Fast algorithms for discontinuity-preserving smoothing

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Abstract

Discontinuities in functions frequently encode significant information: for instance, they represent the boundaries of cellular structures in microscopic images, they correspond to change points in microarray data, and they define tissue layers in tomographic images. Since classical linear smoothing methods destroy this important information, discontinuity preserving models such as the Potts model, the Mumford-Shah model and the Blake-Zisserman models have been developed. Such free-discontinuity problems are algorithmically challenging as they lead to nonsmooth and nonconvex problems.

In the talk, we start discussing the one-dimensional case, and we look at algorithms for solving these problems efficiently, exactly and numerically stable. The methods involve dynamic programming and recurrence schemes for least squares or least absolute deviations. We in particular discuss a recent advance regarding efficient computation of smoothing splines with discontinuities. Then we turn to the higher dimensional case, where only approximative solutions are possible. We study splitting approaches based on the alternating method of multipliers or on iterative minimization. For the higher order models, where smoothing is based on higher order derivatives, we discuss at a recent splitting approach based on Taylor jets.

The talk is based on joints works with Lukas Kiefer and Andreas Weinmann.

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