Contrôle 2
(SV - PC)
(STE - PC)

02/11/15

Exercice 1 :

Division euclidienne :

$$
\begin{array}{l|l}
2x^5 \qquad\quad -x^3 - 3x^2 + 1 & -x^2 + x \\
\hline
-(2x^5 - 2x^4 \qquad\qquad\qquad\quad) & -2x^3 - 2x^2 - x + 2 \\
\hline
\qquad 2x^4 - x^3 - 3x^2 + 1 & \\
\quad -(2x^4 - 2x^3 \qquad\qquad) & \\
\hline
\qquad\quad x^3 - 3x^2 + 1 & \\
\quad\quad -(x^3 - x^2 \qquad\quad) & \\
\hline
\qquad\qquad -2x^2 + 1 & \\
\qquad\quad -(-2x^2 + 2x) & \\
\hline
\qquad\qquad\quad -2x + 1 &
\end{array}
$$

On vérifie :

$$(-2x^3 - 2x^2 - x + 2)(-x^2 + x) + (-2x+1) = 2x^5 - \cancel{2x^4} + \cancel{2x^4} - 2x^3 + x^3 - x^2 - 2x^2 + \cancel{2x} - \cancel{2x} + 1$$
$$= 2x^5 - x^3 - 3x^2 + 1.$$

Donc $\boxed{f(x) = (-2x^3 - 2x^2 - x + 2)\,g(x) + (-2x+1)}$.

Exercice 2 :

$f :]1, +\infty[\to \mathbb{R}$, $f(x) = \left(\dfrac{1}{2x-1}\right)^x$

On réecrit f :

$$f(x) = e^{x \ln\left(\frac{1}{2x-1}\right)} = e^{x \underbrace{\cancel{\ln(1)}}_{0} - x\ln(2x-1)} = e^{-x\ln(2x-1)}$$

f est donc dérivable sur $]1, +\infty[$ et est de la forme e^u, où $u = v \times w$, avec

$$v(x) = -x \quad \rightarrow \quad v'(x) = -1$$
$$w(x) = \ln(2x-1) \quad \rightarrow \quad w'(x) = \frac{(2x-1)'}{2x-1} = \frac{2}{2x-1}$$

Du coup,

$$u'(x) = v'(x)w(x) + v(x)w'(x) = -\ln(2x-1) + (-x) \times \frac{2}{2x-1} = -\ln(2x-1) - \frac{2x}{2x-1}$$

D'où

$$f'(x) = u'(x)e^{u(x)} = \boxed{-\left(\ln(2x-1) + \frac{2x}{2x-1}\right)e^{-x\ln(2x-1)} = f'(x)}$$

Exercice 3 :

Résoudre dans $\mathbb{R}$:

$$\cos(2x) = \sin(x)$$

On sait que

$$\cos(2x) = \cos^2 x - \sin^2 x$$

Du coup,

$$\cos(2x) = \sin x \iff \cos^2 x - \sin^2 x = \sin x \qquad (E)$$

Or $\forall x \in \mathbb{R}$, $\cos^2 x = 1 - \sin^2 x$. Donc

$$(E) \iff (1 - \sin^2 x) - \sin^2 x = \sin x$$

$\Leftrightarrow \quad 2\sin^2 x + \sin x - 1 = 0$

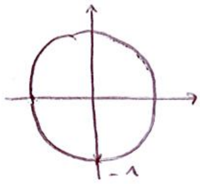
On pose $X = \sin x$. On cherche les racines du trinôme $2X^2 + X - 1$:

$\Delta = 1 + 4 \times 2 = 9 > 0 \longrightarrow$ 2 racines réelles simples

$$X_1 = \frac{-1-3}{4} = -1 \quad \text{et} \quad X_2 = \frac{-1+3}{4} = \frac{1}{2}$$

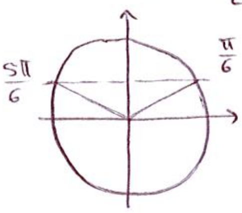
Alors les solutions sont les x tels que

* $\sin x = X_1 = -1$ donc $x = -\frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$



* $\sin x = X_2 = \frac{1}{2}$ donc $x = \frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$

ou $x = \frac{5\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$



Finalement,

$$\boxed{\mathcal{S} = \left\{ -\frac{\pi}{2} + 2k\pi,\ \frac{\pi}{6} + 2k\pi,\ \frac{5\pi}{6} + 2k\pi \,/\, k \in \mathbb{Z} \right\}}$$