Contrôle 2
(MIASHS)
(PC-SV)

06/11/15

Exercice 1:

Division euclidienne:

$$\begin{array}{rrrrr|l}
2x^6 & -3x^4 & & +2x & -1 & x^3+1 \\
-(2x^6 & & +2x^3 & &) & 2x^3-3x-2 \\
\hline
 & -3x^4 & -2x^3 & +2x & -1 & \\
 & -(-3x^4 & & -3x &) & \\
\hline
 & & -2x^3 & +5x & -1 & \\
 & & -(-2x^3 & & -2) & \\
\hline
 & & & 5x & +1 &
\end{array}$$

On vérifie:

$$(2x^3-3x-2)(x^3+1)+(5x+1) = 2x^6+\cancel{2x^3}-3x^4-3x-\cancel{2x^3}-2+5x+1$$
$$= 2x^6-3x^4+2x-1$$

Donc $\boxed{f(x) = (2x^3-3x-2)g(x)+(5x+1).}$

Exercice 2:

$f :]-\infty, 0[\to \mathbb{R}$, $f(x) = \left(\frac{1}{3-2x}\right)^x$

On réécrit f:

$$f(x) = \left(\frac{1}{3-2x}\right)^x = e^{x\ln\left(\frac{1}{3-2x}\right)} = e^{x(\underbrace{\ln 1}_{0}-\ln(3-2x))} = e^{-x\ln(3-2x)}$$

Alors f est dérivable sur $]-\infty,0[$ et est de la forme e^u, où $u = v \times w$, avec

$w(x) = \ln(3-2x) \to w'(x) = \frac{(-2x+3)'}{3-2x} = \frac{-2}{3-2x}$

$v(x) = -x \quad \to \quad v'(x) = -1$

Du coup,
$u'(x) = v'(x)w(x) + v(x)w'(x) = -1\times\ln(3-2x) + (-x)\times\left(\frac{-2}{3-2x}\right) = \frac{2x}{3-2x} - \ln(3-2x)$

D'où

$$f'(x) = u'(x)e^{u(x)} = \boxed{\left(\frac{2x}{3-2x} - \ln(3-2x)\right)e^{-x\ln(3-2x)} = f'(x)}$$

Exercice 3:

Résoudre dans $\mathbb{R}$:

$$\cos(x+\pi) = 2\sin^2 x - 1$$

On sait que $\forall x \in \mathbb{R}$,

$$\cos(x+\pi) = \cos(x)\underbrace{\cos(\pi)}_{=-1} - \sin(x)\underbrace{\sin(\pi)}_{=0} = -\cos x$$

Donc

$$\cos(x+\pi) = 2\sin^2 x - 1 \iff -\cos x = 2\sin^2 x - 1 \quad (E)$$

On $\forall x \in \mathbb{R}$, $\sin^2 x = 1 - \cos^2 x$. Donc

$$(E) \iff -\cos x = 2(1-\cos^2 x) - 1$$

$$\iff 2\cos^2 x - \cos x - 1 = 0$$

On pose $X = \cos x$. On cherche les racines du trinôme $2X^2 - X - 1$:

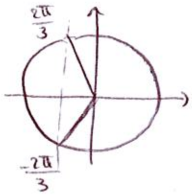
$\Delta = 1 + 4 \times 2 = 9 > 0 \longrightarrow$ 2 racines réelles simples

$$X_1 = \frac{1-3}{4} = -\frac{1}{2} \quad \text{et} \quad X_2 = \frac{1+3}{4} = 1$$

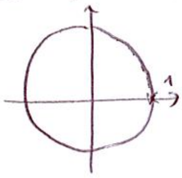
Alors les solutions sont les x tels que

* $\cos x = X_1 = -\frac{1}{2}$ donc $x = \frac{2\pi}{3} + 2k\pi$, $k \in \mathbb{Z}$

ou $x = -\frac{2\pi}{3} + 2k\pi$, $k \in \mathbb{Z}$



* $\cos x = X_2 = 1$ donc $x = 2k\pi$, $k \in \mathbb{Z}$



Finalement,

$$\boxed{\mathcal{S} = \left\{ -\frac{2\pi}{3} + 2k\pi,\ \frac{2\pi}{3} + 2k\pi,\ 2k\pi \ / \ k \in \mathbb{Z} \right\}}$$