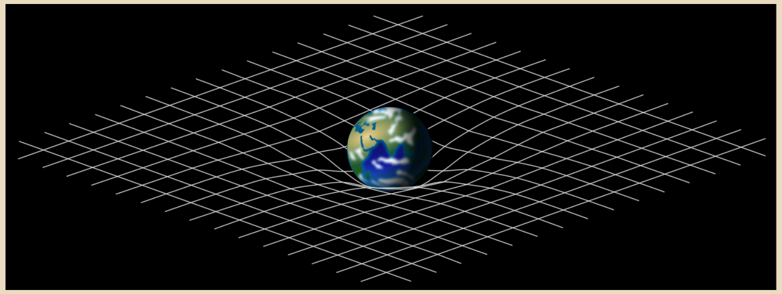
- string compactifications
- integrability and modified IIB supergravity

Einstein's general relativity (1915)

Riemannian geometry

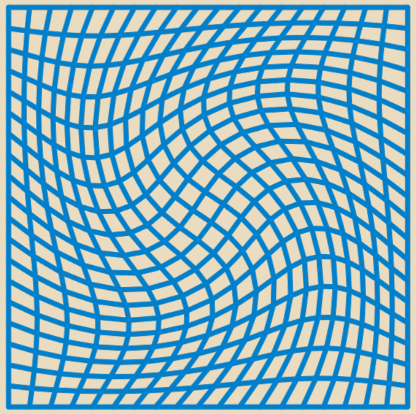
space-time metric $g_{\mu\nu}$

$$\begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix}$$



fundamental symmetry: space-time diffeomorphisms ξ^μ

$$\mathcal{L}_\xi V^\mu \;=\; [\xi, V]^\mu \;=\; \xi^\nu \partial_\nu V^\mu - \partial_\nu \xi^\mu V^\nu$$



dynamics: Einstein-Hilbert action

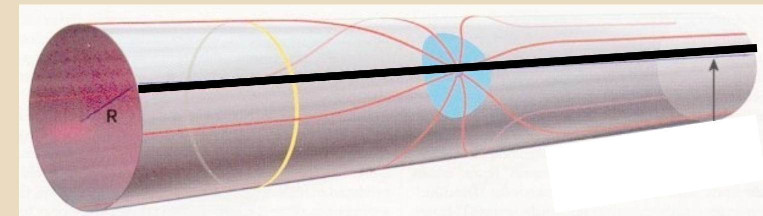
$$S \;=\; \int dx \sqrt{|\det g|}\, R[g]$$ and possible matter couplings

with straightforward generalisation to N space-time dimensions

A) Kaluza-Klein theory (1919)

general relativity in $N+1$ space-time dimensions $\quad (N=4)$

$$G_{MN} = \left(\begin{array}{c|c} e^{a\phi} g_{\mu\nu} + e^{\phi} A_\mu A_\nu & e^{\phi} A_\mu \\ \hline e^{\phi} A_\mu & e^{\phi} \end{array}\right)$$



dynamics $\quad S = \int d^{N+1}x \, \sqrt{|\det G|} \, R[G]$

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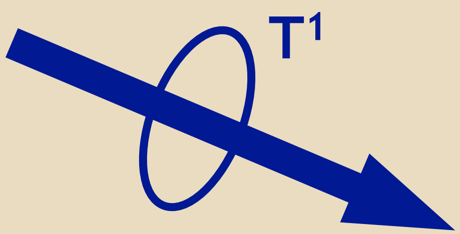
after compactification of the extra dimension:



$$\phi(x,y) \; = \; \sum_{n\in\mathbb{Z}} e^{iny/R} \, \phi_n(x) \; \longrightarrow \; \phi_0(x) \qquad \text{etc.}$$

N-dimensional general relativity with matter:

metric, gauge potential, dilaton $\{g_{\mu\nu}, A_\mu, \phi\}$

$$S = \int d^N x \sqrt{|\det g|} \Big(R - \frac{1}{2} \partial_\mu\phi \, \partial^\mu\phi - \frac{1}{4} \, e^{\alpha\phi} \, F_{\mu\nu} F^{\mu\nu} \Big)$$

Einstein—Maxwell—dilaton theory

A) Kaluza-Klein theory (1919)

general relativity in $N+1$ space-time dimensions

$$G_{MN} = \left(\begin{array}{c|c} e^{a\phi} g_{\mu\nu} + e^{\phi} A_\mu A_\nu & e^{\phi} A_\mu \\ \hline e^{\phi} A_\mu & e^{\phi} \end{array}\right) \quad \in \quad \mathrm{GL}(D+1)$$

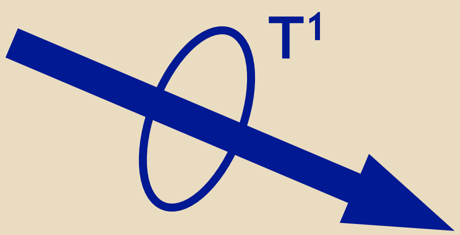
"geometrization of gauge symmetry"

$$\xi^M \;=\; \{\xi^\mu, \Lambda\}$$

dynamics $\quad S = \int d^{N+1}x \, \sqrt{|\det G|} \, R[G]$

---

after compactification of the extra dimension:



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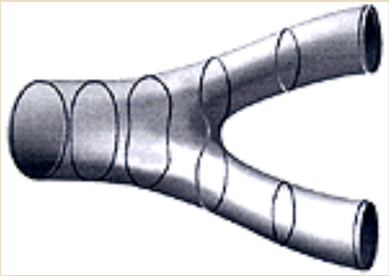
Einstein—Maxwell—dilaton theory

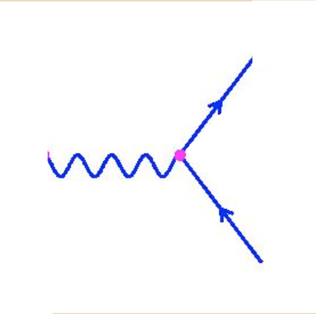
fundamental symmetries: space-time diffeomorphisms ξ^μ , gauge transformations Λ

D) string theory & generalized geometry

1970's: string theory — theory of extended objects (in ten space-time dimensions)



reproducing gravitational and gauge interactions



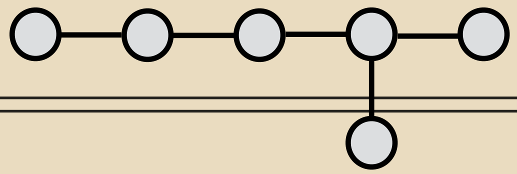
universal sector: Einstein—Kalb-Ramond—dilaton $\{\, g_{\mu\nu}\,,\, B_{\mu\nu}\,,\, \phi\,\}$

$$S = \int dx \sqrt{|g|}\, e^{-2\phi} \left(R + 4\,\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{12}\, H^{\mu\nu\rho}\, H_{\mu\nu\rho} \right)$$

with the three-form flux $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$

fundamental symmetries: space-time diffeomorphisms ξ^μ , gauge transformations Λ_μ

D) string theory & generalized geometry



“geometrization of gauge symmetry”

$\{\xi^\mu, \Lambda_\mu\}$

??

universal sector: Einstein—Kalb-Ramond—dilaton $\{\, g_{\mu\nu}\,,\, B_{\mu\nu}\,,\, \phi \,\}$

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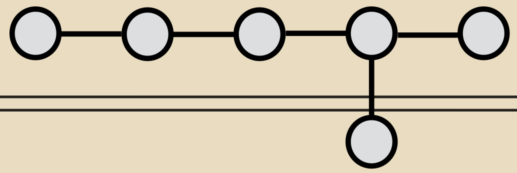
> Kaluza-Klein question :
>
> can this structure be embedded in some “higher-dimensional geometry” ..?
>
> with $\{\xi^\mu, \Lambda_\mu\}$ combining into some “higher-dimensional diffeomorphism” ..?

D) string theory & generalized geometry



physics $\longrightarrow$ **double field theory**

W. Siegel (1993), C. Hull, B. Zwiebach, O. Hohm (2009), ...

$$G_{MN} = \begin{pmatrix} g^{\mu\nu} & -g^{\mu\rho}B_{\rho\nu} \\ B_{\mu\rho}g^{\rho\nu} & g_{\mu\nu} - B_{\mu\rho}g^{\rho\sigma}B_{\sigma\nu} \end{pmatrix} \in \mathrm{SO}(D,D)$$

generalized geometry $\longleftarrow$ mathematics

N. Hitchin, M. Gualtieri (2003), ...

---

universal sector: Einstein—Kalb-Ramond—dilaton $\{\, g_{\mu\nu}\,,\, B_{\mu\nu}\,,\, \phi\,\}$

$$S = \int dx \sqrt{|g|}\, e^{-2\phi} \left(R + 4\,\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{12}\,H^{\mu\nu\rho}\,H_{\mu\nu\rho}\right)$$

Kaluza-Klein question :

can this structure be embedded in some “higher-dimensional geometry” ..?

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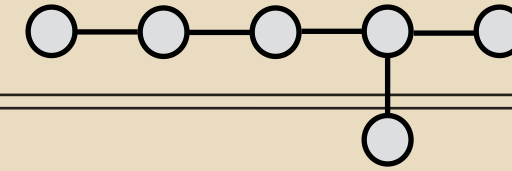
D) string theory & generalized geometry



physics $\longrightarrow$ **double field theory**

generalized geometry $\longleftarrow$ mathematics

$$G_{MN} \;=\; \begin{pmatrix} g^{\mu\nu} & -g^{\mu\rho}B_{\rho\nu} \\ B_{\mu\rho}g^{\rho\nu} & g_{\mu\nu} - B_{\mu\rho}g^{\rho\sigma}B_{\sigma\nu} \end{pmatrix} \;\in\; \mathrm{SO}(D,D)$$

generalized metric: $2D$–dimensional "space" (with a section condition)

generalized diffeomorphisms: unifying $\xi^M \;=\; \{\xi^\mu, \Lambda_\mu\}$

$$\mathcal{L}_\xi V^M \;=\; \xi^N \partial_N V^M - [\partial\,\xi]_{\mathbb{P}}{}^M{}_N\, V^N$$

– compatible with the $\mathrm{SO}(D,D)$ group structure

– closure requires a section condition on the fields: $\eta^{MN}\,\partial_M \otimes \partial_N \;=\; 0$

fields live on D–dimensional slices in the $2D$–dimensional "space"

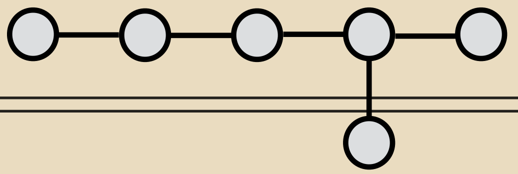
Dorfmann bracket on the generalized tangent bundle $TM \oplus T^*M$

generalized connections and curvature:

– vanishing of the generalized torsion tensor does not fully determine the connection

– notion of a generalized Ricci tensor and Ricci scalar, no generalized Riemann tensor

D) string theory & generalized geometry



physics ⟶ **double field theory**

generalized geometry ⟵ mathematics

$$G_{MN} \;=\; \begin{pmatrix} g^{\mu\nu} & -g^{\mu\rho}B_{\rho\nu} \\ B_{\mu\rho}g^{\rho\nu} & g_{\mu\nu} - B_{\mu\rho}g^{\rho\sigma}B_{\sigma\nu} \end{pmatrix} \;\in\; \mathrm{SO}(D,D)$$

- unified “geometrical” action $S \;=\; \int dx^{2D}\, e^{-2\Delta}\, \mathcal{R}[G,\Delta]$ generalized Ricci scalar

- D–dimensional slices in the D+D–dimensional “space”

  momentum coordinates and dual winding coordinates

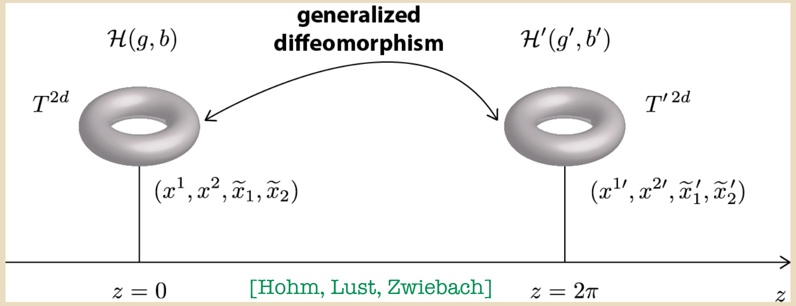
  T-duality covariant formulation

- SO(D,D) covariance of the equations: compact reduction formulas

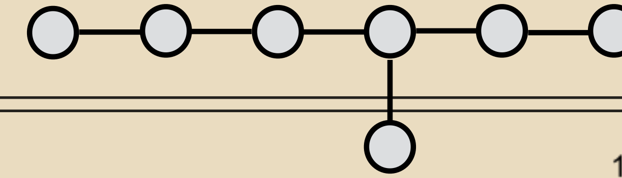
  $$G^{MN}(x,Y) \;=\; U_K{}^M(Y)\, U_L{}^N(Y)\, g^{KL}(x)$$ generalized frame field $U_K{}^M(Y)$

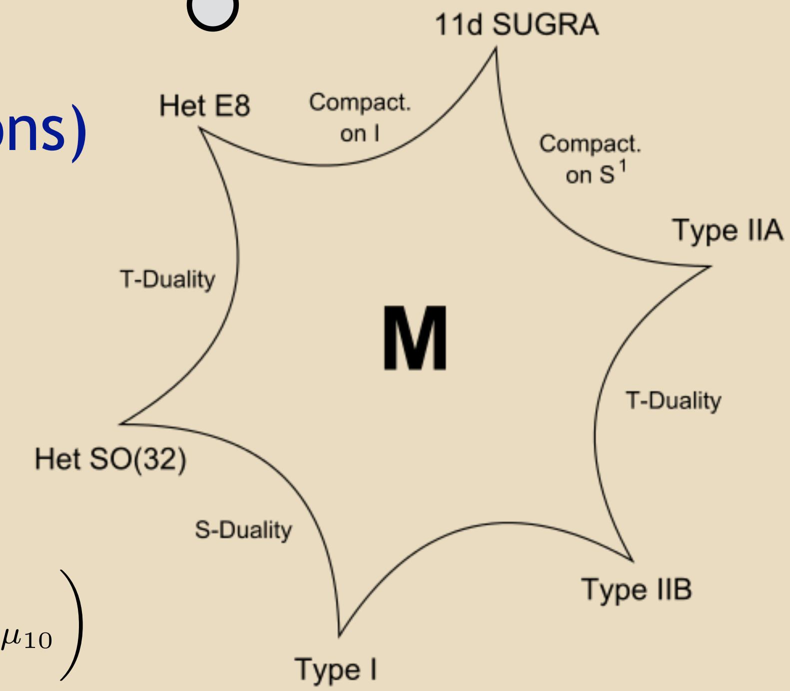
- space for non-geometric compactifications

  patching coordinates and dual coordinates



E) M theory & exceptional geometry





theory of strings and branes (in eleven space-time dimensions) unifying the various string theories

low energy theory: $D = 11$ supergravity $\{g_{\mu\nu}, A_{\mu\nu\rho}\}$

$$\int d^{11}x \left(\sqrt{|g|}\, R - \frac{1}{12} \sqrt{|g|}\, F_{\mu\nu\rho\sigma}\, F^{\mu\nu\rho\sigma} + \frac{1}{2\cdot 36^2}\, \varepsilon^{\mu_0\mu_1\dots\mu_{10}}\, F_{\mu_0\mu_1\mu_2\mu_3}\, F_{\mu_4\mu_5\mu_6\mu_7}\, A_{\mu_8\mu_9\mu_{10}} \right)$$

with the four-form flux $F_{\mu\nu\rho\sigma} = \partial_\mu A_{\nu\rho\sigma} - \partial_\nu A_{\rho\sigma\mu} + \partial_\rho A_{\sigma\mu\nu} - \partial_\sigma A_{\mu\nu\rho}$

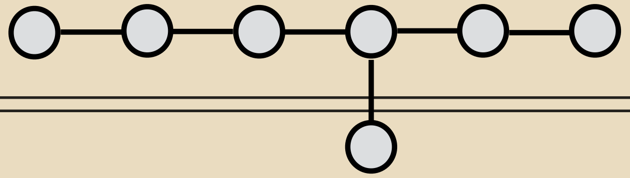
fundamental symmetries: space-time diffeomorphisms ξ^μ , gauge transformations $\Lambda_{\mu\nu}$

can this structure be embedded in some “higher-dimensional geometry” ..?
with $\{\xi^\mu, \Lambda_{\mu\nu}\}$ combining into some “higher-dimensional diffeomorphism” ..?

$\longrightarrow$ **exceptional geometry / exceptional field theory**

[Hull, Berman, Perry, Waldram, Coimbra, Strickland-Constable, Thompson, West, Godazgar, Cederwall, Aldazabal, Grana, Marques, Rosabal, Hohm, H.S., ...]

E) M theory & exceptional geometry



→ **exceptional geometry / exceptional field theory**

generalized metric: E_7: (4+56)–dimensional "space" with a section condition

$$\mathcal{M}_{MN} \;=\; \begin{pmatrix} g^{mk} & g^{mp}\,A_{pkl} & \dots \\ A_{mnp}\,g^{pn} & \dots & \\ \dots & & \end{pmatrix} \;\in\; \mathrm{E}_{7(7)}$$

... combines with dual magnetic fields ...

generalized diffeomorphisms: unifying $\xi^M \;=\; \{\xi^\mu, \Lambda_{\mu\nu}, \dots\}$

$$\mathcal{L}_\xi V^M \;=\; \xi^N \partial_N V^M - [\partial\,\xi]_{\mathbb{P}}{}^M{}_N\, V^N$$

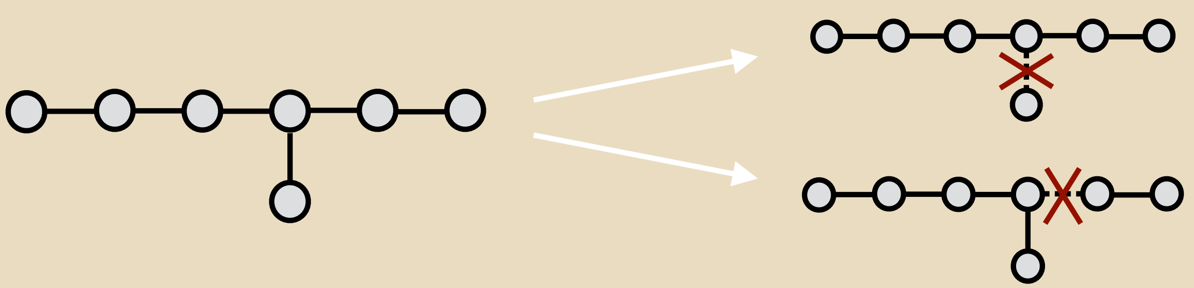
extended Dorfmann bracket on the *extended* tangent bundle

– compatible with the $\mathrm{E}_{D(D)}$ group structure

– closure requires a section condition on the fields: $Y_{KL}{}^{MN}\,\partial_M \otimes \partial_N \;=\; 0$

- fields live on (4+7) / (4+6)–dimensional slices
- inequivalent slices describe IIA and IIB supergravity, respectively

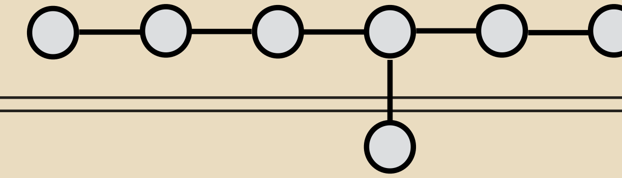
c.f. embedding of A_D subgroups



IIA

IIB

E) M theory & exceptional geometry



→ exceptional geometry / exceptional field theory

generalized metric: E_7: (4+56)–dimensional "space" with a section condition

generalized diffeomorphisms: $\mathcal{L}_\xi V^M = \xi^N \partial_N V^M - [\partial\,\xi]_{\mathbb{P}}{}^M{}_N V^N$

generalized torsion, connection, curvature (analogue of Ricci tensor)

unique action invariant under generalized diffeomorphisms:

$E_{7(7)}$ covariant exceptional field theory

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

$D = 4+56$ with section condition

four-dimensional field theory with fields in infinite-dimensional representations, and infinite-dimensional gauge structure

- upon solving the section condition: reformulation of the original theories
- IIA and IIB supergravity accommodated in the same framework

manifestly duality covariant formulation of maximal supergravity

$E_{7(7)}$ covariant exceptional field theory

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

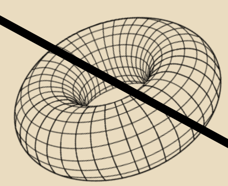
$D = 4+56$ with section condition

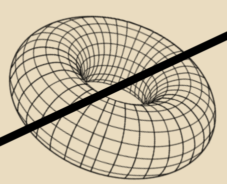
GL(7) solution to section condition

GL(6) solution to section condition

D=11 sugra

IIB sugra





T^7

T^6

D=4 maximal sugra

global $E_{7(7)}$

reduction of these theories to D=4 dimensions
yields maximal supergravity with (hidden) global $E_{7(7)}$ symmetry

[Cremmer, Julia 1979]

$E_{7(7)}$: exceptional field theory — applications

manifestly duality covariant formulation of maximal supergravity

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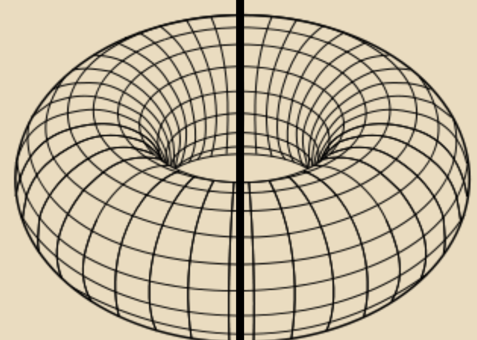
$D = 4+56$ with section condition

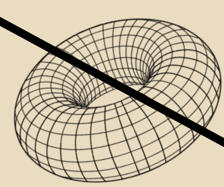
GL(7) solution to section condition

GL(6) solution to section condition

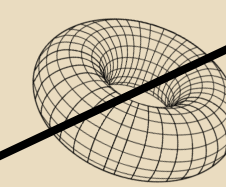


D=11 sugra

IIB sugra



T^7



T^6

D=4 maximal sugra

global $E_{7(7)}$

reduction of these theories to D=4 dimensions

yields maximal supergravity with ~~(hidden)~~ global $E_{7(7)}$ symmetry

exceptional field theory explains the symmetry enhancement

$E_{7(7)}$: exceptional field theory — applications

manifestly duality covariant formulation of maximal supergravity

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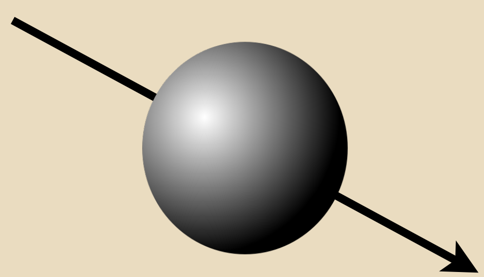
$D = 4+56$ with section condition

GL(7) solution to section condition

D=11 sugra



$S^7 \times AdS_4$

D=4 maximal sugra

gauge group SO(8)

also allows a compact description of non-trivial reductions

$E_{7(7)}$: exceptional field theory — applications

manifestly duality covariant formulation of maximal supergravity

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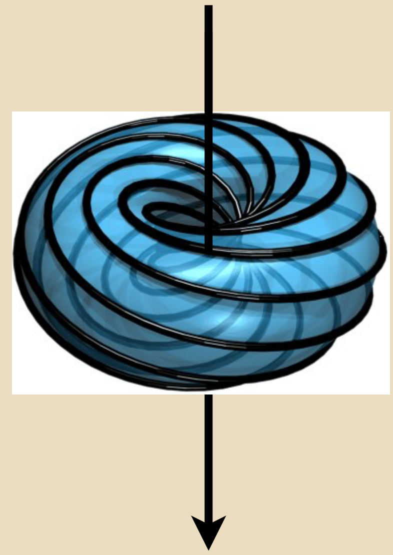
$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^{N}_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

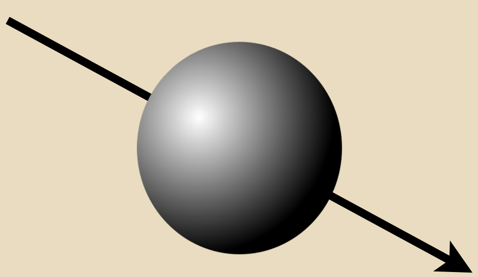
$D = 4+56$ with section condition

GL(7) solution to section condition

D=11 sugra



captured by a twisted torus (Scherk-Schwarz) reduction of ExFT



$S^7 \times AdS_4$

D=4 maximal sugra

gauge group SO(8)

[Kaloper, Myers, Dabholkar, Hull, Reid-Edwards, Dall'Agata, Prezas, HS, Trigiante, Hohm, Kwak, Aldazabal, Baron, Nunez, Marques, Geissbuhler, Grana, Berman, Musaev, Thompson, Rosabal, Lee, Strickland-Constable, Waldram, Dibitetto, Roest, Malek, Blumenhagen, Hassler, Lust, Cho, Fernández-Melgarejo, Jeon, Park, Guarino, Varela, Inverso, Ciceri, ..., ...]

also allows a compact description of non-trivial reductions

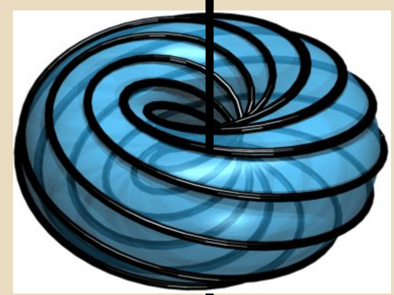
$E_{7(7)}$: exceptional field theory — applications

$E_{7(7)}$ covariant exceptional field theory

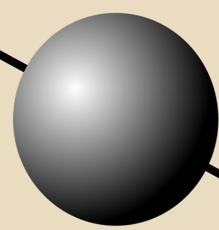
$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

GL(7) solution to section condition

D=11 sugra



$$\begin{aligned}
\mathcal{M}_{MN}(x,Y) &= U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y) \\
\mathcal{A}_\mu{}^M(x,Y) &= \rho^{-1}(Y)\, (U^{-1})_K{}^M(Y)\, A_\mu{}^K(x) \\
\mathcal{B}_{\mu\nu\,\alpha}(x,Y) &= \rho^{-2}(Y)\, U_\alpha{}^\beta(Y)\, B_{\mu\nu\,\beta}(x)
\end{aligned}$$



D=4 maximal sugra

gauge group SO(8)

- reduction via generalized Scherk-Schwarz ansatz in ExFT

  in terms of an $E_{7(7)}$–valued twist matrix $U_M{}^N(Y)$ and scale factor $\rho(Y)$

  - system of consistency equations $\left[(U^{-1})_M{}^P (U^{-1})_N{}^L\, \partial_P U_L{}^K\right]_{\mathbf{912}} \stackrel{!}{=} \rho\, X_{MN}{}^K$
  - generalized parallelizability
  - no general classification of its solutions (Lie algebras vs Leibniz algebras)

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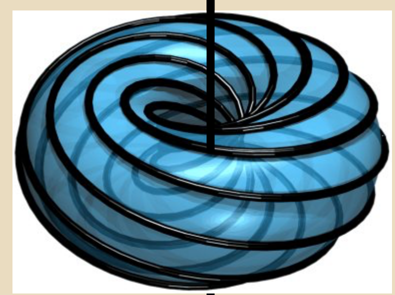
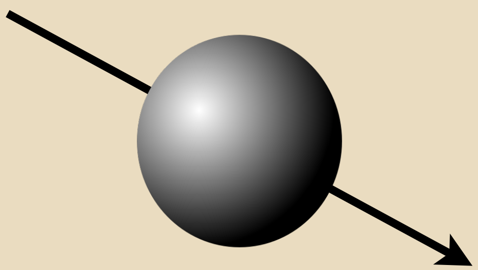
GL(7) solution to section condition



$$\begin{aligned} \mathcal{M}_{MN}(x,Y) &= U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y) \\ \mathcal{A}_\mu{}^M(x,Y) &= \rho^{-1}(Y)\, (U^{-1})_K{}^M(Y)\, A_\mu{}^K(x) \\ \mathcal{B}_{\mu\nu\,\alpha}(x,Y) &= \rho^{-2}(Y)\, U_\alpha{}^\beta(Y)\, B_{\mu\nu\,\beta}(x) \end{aligned}$$

D=11 sugra



D=4 maximal sugra
gauge group SO(8)

- twist matrix $U \in \mathrm{SL}(8)$ associated to SO(8) structure constants

coordinates $\quad \underset{56}{\{Y^M\}} \longrightarrow \underset{28+28}{\{Y^{AB}, Y_{AB}\}} \longrightarrow \underset{7\;+21+28}{\{Y^{i8}, Y^{ij}, Y_{AB}\}}$

twist matrix $\quad U = \begin{pmatrix} \delta_i{}^j\, a(y^2) & y^i\, b(y^2) \\ y^i\, c(y^2) & d(y^2) \end{pmatrix} \qquad y^i \equiv Y^{i8} \qquad y^2 \equiv y^i y^i$

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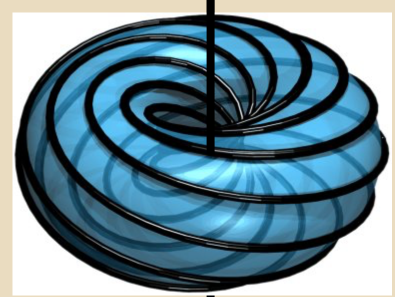
$E_{7(7)}$ covariant exceptional field theory

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\rm top} - V(\mathcal{M}, e).$$

dictionary

GL(7) solution to section condition



$$\begin{aligned}
\mathcal{M}_{MN}(x,Y) &= U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y) \\
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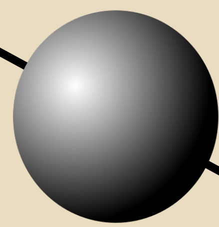
D=11 sugra



D=4 maximal sugra

gauge group SO(8)

- encodes complicated reduction formulas for the original fields

$$ds^2 = \Delta^{-2/3}(x,y)\, \mathrm{g}_{\mu\nu}(x)\, dx^\mu dx^\nu + G_{mn}(x,y)\, \left(dy^m + \mathcal{K}_{[ab]}{}^m(y) A_\mu^{ab}(x) dx^\mu\right) \left(dy^n + \mathcal{K}_{[cd]}{}^n(y) A_\nu^{cd}(x) dx^\nu\right)$$

$$G^{mn}(x,y) = \Delta^{2/3}(x,y)\, \mathcal{K}_{[ab]}{}^m(y) \mathcal{K}_{[cd]}{}^n(y)\, M^{ab,cd}(x)$$

$$C_{klmn} = \tilde{C}_{klmn} + \frac{1}{16} \tilde{\omega}_{klmnp} \Delta^{4/3} m_{\alpha\beta} \tilde{G}^{pq} \partial_q (\Delta^{-4/3} m^{\alpha\beta}),$$

$$C_{\mu kmn} = \frac{\sqrt{2}}{4} \mathcal{Z}_{[ab]kmn} A_\mu{}^{ab},$$

$$C_{\mu\nu mn} = \frac{\sqrt{2}}{4} \mathcal{K}_{[ab]}{}^k \mathcal{Z}_{[cd]kmn} A_{[\mu}{}^{ab} A_{\nu]}{}^{cd},$$

$$C_{m\mu\nu\rho} = -\frac{1}{32} \mathcal{K}_{[ab]m} \left(2\sqrt{|\mathrm{g}|} \varepsilon_{\mu\nu\rho\sigma\tau} M_{ab,N} F^{\sigma\tau N} + \sqrt{2} \varepsilon_{abcdef} \Omega^{cdef}_{\mu\nu\rho}\right) - \frac{1}{4} \sqrt{2} \mathcal{K}_{[ab]}{}^k \mathcal{K}_{[cd]}{}^l \mathcal{Z}_{[ef]mkl} (A_{[\mu}{}^{ab} A_\nu{}^{cd} A_{\rho]}{}^{ef}),$$

$$\begin{aligned}
C_{\mu\nu\rho\sigma} = &-\frac{1}{16} \mathcal{Y}_a \mathcal{Y}^b \left(\sqrt{|\mathrm{g}|} \varepsilon_{\mu\nu\rho\sigma\tau} D^\tau M_{bc,N} M^{Nca} + 2\sqrt{2} \varepsilon_{cdefgb} F_{[\mu\nu}{}^{cd} A_\rho{}^{ef} A_{\sigma]}{}^{ga}\right) \\
&+ \frac{1}{4} \left(\sqrt{2} \mathcal{K}_{[ab]}{}^k \mathcal{K}_{[cd]}{}^l \mathcal{K}_{[ef]}{}^n \mathcal{Z}_{[gh]kln} - \mathcal{Y}_h \mathcal{Y}^j \varepsilon_{abcegj} \eta_{df}\right) A_{[\mu}{}^{ab} A_\nu{}^{cd} A_\rho{}^{ef} A_{\sigma]}{}^{gh} + \Lambda_{\mu\nu\rho\sigma}(x).
\end{aligned}$$

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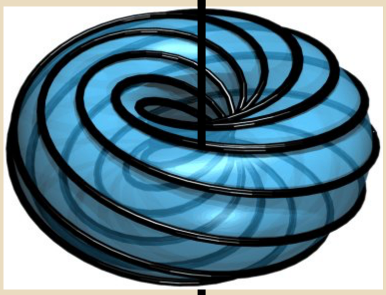
GL(7) solution to section condition

D=11 sugra



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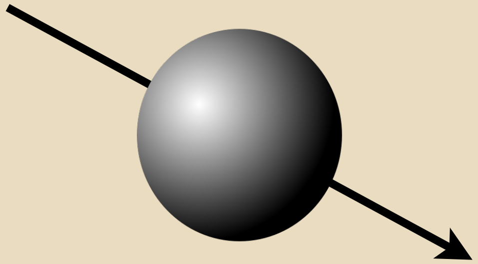
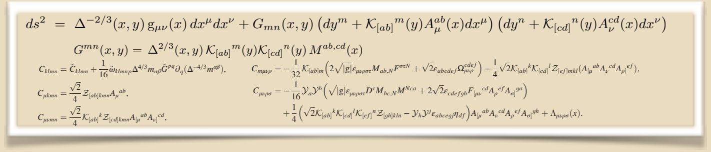


D=4 maximal sugra

gauge group SO(8)

- encodes complicated reduction formulas for the original fields
  - solves many old problems on *consistent truncations*
  - IIB supergravity on $AdS_5 \times S^5$ (holography)
  - heterotic string on group manifolds (string vacua)



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truncate the equations of D=11 supergravity to a lower-dimensional theory, such that any solution to the truncated equations defines a solution of D=11 supergravity

typically: $\longrightarrow$ commuting Killing vectors (tori)

around curved backgrounds: non-trivial!
(in general: impossible)

D

$$\mathcal{M}_{MN}(x,Y) = U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y)$$

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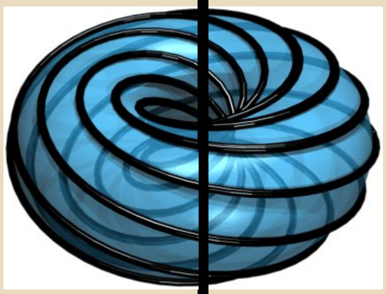

D=4 maximal sugra

gauge group SO(8)

- similar: twist matrix $U \in \mathrm{SL}(8)$ associated to $\mathrm{SO}(p,q)$ and $\mathrm{CSO}(p,q,r)$ structure constants

  built from Killing vectors on SO(p,q)/SO(p–1,q)

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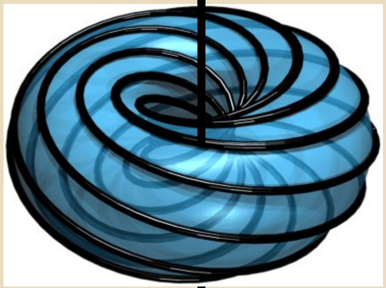

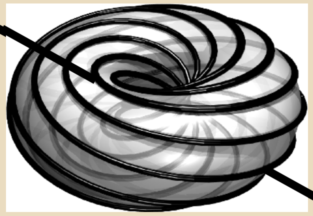

D=4 maximal sugra
gauge group SO(p,q)

D=4 maximal sugra
gauge group SO(8)

D=4 maximal sugra
gauge group CSO(p,q,r)

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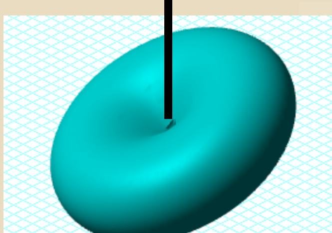
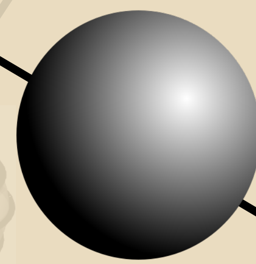
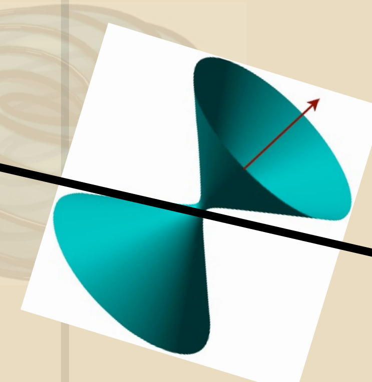
dictionary

D=11 sugra







D=4 maximal sugra
gauge group SO(p,q)

D=4 maximal sugra
gauge group SO(8)

D=4 maximal sugra
gauge group CSO(p,q,r)

- similar: twist matrix $\mathring{U} \in \mathrm{SL}(8)$ associated to $\mathrm{SO}(p,q)$ and $\mathrm{CSO}(p,q,r)$ structure constants
  - background: (warped) hyperboloids
  - new compactifications and solutions
  - spectra and moduli spaces

conclusion

generalized / exceptional geometry

- geometrization of gauge interactions

exceptional field theory

- unique theory with generalized diffeomorphism invariance in all coordinates (modulo section condition)
- upon an explicit solution of the section condition the theory reproduces full D=11 and full D=10 IIB supergravity
- reproduces the *modified* supergravities arising in the study of integrable deformations of string sigma models [with M. Magro]
- new tools for
  - supersymmetric backgrounds
  - consistent truncations
  - moduli spaces
  - non-geometric compactifications

outlook — what's next ?

further applications

- systematics of compactifications (integration of Leibniz algebras)
- higher order corrections (counterterms of N=8 supergravity)
  candidat for a finite theory of quantum gravity [Bern,Dixon,Roiban, ...]
- loop calculations
  duality invariant graviton amplitudes [Bossard,Kleinschmidt]

hints towards a more fundamental formulation

- infinite-dimensional extensions
- inclusion of massive string modes
- underlying symmetries of string and M-theory

other exceptional groups

- $E_6: \ (5+27)$–dimensional space with section condition
- $E_7: \ (4+56)– \quad \dots$
- $E_8: \ (3+248)– \quad \dots$
- $E_9: \ (2+\infty)– \quad \dots$ [Bossard,Cederwall,Kleinschmidt,Palmkvist,HS]
- ... [Damour,Henneaux,Nicolai] [West]