Avoiding long abelian repetitions in words

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Subject

A word is a (possibly infinite) sequence of letters from a fixed alphabet. A square in a word of the form uu. Is it possible to construct an infinite word on binary alphabet without squares (as subword of consecutive letters) ? Obviously not... And what about 3 letters ? Axel Thue answered positively to that question in a paper in 1906, which is considered as one of the origin of word combinatorics. Since then, the study of repetitions is a central focus in this domain.

In 1957, Paul Erdös asked whether abelian squares can be avoided by an infinite word on an alphabet of 4 letters (uv is an abelian square if u is a permutation of the letters of v). Keränen answered positively to Erdös's question in 1992, with an intensive use of computer. Erdös also raised whether arbitrarily long squares can be avoided in binary words. Some conjectures are still open on the size the smallest alphabet required to avoid arbitrarily long abelian squares (resp. cubes).

Recently Karhumäki et al. introduced a new notion of word equivalence, the k-abelian equivalence, which is a generalization of both abelian and standard equivalence (and bring the gap between them). We now know, for every k, the size of the smallest alphabet required to avoid every k-abelian squares (resp. cubes).

The subject of this stage is to focus on the question of repetition avoidability (squares/cubes) of arbitrary long k-abelian squares (resp. cubes). One goal is to answer to the following questions:

- Is there a $k \in \mathbb{N}$ such that one can avoid arbitrarily long abelian cubes on binary words ?
- Is there a $k \in \mathbb{N}$ such that one can avoid arbitrarily long k-abelian-squares on binary words ?

Some other conjectures can be found in [4]. For the definition of k-abelian equivalence, see [2]. For an overiew and recent results on abelian and k-abelian avoidability, see [4].

Candidate

- Background in discrete mathematics / combinatorics.
- Good programming skills (many proofs are computer aided in this area).

References

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