

VITAE FOR NATHALIE REVOL

Nathalie REVOL

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DIPLOMAS

- **PhD thesis in Applied Mathematics**, entitled *Complexity of the parallel evaluation of arithmetic circuits*, defended in 1994 at INPG (Institut National Polytechnique de Grenoble), supervised by Jean Della Dora and Jean-Louis Roch, with highest distinction.
- **Master of Applied Mathematics**, of the University Joseph Fourier (Grenoble), 1990, with distinction.
- **Ingénieur diploma** of the ENSIMAG (École Nationale Supérieure d'Informatique et de Mathématiques Appliquées de Grenoble) in 1990, with distinction.

EDUCATION

- **PhD thesis** in applied mathematics at INPG, November 1990 - August 1994.
- **Monitorat** in applied mathematics at ENSIMAG (kind of teaching assistant), November 1990 - August 1993.
- **DEA** in applied mathematics at University Joseph Fourier, Grenoble, 1989 - 1990.
- **ENSIMAG** (École Nationale Supérieure d'Informatique et de Mathématiques Appliquées de Grenoble), 1987 - 1990. 4th year in Computer Science and 5th year in Scientific Computing.
- **Classes préparatoires** at lycée du Parc, Lyon, 1985 - 1987.

RESEARCH AND TEACHING EXPERIENCE

- **Research scientist** at INRIA (Institut National de Recherche en Informatique et en Automatique), since September 2002, within the Arenaire team (until 2011) and now AriC, LIP (Laboratoire de l'Informatique du Parallélisme), École Normale Supérieure de Lyon.
- **Associate professor** at University of Sciences and Technologies of Lille, Laboratoire ANO (Analyse Numérique et Optimisation), from January 1996 to August 2002.
- **INRIA sabbatical leave** within the project Arenaire, LIP, École Normale Supérieure de Lyon, from September 2000 to August 2002.
- **Attachée temporaire d'enseignement et de recherche (ATER)** (temporary research and teaching assistant, with more teaching than during "Monitorat"), ENSIMAG, September 1993 - August 1995.
- **Moniteur** (teaching assistant) at ENSIMAG, November 1990 - August 1993.

1 Summary of research activities

1.1 Research conducted in Lyon: computer arithmetic and numerical quality

The common expertise of the Arenal, and then AriC, team in Lyon about floating-point arithmetic led to [20,19,17].

In my work, the two main questions regarding the issue of numerical quality of computations are to determine how far a computed result lies from the exact result, and to enclose this unknown exact result. My main tool is interval arithmetic, where operations are performed on intervals instead of numbers.

Definition and standardization of interval arithmetic (2000-2017).

Interval arithmetic is used and implemented for decades [18,14,15]. However, only the usual arithmetic operations have reached a consensual definition. Comparisons or the handling of exceptional cases, such as $\sqrt{[-2, 3]}$ for instance, have received many different interpretations [43]. In 2008, the interval arithmetic community felt that interval arithmetic was mature enough to be standardized [48]. With R. B. Kearfott, we have created and chaired the IEEE 1788 working group for the standardization of interval arithmetic (cf. <http://grouper.ieee.org/groups/1788/>). The development of the standard [31,42,27] is now complete.

Arbitrary precision interval arithmetic, the MPFI library (2000-).

MPFI, *Multiple Precision Floating-point Interval arithmetic* library developed mainly by F. Rouillier and myself, is a C library implementing interval arithmetic [6,49], based upon MPFR (cf. <http://www.mpfr.org/>). In MPFI, intervals are represented by their endpoints which are floating-point numbers with arbitrary precision. (In MPFI, we made the choice of returning `Invalid` for $\sqrt{[-2, 3]}$.)

The main features of interval arithmetic are exemplified by the interval Newton algorithm, that determines all zeros of a function over a given, possibly large, input interval. It has been adapted for arbitrary precision interval arithmetic, implemented using MPFI and experimented [9,10]: the computing precision can be automatically and dynamically adapted to fulfill the computing needs and reach the required accuracy, without re-starting the whole execution of the program.

As stated by Kahan in 2006, when it replaces floating-point arithmetic in a code without further modification, arbitrary precision interval arithmetic is a first and safe step towards assessing the numerical quality of a computation, by returning a tight interval that encloses the exact result [46].

Variants of interval arithmetic (2001-).

Other, more elaborate, ways to guarantee the numerical quality of a computation use "variants" of interval arithmetic that partly cure the so-called *dependency problem* of interval arithmetic. Arithmetic on Taylor models is such a variant. Taylor models are a representation of a function: they consist of a Taylor expansion plus an interval remainder, which encloses the truncation error.

F. J. Chaves Alonso, during his PhD thesis co-supervised with M. Daumas, implemented Taylor models arithmetic and the proof of containment of each operation within PVS, a formal proof checker. Formal proof adds an extra level of guarantee to results computed using interval arithmetic. He then developed strategies to automate the proofs. This tool is used to prove bounds on expressions and inequalities.

M. Berz et al. implemented Taylor models arithmetic using floating-point arithmetic and used the interval remainder to enclose also the roundoff errors. I proved that the arithmetic operations defined by M. Berz and K. Makino correctly take into account the roundoff errors due to the computations performed with floating-point arithmetic [32,7]. Again, these results have been formally checked using Coq [3].

Verification of floating-point computations in linear algebra (2007-).

We focused on the verification of the solution of a linear system, computed using floating-point arithmetic. Compared to the approach proposed by Rump, we managed to reduce the execution time by resorting to optimized floating-point BLAS3 routines as often as possible, by relaxing (enlarging) intervals when possible [44], and at the same time we managed to improve the accuracy of the results by using double-double computations only for well-chosen variables (following the approach by Demmel), without significant time

penalty. This work has been done with H. D. Nguyen [4] during his PhD thesis supervised by G. Villard and myself.

With H.D. Nguyen, we proposed new algorithms for the product of matrices, either with floating-point or with interval coefficients, that offer a tradeoff between the accuracy (the width) of the result and the execution time [44], in line with results by Rump et al. Bounds on the overestimation due to roundoff errors have been determined for several such algorithms by P. Théveny during his PhD thesis. These bounds are tight and have been experimentally confirmed [30].

Further experiments also illustrate the influence of the numerical condition number on the accuracy of the interval results [16]. Floating-point and interval behaviors remain similar also for so-called "compensated" algorithms.

Parallel computing and numerical reproducibility (2013-2014).

Getting an efficient, or even correct, implementation of the algorithms for interval matrix multiplication requires "taylor-made" optimizations of the code, as the behavior of numerical codes on emerging architectures such as multicores is not reproducible. With P. Théveny, we obtained an implementation comparable to the MKL, within a factor less than 2, for interval matrices [2]. We thoroughly examined why numerical computations in general, and interval computations in particular, are not reproducible when executed in parallel [1,29,28] and what can be done to overcome this difficulty [13].

Automation (2006-2011).

The tuning of the best evaluation formula or variant of interval arithmetic, or of the right computing precision, is crucial for performances, be they execution time or accuracy of the results. However, hand-tuning is tedious, error-prone and time-consuming. A first project towards more automation was the ANR project EVA-Flo. This project focused on the way a mathematical formula is evaluated in floating-point arithmetic. The approach was threefold: study of algorithms for approximating and evaluating mathematical formulae, validation of such algorithms, and automation of the process or rather of several steps of this process. The LEMA language [45] was intended to enable the communication between the different tools in charge of each of these steps.

Floating-point arithmetic and elementary functions (2000-2006).

The IEEE 754-1985 standard specifies floating-point formats and arithmetic operations. We have proposed an extension of this standard to elementary functions [8]. An issue related to the standardization of floating-point arithmetic, and more specifically of mathematical functions, is the implementation of special functions with correct rounding; we proposed an algorithm for the error functions erf and erfc in arbitrary precision in [47]. We have also proposed an algorithm for the reduction of the arguments of trigonometric functions, which is always accurate and, for most cases, it is also fast [5].

Another aspect of this work includes the joint development, with P. Pelissier and P. Zimmermann, of a software named `mpcheck` that tests the quality of the implementation of elementary functions (accuracy, output range...) and also its use to test various mathematical libraries.

Parallelization of automatic speech recognition (2000).

Y.O. Mohamed El Hadj, PhD student supervised with E.M. Daoudi, and myself have determined a static estimate of the computational load and thus a static load distribution among the processors, which yields efficiencies ranging from 65% to 80% on a cluster of 12 PCs [54].

1.2 Research conducted in Lille: arithmetic and parallelism

Interval arithmetic and parallelism (1999-2000). My target application with interval arithmetic is the global optimization of a continuous function; this problem can be solved by Hansen's algorithm, which is a Branch&Bound algorithm and has an exponential worst-case complexity. In order to reduce its execution time, we have studied its parallelization: it is quite difficult because the execution graph is not predictable. We obtained good performances with a homogeneous architecture [50] and adapted to heterogeneous dis-

tributed architectures, using MC-PM² [52,51] developed by B. Planquelle,.

Interval arithmetic: constrained global optimization (2000). The problem is to find the global optimum of a continuous function on a domain defined by constraints. A first approach consists in simplifying Hansen’s algorithm in order to keep a smaller dimension for the working vector space (with N. Baeyens), than the classical approach based on the Lagrangian.

Multiple precision arithmetic (1997). With J.-C. Bajard (LIM, Marseille), we wrote a chapter on multiple precision arithmetic [21] for the book on computer arithmetic coordinated by J.-M. Muller. With students, we have experimented several multiple precision algorithms.

Floating-point arithmetic (1998). With J.-C. Yakoubsohn (LAO, Toulouse), we have proposed accelerated Shift-and-Add algorithms for the computation of elementary functions: the last steps are replaced by one step of Euler or Runge-Kutta method. The benefits are between 33% and 75% of the computation time [11].

1.3 Research conducted for the PhD

I have done my PhD entitled *Complexity of the parallel evaluation of arithmetic circuits* within the “Computer Algebra and Parallelism” team of the LMC laboratory in Grenoble, supervised by J.-L. Roch and J. Della Dora. I defended my thesis in August 1994.

Subject. The underlying idea was that the algebraic properties of arithmetic operations can be used in order to extract the intrinsic parallelism of some programs (arithmetic expressions, straight-line programs) such as programs of scientific computing and linear algebra in particular [53].

Result: a new algorithm for the parallel evaluation of arithmetic programs over lattices. The main result of my PhD is a new algorithm for the parallel evaluation of straight-line programs with operations taking place in a lattice [12]. A distributive lattice is an important algebraic structure, since the Boolean algebra, which is fundamental in complexity, belongs to it.

Towards a compiling tool. This algorithm enables one to transform sequential programs into parallel programs with a complexity as good as that of the tailor-made parallel programs for the sort problem, exact arithmetic operations and some algorithms in computer algebra. I have implemented a simulator and then a compiler-parallelizer for arithmetic expressions with T. Gautier (DEA).

1.4 Conclusion

This work shows how numerical analysis, arithmetic and parallelism can interact. It illustrates the different steps done in scientific computing, to design and implement an efficient algorithm on a computer, circumventing numerical problems by choosing the proper arithmetic and the efficiency problems by taking benefit from optimized routines and high-performance architectures. The goal is to offer guarantees on the results of numerical computations and to prove that the overhead to get guarantees remains small.

2 Publications

Most of these publications can be downloaded as research reports (extended versions of the published articles), along with slides of the presentations, from my Web page : <http://perso.ens-lyon.fr/nathalie.revol/>

Journals

- [1] *Numerical reproducibility and parallel computations: Issues for interval algorithms*, N. Revol and P. Théveny, IEEE Transactions on Computers, vol. 63, no 8, pp 1915-1924, 2014.
- [2] *Parallel Implementation of Interval Matrix Multiplication*, N. Revol and P. Théveny, Reliable Computing, vol. 19, no. 1, pp 91-106, 2013.
- [3] *A Validated Real Function Calculus*, P. Collins, M. Niqui and N. Revol, Mathematics in Computer Science, vol. 5, issue 4, pp 437-467, 2011.
- [4] *Solving and Certifying the Solution of a Linear System*, H. D. Nguyen and N. Revol, Reliable Computing, vol. 15, no. 2, pp 120-131, 2011.
- [5] *A new range-reduction algorithm*, N. Brisebarre, D. Defour, P. Kornerup, J.-M. Muller and N. Revol, IEEE Transactions on Computers, vol. 54, no. 3, pp 331-339, 2005.
- [6] *Motivations for an arbitrary precision interval arithmetic and the MPFI library*, N. Revol and F. Rouillier, Reliable Computing, vol. 11, no. 4, pp 275-290, 2005.
- [7] *Taylor models and floating-point arithmetic: proof that arithmetic operations are validated in COSY*, N. Revol, K. Makino and M. Berz, Journal of Logic and Algebraic Programming, vol. 64, pp 135-154, 2005.
- [8] *Proposal for a standardization of mathematical function implementation in floating-point arithmetic*, D. Defour, G. Hanrot, V. Lefèvre, J.-M. Muller, N. Revol and P. Zimmermann, Numerical Algorithms, vol. 37, no 1-4, pp 367-375, 2004.
- [9] *Interval Newton iteration in multiple precision for the univariate case*, N. Revol, Numerical Algorithms, vol. 34, no. 2, pp 417-426, 2003.
- [10] *Validating polynomial computations with complementary automatic methods*, P. Langlois and N. Revol, to appear in Mathematics and Computers in Simulation. <http://www.inria.fr/rrrt/rr-4205.html>
- [11] *Accelerated Shift-and-Add algorithms*, N. Revol and J.-C. Yakoubsohn, Reliable Computing, vol. 6, no. 2, pp. 1-13, 2000. <ftp://ano.univ-lille1.fr/pub/1999/ano395.ps.Z>
- [12] *Parallel evaluation of arithmetic circuits*, N. Revol and J.-L. Roch, Theoretical Computer Science A, vol. 162, pp 133-150, 1996.

National journals

- [13] *First steps towards more numerical reproducibility*, F. Jézéquel, Ph. Langlois and N. Revol, ESAIM: M2AN (Mathematical Modelling and Numerical Analysis), vol. 45, pp 229-238, 2014.
- [14] *JDEV 2013 : Développer pour calculer*, partie *Des outils pour calculer avec précision* et partie *Comment calculer avec des intervalles* (in french), F. Langrognet, F. Jézéquel and N. Revol, HPC magazine, 2013.
- [15] *Arithmétique par intervalles* (in french), N. Revol, Calculateurs Parallèles, vol. 13, pp 387-426, 2001.

Books and chapters of books

- [16] *Influence of the Condition Number on Interval Computations: Illustration on Some Examples*, N. Revol, in honour of Vladik Kreinovich's 65th birthday. LNCS Festschrift, to appear.
- [17] *Handbook of Floating-Point Arithmetic*, J.-M. Muller, N. Brunie, F. de Dinechin, C.-P. Jeannerod, M. Joldes, V. Lefèvre, G. Melquiond, N. Revol, and S. Torres, Birkhäuser Boston, 2nd edition, 2018.
- [18] *Analyser et encadrer les erreurs dues à l'arithmétique flottante* (in french), C.-P. Jeannerod and N. Revol in Informatique mathématique: une photographie en 2017, CNRS Editions, pp. 115-144, 2017.
- [19] *Handbook of Floating-Point Arithmetic*, J.-M. Muller, N. Brisebarre, F. de Dinechin, C.-P. Jeannerod, V. Lefèvre, G. Melquiond, N. Revol, D. Stehlé and S. Torres, Birkhäuser Boston, 2010.
- [20] *Digital Arithmetic*, F. de Dinechin, M. D. Ercegovac, J.-M. Muller and N. Revol, in *Encyclopedia of Computer Science and Engineering*, pp 935-948, Benjamin W. Wah editor, Hoboken, NJ, 2009.
- [21] *Arithmétique multi-précision*, J.-C. Bajard and N. Revol, chapter of *Qualité des calculs sur ordinateur – Vers des arithmétiques plus fiables ?*, directed by M. Daumas and J.-M. Muller, Masson, 1997.

Editorial work

- [22] Special section on *Computer Arithmetic*, J. Hormigo, J.-M. Muller, S. Oberman, N. Revol, A. Tisserand and J. Villalba-Moreno editors, IEEE Transactions on Computers, vol. 66, no. 12, 2017.
- [23] Special issue following SCAN 2010, G. Alefeld and N. Revol editors, Computing, vol. 94, no. 2-4, 2012.
- [24] Special issue on *Reliable Implementation of Real Number Algorithms: Theory and Practice*, P. Hertling, Ch. M. Hoffmann, W. Luther and N. Revol editors, Lecture Notes in Computer Science 5045, 2008.
- [25] Special issue on *Real Numbers and Computers*, M. Daumas and N. Revol editors, Theoretical Computer Science, vol. 351, no. 1, 2006.
- [26] Special issue on *Linear Algebra and Arithmetic*, S. El Hajji, N. Revol and P. Van Dooren editors, Journal of Computational and Applied Mathematics, vol. 162, no. 1, 2004.

Invited conferences

- [27] *Introduction to the IEEE 1788-2015 Standard for Interval Arithmetic*, N. Revol plenary conference at NSV17: 10th International Workshop on Numerical Software Verification 2017, Springer LNCS 10381, pp. 14-21, Heidelberg, Germany, 22-23 July 2017. (DOI: 10.1007/978-3-319-63501-9)
- [28] N. Revol, ICERM Workshop on *Challenges in 21st Century Experimental Mathematical Computation*, Brown University, Providence, RI, USA, 2014.
- [29] *Numerical reproducibility in HPC: issues in floating-point arithmetic and in interval arithmetic*, N. Revol and P. Théveny, McMaster University, Hamilton and University of Toronto, Canada, 2013.
- [30] *Tradeoffs between Accuracy and Efficiency for Interval Matrix Multiplication*, N. Revol, H. D. Nguyen and P. Théveny, Numerical Software 2012: Design, Analysis and Verification, Spain, 2012.
- [31] *IEEE-1788 standardization of interval arithmetic: work in progress (a personal view)*, N. Revol, IFIP Working Group 2.5 on Numerical Software, Spain, 2012.
- [32] *Proof that arithmetic operations are validated in COSY*, N. Revol, mini-workshop on Taylor models, USA, 2002.

Invited conferences in France

- [33] *Numerical Reproducibility and Parallel Computations: Issues for Interval Algorithms*, N. Revol, Polytechnique, 2014.
- [34] *Numerical reproducibility in HPC: issues in floating-point arithmetic and in interval arithmetic*, N. Revol and P. Théveny, RAIM: Rencontres Arithmétiques de l'Informatique Mathématique, Paris, 2013.
- [35] *IEEE-1788 standardization of interval arithmetic: work in progress (a personal view)*, N. Revol, RAIM: Rencontres Arithmétiques de l'Informatique Mathématique, Dijon, 2012.
- [36] *Interval arithmetic to handle uncertainties and to assess numerical quality*, N. Revol, "Précision et incertitudes", SMAI GAMNI&MAIRCI, Paris, 2012.
- [37] *Introduction to interval arithmetic and assessment of the numerical quality*, N. Revol, "Précision numérique", CNES, Toulouse, 2012.
- [38] *Entre mathématiques et informatique, les algorithmes* (in french), N. Revol, colloque de l'IREM (Institut de Recherche sur l'Enseignement des Mathématiques), France, 2002.
- [39] *Quelques exemples d'arithmétiques sur ordinateur* (in french), N. Revol, Forum des jeunes mathématiciennes et des jeunes informaticiennes, France, 2002.
- [40] *Complexité parallèle et straight-line programs* (in french), N. Revol, Journées Toulouse-Limoges "Around Straight-Line Programs", France, 1997.
- [41] *Outils graphiques pour l'étude des systèmes dynamiques à l'ENSIMAG* (in french), N. Revol, mini-workshop on "Pedagogical tools", CANU (Colloque d'Analyse NUMérique), France, 1994.

Conferences with referees and proceedings

- [42] *Latest developments on the IEEE 1788 effort for the standardization of interval arithmetic*, N. Revol, ICVRAM & ISUMA - Second International Conference on Vulnerability and Risk Analysis and Management & Sixth International Symposium on Uncertainty Modelling and Analysis, United Kingdom, (10 pages, ASCE

publisher), 2014.

[43] *Standardized Interval Arithmetic and Interval Arithmetic Used in Libraries*, N. Revol, ICMS 2010 (3rd International Congress in Mathematical Software), Japan, LNCS 6327, pp 337–341, 2010.

[44] *High performance linear algebra using interval arithmetic*, H. D. Nguyen and N. Revol, PASCO 2010 (4th International Workshop in Parallel and Symbolic Computation), France, ACM Digital Library pp 171–172, 2010.

[45] *LEMA: Towards a Language for Reliable Arithmetic*, V. Lefèvre, P. Théveny, F. de Dinechin, C.-P. Jeannerod, C. Moulleron, D. Pfannholzer and N. Revol, PLMMS 2010 (Programming Languages for Mechanized Mathematical Systems), France, ACM Communications in Computer Algebra, vol. 44, pp 41–52, 2010.

[46] *Floating-point geometry*, J.-C. Bajard, D. Michelucci, Ph. Langlois, G. Morin and N. Revol, (12 pages), SPIE Optics+Photonics, San Diego, California USA, 2008.

[47] *Computation of the error function erf in arbitrary precision with correct rounding*, S. Chevillard and N. Revol, RNC8 (Real Numbers and Computers), Spain, pp 27–36, 2008.

[48] *Discussions on an Interval Arithmetic Standard at Dagstuhl Seminar 08021*, R. B. Kearfott, J. D. Pryce and N. Revol, Germany, LNCS 5492, pp 1–6, 2009.

[49] *Multiple precision interval packages: comparing different approaches*, M. Grimmer, K. Petras and N. Revol, Germany, LNCS 2991, pp 64–90, 2004.

[50] *A methodology of parallelization for continuous verified global optimization*, N. Revol, Y. Denneulin, J.-F. Méhaut, B. Planquelle, PPAM’01, Poland, 2001, LNCS 2328, pp 803–810, 2002.

[51] *Multi-protocol communications and high performance networks*, B. Planquelle, J.-F. Méhaut and N. Revol, EuroPar’99, France, 1999, LNCS 1685, pp. 139–143.

[52] *MC-PM²: Multi-cluster approach with PM²*, B. Planquelle, J.-F. Méhaut and N. Revol, PDPTA (Parallel and Distributed Processing Techniques and Appl.), USA, 1999, pp. 779–785, vol. II, CSREA Press.

[53] *Automatic parallelization of numerical programs in fields and lattices*, N. Revol and J.-L. Roch, Numerical Methods and Applications, Bulgarie, 1994, pp 265–273, World Scientific.

Unpublished research reports

[54] *Parallelization of automatic speech recognition*, Y.O. Mohamed El Hadj, N. Revol and A. Meziane. <http://www.inria.fr/rrrt/rr-4110.html>.

Lectures in schools

[55] *Analyser et encadrer les erreurs dues à l’arithmétique flottante* (in french), C.-P. Jeannerod and N. Revol, École Jeunes Chercheurs et Jeunes Chercheuses en Informatique Mathématique, Lyon, January 2017.

[56] *Arithmétique flottante et intervalles*, N. Revol and P. Théveny, École CNRS *Précision et reproductibilité en calcul numérique*, Fréjus, March 2013 and May 2017.

[57] *Précision et arithmétique flottante : outils, bibliothèques*, N. Revol and P. Théveny, JDEV : Journées du Développement Logiciel, École Polytechnique, September 2013.

[58] *Introduction à l’arithmétique par intervalles*, N. Revol and P. Théveny, professional training entitled “Contrôler et améliorer la qualité numérique d’un code de calcul industriel”, Collège de l’X, Paris, November 2013.

[59] *Calcul numérique sur des ensembles, vérification de calculs numériques : un outil de choix, l’arithmétique par intervalles*, N. Revol, SIESTE 2010-2011 (students seminar), ENS Lyon, France, April 2011.

[60] *Arithmétique des ordinateurs : calculer de façon rapide, fiable, précise*, N. Revol, SIESTE 2007-2008 (students seminar), ENS Lyon, France, September 2007.

[61] *Introduction à l’arithmétique d’intervalles*, N. Revol, École Jeunes Chercheurs en Algorithmique et Calcul Formel, Grenoble, March 2004.

[62] *Arithmétique d’intervalles*, N. Revol, Spring school of Theoretical Computer Science on Computer Arithmetic, Prapoutel, March 2001.

[63] *Parallélisation d’applications irrégulières : exemples en optimisation combinatoire et en optimisation globale par intervalles*, N. Revol, Autumn school on Parallel and Distributed Computing ParDi, Oujda, Morocco, October 1999. <ftp://ano.univ-lille1.fr/pub/2000/ano417.ps.Z>

3 Software writing and distribution

They are available from my Web page: <http://perso.ens-lyon.fr/nathalie.revol/software.html>.

MPFR++: C++ interface for the MPFR library (<http://www.mpfr.org/>) developed by the INRIA project Caramel (Nancy), 2 900 lines. Used in the Arithmos project, Antwerpen, Belgium. No more maintained.

MPFI (*Multiple Precision Floating-point Interval arithmetic library*): C/C++ library still under development, joint work with F. Rouillier, 7 500 lines. Used by F. Rouillier to isolate real roots of polynomials. Other users in Europe (France, Germany, Belgium, United Kingdom) and farther (USA, Colombia,...).

4 Supervision of research activities

Where no percentage is given I am the sole supervisor.

B. Planquelle has worked on a PhD in computer science (MESR grant, 1997-2002) entitled *Multithreaded environment for distributed and heterogeneous architectures*, he was supervised by J.-M. Geib, J.-F. Méhaut and myself (33% each). He has extended the multithreaded runtime environment PM² for multi-clusters and has parallelized Hansen's algorithm for interval global optimization on a shared-memory architecture. He was not able to bear the heavy working load of finishing his PhD and writing his manuscript (he is strongly disabled) and he gave up his PhD. He is now computer scientist at rectorat of Lille.

Y.O. Mohamed El Hadj (PhD thesis in computer science, U. Oujda - Morocco, European contract INCO-DAPPI grant, 1998-2001, co-supervised 25% by me and 75% by E.M. Daoudi) worked on the parallelization of automatic speech recognition. He is now associate professor in Riyadh, Saudi Arabia.

F.J. Cháves Alonso (PhD thesis in computer science, European contract MathLogAps grant, 2004-2007, co-supervised 75% by me and 25% by M. Daumas) worked on the implementation and certification of the Taylor models arithmetic on the formal proof assistant PVS. He implemented arithmetic operations and several transcendental functions (exp, atan, sin) on Taylor models and proved the containment property in PVS. He developed strategies to build the Taylor model corresponding to a given expression and to prove the containment property or bounds on this expression, hiding the step-by-step process. He is now Software Developer at Liftit, and Lecturer at Escuela Colombiana de Ingeniera, Bogota, Colombia.

H.D. Nguyen (PhD thesis in computer science, INRIA Cordi-S grant, 2007-2011, co-supervised 85% by me and 15% by G. Villard) worked on problems in linear algebra and interval arithmetic. He reached a trade-off for the product of interval matrices: his algorithm is more accurate than the fastest algorithm and only a bit slower. He also worked on the verification of the solution of linear systems: starting from an approximate solution, computed using floating-point arithmetic, he improved its accuracy. Simultaneously he computed and refined an enclosure of the error between the computed solution and the exact one. His results exhibit very good time and accuracy. After a postdoc at the U. California at Berkeley, supervised by J. Demmel, he got a position as consultant in the Silicon Valley.

P. Théveny (expert engineer, 2009-2010) has been hired for the ANR project EVA-Flo. He worked on the design and the implementation of the LEMA language, that extends MathML for the representation of floating-point data. He also updated the MPFI library. After a master in computer science, P. Théveny obtained a grant for a PhD. thesis at ENS de Lyon.

D. Pfannholzer (unfinished PhD thesis in computer science, MEFI Mediacom-Nano 2012 grant, 2009-2011, co-supervised 50%-50% by F. de Dinechin and myself) worked on the automated generation of code for elementary functions and more precisely for the exponentials. He automated the range reduction, which was not yet tackled. After two years, he received a very good job offer and did not complete his PhD.

P. Théveny (PhD thesis, MESR grant, 2011-2014) worked on the parallel implementation of some algorithms for linear algebra using interval analysis. For the product of matrices with interval coefficients, several algorithms, that exhibit different tradeoffs between accuracy and speed, have been compared. Their accuracy has been extensively analyzed. Their speed is high, through a parallel implementation by blocks, with the optimization of block sizes and use of vectorization. It reaches performance comparable to the Intel MKL's one. This work also led to thoroughly analyze numerical reproducibility issues and to propose recommendations to circumvent these issues. After a postdoc with J. Langou, U. Colorado at Denver, P. Théveny got a position with the finance branch of the Swiss Post.

T. Gautier (master in applied math. 1992) has done his final project with J.-L. Roch and myself (50% each) on the design of the compiler-parallelizer and on the specialization of programs. **S. Czech (master in computer science 1998)** has worked on the implementation of Hansen's algorithm with M. Petitot, Y. Denneulin (LIFL) and myself (33% each). **N. Baeyens (master in math. 2000)** has done his final project on the handling of constraints in interval global optimization. **N. Dessart (master in computer science, 2004)** worked on the comparison of various algorithms solving linear systems with arbitrary precision interval arithmetic. She is now associate professor at U. Antilles at Martinique.

I have also supervised 2 Master projects (4th year) in applied math., 3 Master projects in computer science, 2 undergraduate training periods (3rd year) in computer science and 5 third year projects in IUP GMI (4th year, professional degree in computer science).

5 Responsibilities and collaborations

Co-chair, with R. B. Kearfott, of the IEEE 1788 working group for the standardization of interval arithmetic. The standard defines intervals, their representation, operations including exceptional cases, and the handling of exceptions. Several mathematical models have been considered, the "set-based" model is entirely defined. The document is 100 pages long and has been adopted in July 2015 as *The IEEE 1788-2015 Standard for Interval Arithmetic*. A simplified version, where only 64-bit floating-point numbers are considered for the representation of intervals by their endpoints, has been adopted in 2017 as *The IEEE 1788.1-2017 Standard for Interval Arithmetic (Simplified)*.

PI of the ANR project "EVA-Flo: Evaluation et Validation Automatique pour le calcul Flottant", (2006-2010, <http://www.ens-lyon.fr/LIP/Arenaire/EVA-Flo/>), involving 4 teams: Arenaire (LIP-U. Lyon, ENS Lyon), Dali (U. Perpignan), Fluctuat (LIST, CEA Saclay), Tropics (INRIA Sophia Antipolis). The goal of the project was to evaluate and to validate small pieces of critical codes and to automate this process.

European contract INCO-DC DAPPI. This contract between 3 partners (Mons-Belgium, Lille-France and Oujda-Morocco) focused on recent advances in parallelism. It consisted in organizing a post-graduate school where I gave a lecture, a conference in May 2001 and in co-supervising 2 PhD, one between Oujda and Mons and the other between Oujda and Lille (Y. O. Mohamed El Hadj).

European contract MathLogAps: Early Stage Research Training Site in MATHEMATICAL LOGIC and APplicationS (2004-2008), including 3 partners in Leeds, Munich and Lyon: it was devoted to the training of students in mathematical logic. This contract funded the PhD thesis of F. J. Cháves Alonso, co-supervised

with M. Daumas. It offered the opportunity to visit other centers for the PhD students, and to gather during one workshop per year (I co-organized the workshop in 2007).

International collaborations: non institutional with R. B. Kearfott (U. Louisiana at Lafayette) and J. D. Pryce (U. Cardiff, UK) for the IEEE 1788 working group, and with M. Berz (U. Michigan) and K. Makino (U. Illinois), with an invitation to the mini-workshop on Taylor models, Miami, Florida, USA, 2002. Invitation of D. Saunders (U. Delaware at Newark, USA) by ENS Lyon in May-June 2016, of Ned Nedialkov (U. McMaster, Ontario, Canada) by INRIA in June-July 2008 and of T. Csendes (Univ. Szeged, Hungary), at U. Lille in January 2000.

National collaborations. Participant of the *PEPS project "Quarenum: Qualité et Reproductibilité Numériques dans le Calcul Scientifique Haute Performance"* (2013) involving 4 partners (Paris 6, Perpignan, Lyon, Onera), of the *ANR project "HPAC: High-Performance Algebraic Computations"* (2011-2014) involving 4 partners (Grenoble, Paris 6, Montpellier and Lyon), of the *INRIA-STMicroelectronics project "Mediacom - Nano 2012"* (2009-2012) in relation with the PhD grant of D. Pfannholzer, of the AHA (Adaptive and Hybrid Algorithms, cf. <http://aha.gforge.inria.fr/>) group (INRIA-IMAG) on the combination of algorithms that can adapt to the context (data or architecture). I was co-responsible with S. Lesecq of the working group on *Set computing in automatics* in 2004-2005 (<http://www-lag.ensieg.inpg.fr/gt-ensembliste>). I took part in the working group on computer arithmetic *AriNews* (<http://listes.ens-lyon.fr/wws/info/arinews.lip>), in the working group on the validation of numerical computations on embedded systems, in the working group *Collaboration of solvers* of the GDR ALP and in the cooperative actions of the INRIA *Certified computer arithmetic* and *ResCapa*.

Guest co-editor of a special issue of the *Journal of Computational and Applied Mathematics* on Linear Algebra and Arithmetic, with S. El Hajji and P. Van Dooren, 2004, of a special issue of *Theoretical Computer Science* on Real Numbers and Computers, with M. Daumas, 2006, of a volume of *LNCS* on Reliable Implementation of Real Numbers Algorithms: Theory and Practice, with P. Hertling, Ch.M. Hoffmann and W. Luther, 2008, of a topical issue following SCAN 2010 of *Computing* with G. Alefeld, 2012, of a special section of *IEEE Transactions on Computers* with J. Hormigo, J.-M. Muller, S. Oberman, A. Tisserand and J. Villalba-Moreno, 2017.

Chair of the program committee of *SCAN 2010: 14th GAMM - IMACS International Symposium on Scientific Computing, Computer Arithmetic and Validated Numerics* (27-30 September 2010, Lyon, France), and of *ARITH 23: 23rd IEEE Symposium on Computer Arithmetic* (10-13 July 2016, Silicon Valley, California, USA) with J. Hormigo and S. Oberman.

Member of the program committees of *Real Numbers and Computers* (27-29 April 1998, Paris), of the *3rd seminar on numerical techniques for industrial problems* (11-12 March 1999, Rennes), of *Linear algebra and arithmetic* (18-31 May 2001, Rabat, Morocco), of the seminar on *Reliable Implementation of Real Number Algorithms: Theory and Practice* (8-13 January 2006, Dagstuhl, Germany), of *SCAN'06* (26-29 September 2006, Duisburg, Germany), of *CCA'08: Computability and Complexity in Analysis* (20-22 August 2008, Hagen, Germany), of *NSV3: Numerical Software Verification* (15 July 2010, Edinburgh, Scotland, United Kingdom), of *PASCO'10: 4th International Workshop in Parallel and Symbolic Computation* (21-23 July 2010, Grenoble, France), of *ICMS 2010: The Third International Congress on Mathematical Software conference* (13-17 September 2010, Kobe, Japan), of *NSV-2011: Fourth International Workshop on Numerical Software Verification* (14 July 2011, Cliff Lodge, Snowbird, Utah), of *SIAM Conference on Applied Algebraic Geometry 2011* (6-9 October 2011, Raleigh, North Carolina, USA), of *MACIS 2011: Fourth International Conference on Mathematical Aspects of Computer and Information Sciences* (19-21 October 2011, Beijing, China), of *SCAN 2012* (23-29 September 2012, Novosibirsk, Federation of Russia), of *SCAN 2014* (21-26 September 2014, Würzburg, Germany), of *SCAN 2016* (26-29 September 2016, Uppsala, Sweden), of *CRE 2017: Computational Reproducibility at Exascale 2017* and of *Correctness 2017: First International Workshop on Software Correctness for HPC Applications*, workshops of SuperComputing 2017 (Denver, Colorado,

USA, 12 November 2017), of *ARITH 24: 24th IEEE Symposium on Computer Arithmetic* (London, United Kingdom, 24-26 July 2017), of *ARITH 25: 25th IEEE Symposium on Computer Arithmetic* (Amherst, Massachusetts, USA, 25-27 June 2018), of *SCAN 2018* (10-14 September 2018, Tokyo, Japan), of *Correctness 2018*, workshop of SuperComputing 2018 (Dallas, Texas, USA, 11-16 November 2018).

Chair of the organizing committee for *SCAN 2010: 14th GAMM - IMACS International Symposium on Scientific Computing, Computer Arithmetic and Validated Numerics* (27-30 September 2010, Lyon, France): 123 participants from 20 countries; for *13e Forum des Jeunes Mathématicien-ne-s* (13-15 novembre 2013, Lyon, France): 59 participants, in charge of the organization and of the program about gender issues, consisting in an introductory conference, a job-meeting, a theater play, a mentoring session; in charge of the organization of *SWIM 2016: 9th Summer Workshop on Interval Methods*, (19-22 June 2016, Lyon, France): 41 participants from 11 countries.

Organization of conferences: *Computer algebra week* (19-23 June 1994, Marseille) with G. Villard and E. Tournier, *2nd seminar on new techniques for sparse matrices in industrial problems* (28-29 April 1997, Lille) with C. Le Calvez, *Linear algebra and arithmetic* (18-31 May 2001, Rabat, Morocco), *Numerical Algorithms* (1-5 October 2001, Marrakesh, Morocco), *Real Numbers and Computers* (3-5 September 2003, Lyon, France), *Forum des Jeunes Mathématiciennes - Mathématiques, Informatique et Sciences du Vivant* (30-31 January 2004, Paris, France), session on Computer Arithmetic, *School for Young Researchers on Algorithms and Computer Algebra*, (29th March-2nd April 2004, Grenoble, France), session on Computer Arithmetic, *Rencontres Arithmétique de l'Informatique Mathématique* (22-25 January 2007, Montpellier, France), *3rd MATHLOGAPS Workshop* with F. Wagner and E. Jaligot (24-30 June 2007, Aussois, France), *RAIM 2009: 3es Rencontres " Arithmétique de l'Informatique Mathématique "* (26- 28 octobre 2009, Lyon, France), *L(yo)nBox 2012: LinBox high performance kernels meeting* (16-21 July 2012, Lyon, France), *EJCP : École Jeunes Chercheurs et Jeunes Chercheuses en Programmation* (25-29 June 2018, Lyon, France).

Member of board of examiners for PhD theses: M. Charikhi in January 2005 (U. Paris 6), L. Lamarque in December 2006 (U. Bourgogne), O. Mullier in May 2014 (Polytechnique), Q. Li in November 2015 (U.T. Compiègne), R. Nheili in December 2016 (U. Perpignan) and R. Picot in February 2018 (U. Paris 6). Member of the "comité de suivi" of C. Jeangoudoux (U. Paris 6 and Safran, defense planned for March 2019). **External referee** for B. Zheng in December 2013 (U. McMaster, Hamilton, Canada).

Member of the visiting committee for the computer science and mathematics departments of Uppsala U., Sweden, 2017.

Member of the recruiting committees for permanent junior research scientists at INRIA Rhône-Alpes (2007, 2009, 2010, 2011, 2013 and 2014), member of *commissions de spécialistes* (committees devoted to the recruitment of new lecturers) in applied mathematics at the U. of Sciences and Technology of Lille (2000-2004), in applied mathematics at the U. Joseph Fourier of Grenoble (2000-2006), in computer science at the École Normale Supérieure de Lyon (2004-2008), member of *comités de sélection* in computer science at the U. Paris 6 in 2010, in computer science at the U. Claude Bernard Lyon 1 in 2010, in computer science at the U. Paris Sud in 2014 and in computer science at the U. Montpellier in 2015. I have been a member of the committee for post-doc at INRIA Grenoble - Rhône-Alpes from 2007 to 2014.

Examiner for the admission at ENS de Lyon: oral examinations for 2nd concours, 2005 and 2006.

Animation and administration at USTL: creation of a new master in applied mathematics at USTL, member of the department committee, creation and animation of the working group of the ANO laboratory with J.-P. Chehab, **of MEA:** in 2004-2005, with S. Lesecq, chairs of the CNRS working group "Méthodes Ensemblistes pour l'Automatique - Set methods for control theory", which is part of GDR MACS (Modélisation, Analyse et Conduite des Systèmes dynamiques), **at LIP - ENS Lyon:** member of the lab council between 2007 and 2011 and between 2013 and 2016.

6 Teaching

During my PhD I have taught at ENSIMAG (1990-1995) and between 1996 and 2000 I was associate professor at the University of Sciences and Technologies of Lille. A major part of my lectures was devoted to adult classes. Since 2000 I have been working in Lyon and giving or organizing several lectures at École Normale Supérieure and Université Claude Bernard Lyon 1.

Teaching at ENSIMAG

Scientific programming week, 4th year, 30h, 1990-1995.

Scientific computing: projects in ODEs and matrix computations, 4th year, 24h, 1990-1995.

Ordinary differential equations, exercise classes, 4th year, 12h, 1990-1994.

Numerical analysis, lectures and exercise classes, 3rd year, 56h, 1993-1995.

Dynamical systems (labs, 1993) and *Computer algebra* (lecture and labs, 1995), classes for the lecturers in “mathématiques spéciales”. Handouts available from my Web page <http://perso.ens-lyon.fr/nathalie.revol/> (in french).

Teaching at Université des Sciences et Technologies de Lille

Programming in Ada 1 and 2, 2nd year in math and CS, 44h and 52h lectures, exercises and labs, 1996-1997.

Mathematics, 2nd year in biology (adult class), 80h lectures and exercises, 1996-1999.

Initiation to Unix, Matlab and Maple, master math, 20h lectures and labs, 1996-1999.

Operational research, 3rd year professional diploma in CS and in CS applied to management (adult class), 50h lectures, exercises and projects, 1996-1999.

Numerical algorithms, 4th year in CS, 28h exercises and labs, 1998-2000.

Scientific computing, 4th year in applied math, 36h exercises, 1999-2000.

Symbolic and numerical computation, modelling, preparation to the competition for math. teachers (agrégation), 30h lectures and exercises, 1999-2000.

Discrete optimization, 5th year professional diploma in math. engineering, 24h lectures and projects, 2000.

Teaching at École Normale Supérieure de Lyon and at ISFA

Algorithms and arithmetics, 5th year diploma in computer science and also in applied math., 24h with G. Villard, 2001-2003.

Applications of computer science, series of lectures for the doctoral students in sciences (computer science, mathematics, chemistry, biology. . .) 2001-2007.

Validation in scientific computing, master in computer science, 24h, 2006-2007.

Certified linear algebra, master in computer science, 24h with C.-P. Jeannerod and N. Louvet, 2009-2010.

Algorithms for certified linear algebra, master in computer science, 24h with C.-P. Jeannerod and N. Louvet, 2011-2012.

Numerical Algorithms, master pro in computer science, 12 to 20h (depending on the year) with C.-P. Jeannerod, V. Lefèvre and N. Louvet, 2012-2019.

In charge of the organization of the research schools (3 in 2017-2018, 2 in 2018-2019) for M1 and M2 students in computer science, cf. <http://www.ens-lyon.fr/DI/er-registration/?lang=en>.

Topical schools I have been giving lectures at an Autumn school on Parallel and Distributed Computing, Oujda, Morocco, October 1999, at a Spring school on Computer Arithmetic, Prapoutel, France, March 2001, at a School for Young Researchers, Grenoble, France, March 2004, at Spring schools on Accuracy and Reproducibility in Numerical Computations, Fréjus, France, March 2013 and May 2017, for professional training entitled “Controlling and improving the numerical quality of an industrial computing code”, Paris, France, organized every year by EXED, former Collège de l’X (continuing education office by Polytechnique) since 2013, at JDEV : Journées du Développement Logiciel, École Polytechnique, September 2013, and at École Jeunes Chercheurs et Jeunes Chercheuses en Informatique Mathématique, Lyon, January 2017.

7 Dissemination of scientific knowledge ($\simeq 20\%$ of my time)

I have been giving talks in collèges and lycées (6th to 12th grades) as an incentive to choose scientific careers, aiming at girls in particular: I gave over 45 talks since 2003 and I met over 3000 pupils. I attend "Mondial des Métiers" every year in Bron, I intervened at some other forums such as Dijon 2004, Cité des Sciences Paris in 2004 or "40 ans d'Inria" in Lille in 2007. More frequently and regularly, I have been at collèges and lycées spread in the whole Rhône-Alpes region (2-3 times per year): Crest, Buis-les-Baronnies, Charlieu, Gex, Rillieux-la-Pape, Oyonnax, Saint-Étienne, Décines, Boen-sur-Lignon, Villefranche-sur-Saône, Lyon, Brignais, Bourg-en-Bresse, Amplepuis, Montélimar, Lentilly, Villeurbanne, Cessy, Neuville-sur-Saône, Châtillon d'Azergues, Ceyzériat, Nantua, Saint-Romain-en-Gal. I have been taking part in events at Musée des Confluences, Insa...

I give scientific conferences, mostly about arithmetic, number systems and also magic and computer science, for pupils from primary school to high school or to a larger audience, more frequently since 2012 (2-3 conferences per year). These conferences were given mainly but not exclusively for the Science Fair in Lyon in October since 2004 and for the Mathematics Week in March in several socially disadvantaged lycées (sometimes chosen by rectorat), but also for specific events: Die in 2004, Thuir in 2012, twice in Bron, Annecy in 2014, for MathC2+ in Montbonnot in 2014, as the opening conference of Maths en Jeans congress in Lyon in 2014, for the Rallye des maths in Lyon in 2015. I gave over 25 talks and met around 2000 persons. I also took part in the "Tournée de π " in Lyon in 2017, taking place in Transbordeur theater with over 400 participants. I did magic tricks based on computer science (for instance, "telepathy" based on error correcting codes) at the Bibliothèque Lyon 8e in 2015, the Bibliothèque Lyon Part Dieu in 2017 and even in the street for the "Forum des Associations de Lyon 7e" in 2015, and I also helped a class of 10th grade of lycée Récamier, Lyon, to prepare a "magic" show.

I take part in "Coding gouters", either organized by Inria for the administrative staff in 2015 or by Mixteen twice a year at MMI: Maison des Mathématiques et de l'Informatique, Lyon.

I created and taught a curriculum for primary-school pupils (9-10 years old), based on the 4 chapters of the curriculum for high-school: 8 sessions of 45mn for 1 class the first year (cf. <https://pixees.fr/category/thematique/methodologie-de-la-mediation/partage-de-bonnes-pratiques/?cat=491>) and 2×12 sessions of 45mn to 1h30 for 2 classes the second year, in 2014-2016.

I have been giving conferences or taking part in "teaching the teachers": plenary conference at APMEP in 2003, 3 conferences in 2014-2015 related to the "ISN: Informatique et Sciences du Numérique" option at lycée, week sponsored by Google in 2016. In 2016, I met members of FERS (Fondation Entreprise Réussite Scolaire) and Ébulliscience to help them in the development of activities on computer science for primary schools.

Sharing experiments and experience about dissemination of scientific knowledge

I taught *Dissemination of Scientific Knowledge* to the 4th year of ENS de Lyon and doctoral school of U. of Lyon, 20h for doctoral school in 2016 and 2017, 10h for 4th year in 2018.

I have been invited to discuss my experience and experiments at various conferences and meetings: "Passeurs de Science" meeting of SIF in 2017 in Lyon, ESOF 2018 in Toulouse, 3es Rencontres Nationales de la Robotique Éducative in 2018 in Lyon for the more recent ones. I have given a TEDxINSA conference in 2016. I have written about "Informatique sans ordinateur" for Tangente (journal for mathematics teachers - hors série) of December 2017.

I belonged to the national committee for the national phase of the "Faites de la science" competition in 2010. I belonged to the scientific committee of CapMaths consortium from 2012 to 2016. I belong to the steering committee of the MMI (Maison des Mathématiques et de l'Informatique) of LabEx MILyon since

2011 and I participated in the creation of the recent exhibitions (Magimatique 1 and 2, Comme par hasard). I belong to the group "ISO : informatique sans ordinateur" that creates "unplugged" activities for schools since 2015; this working group meets twice a year in Lyon. I belong to the editorial board of interstices, a Website for dissemination of computer science, since 2016. I took part in the working group that created the "7 familles de l'informatique" playcards in 2018.

8 Experience diversity and geographic mobility

I did my PhD in Grenoble on the complexity of the parallel evaluation of arithmetic circuits. In Lille, I have worked on multiple precision arithmetic, on floating-point arithmetic, then on interval arithmetic with M. Petitot and on the parallelization of an algorithm based on this arithmetic with J.-F. Méhaut, Y. Denneulin and B. Planquelle. With Y.O. Mohamed El Hadj, we studied the parallelization of automatic speech recognition. Since October 2000, I work at INRIA within the Arenal project (LIP, Lyon), first on sabbatical leave and now as research scientist. My research covers many aspects of multiple precision and interval arithmetic (definition, standardization, implementation) and their use for reliable computing, be it for computing on large sets or for assessing the numerical quality of results computed using floating-point arithmetic.