Homework assignment 1

The due date is October 25th 2015.

You can either send me your homework by email at nathalie.aubrun@ens-lyon.fr or put it in my mailbox – in the printers room on the third floor. If you have given your work but have not received a confirmation email by October 28th, you should worry about it...

Exercise 1 Let $d \in \mathbb{N}^*$ and $A$ a finite alphabet. Let $X \subset A^{Z_d}$ be a sofic subshift. Prove that there exists a nearest neighbor SFT $Y \subset B^{Z_d}$ and a letter-to-letter sliding block code $\Phi$ such that $X = \Phi(Y)$.

Exercise 2 Let $d \in \mathbb{N}^*$. The set of configurations $X_{\text{sym}} \subset \{0, 1, \star\}^{Z_d}$ is defined as follows

- if $x \in \{0, 1\}^{Z_d}$, then $x \in X_{\text{sym}}$;
- if $x \in X_{\text{sym}}$ and $x_i = x_j = \star$, then $i = j$;
- if $x \in X_{\text{sym}}$ and $x_i = \star$, then $x_{i+j} = x_{i-j}$ for every $j \in Z_d$.

The objective of this exercise is to prove that $X_{\text{sym}}$, called the symmetric subshift, is not a sofic subshift.

1. Check that the set of configurations $X_{\text{sym}}$ is a subshift for any integer $d \in \mathbb{N}^*$.
2. Explain the name symmetric subshift.
3. Give a set of forbidden patterns that defines $X_{\text{sym}}$.
4. Prove that $X_{\text{sym}}$ is not an SFT.
5. Prove that $X_{\text{sym}}$ is not a sofic subshift.

Exercise 3 In the exercise we only consider subshifts of dimension 1. A subshift is effectively closed if there exists a recursively enumerable set of forbidden patterns that defines it.

1. Prove that a sofic subshift is effectively closed.
2. Prove that there exist effectively closed subshifts that are not sofic.
3. Prove that there exist subshifts that are not effectively closed.
4. Prove that if $X$ is effectively closed, then there exists a recursive set of forbidden patterns that defines it.

Exercise 4 Consider the following decision problem: take as input a finite set of Wang tiles, and output Yes if the set of tiles can tile the plane in a periodic way, No otherwise. Prove that this problem is undecidable.