TD03 - Riemannian metric, isometries

Exercise 1 (Standard metrics)**.** Let g_0 denote the standard Riemannian metric on $\mathbb{R}^n$, that is for all $x \in \mathbb{R}^n$, $(g_0)_x$ is the standard Euclidean inner product.

1. Are the translations isometries of $(\mathbb{R}^n, g_0)$? Define a natural Riemannian metric on the flat torus $\mathbb{T}^n = \mathbb{R}^n/\mathbb{Z}^n$.
2. Let $i : \mathbb{S}^n \to \mathbb{R}^{n+1}$ denote the canonical inclusion, we set $g = i^*(g_0)$. Do elements of the orthogonal group $O_{n+1}(\mathbb{R})$ induce isometries of $(\mathbb{S}^n, g)$? Define a natural Riemannian metric on the real projective space $\mathbb{RP}^n$.

Exercise 2 (Dimension 1)**.**

1. Let M be a connected smooth manifold without boundary of dimension 1. Are any two Riemannian metrics on M conformal to one another?
2. Same question if $\dim(M) \geqslant 2$.
3. What are the connected Riemannian manifolds without boundary of dimension 1 up to isometries?