The Sound of Silence

Geometric Filtering of EEG Signal.
Detection of High-Frequency Oscillations.

L3 / Bachelor Internship (2018)

1. **High-Frequency Oscillations**

   Definition, Clinical Relevance, Detection
2. **Time-Frequency Analysis**

   Motivation, Basic Principles and Modern Approaches
3. **T.F. Analysis for HFO Detection**

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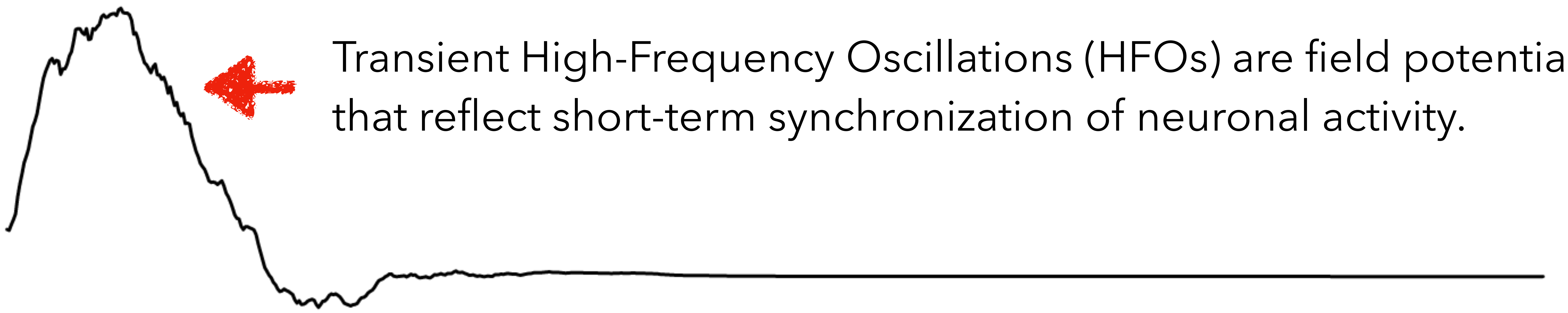
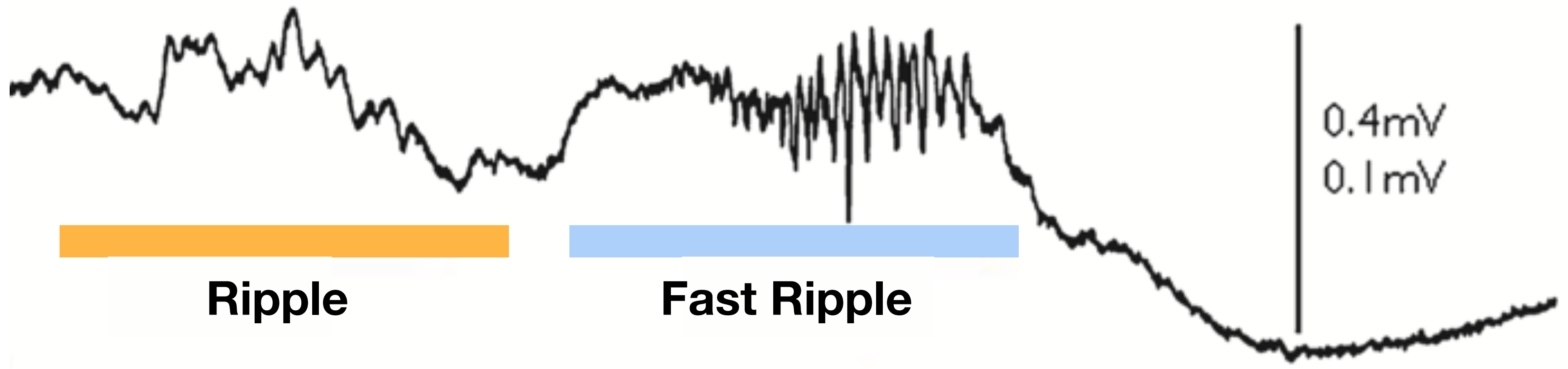
High Frequency Oscillations



Transient High-Frequency Oscillations (HFOs) are field potentials that reflect short-term synchronization of neuronal activity.

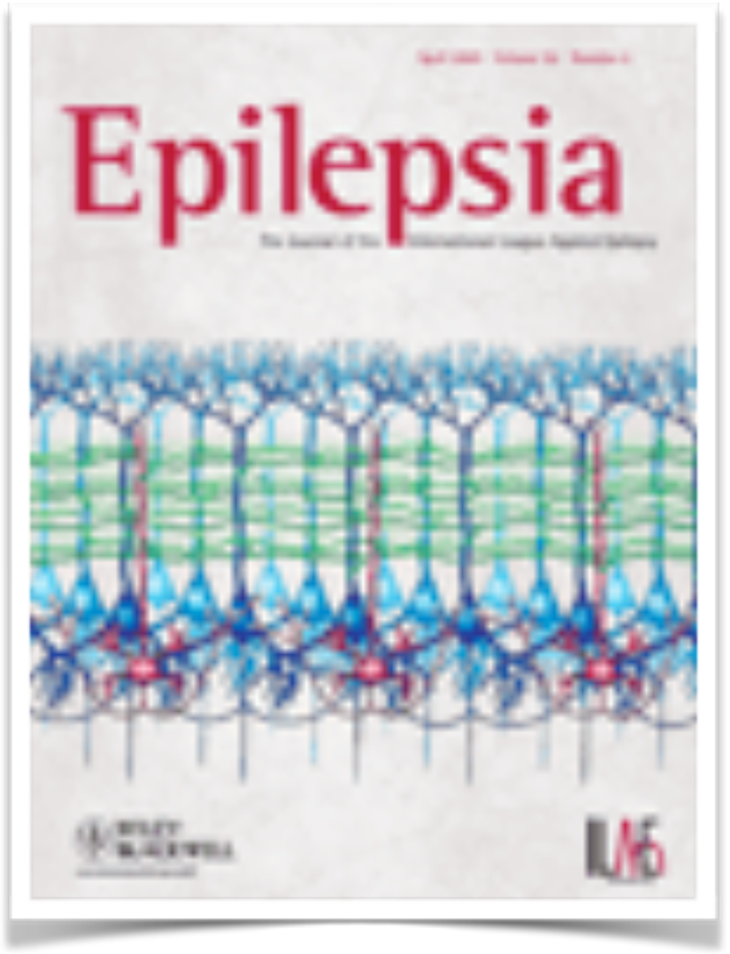


Ripple (80-250Hz)

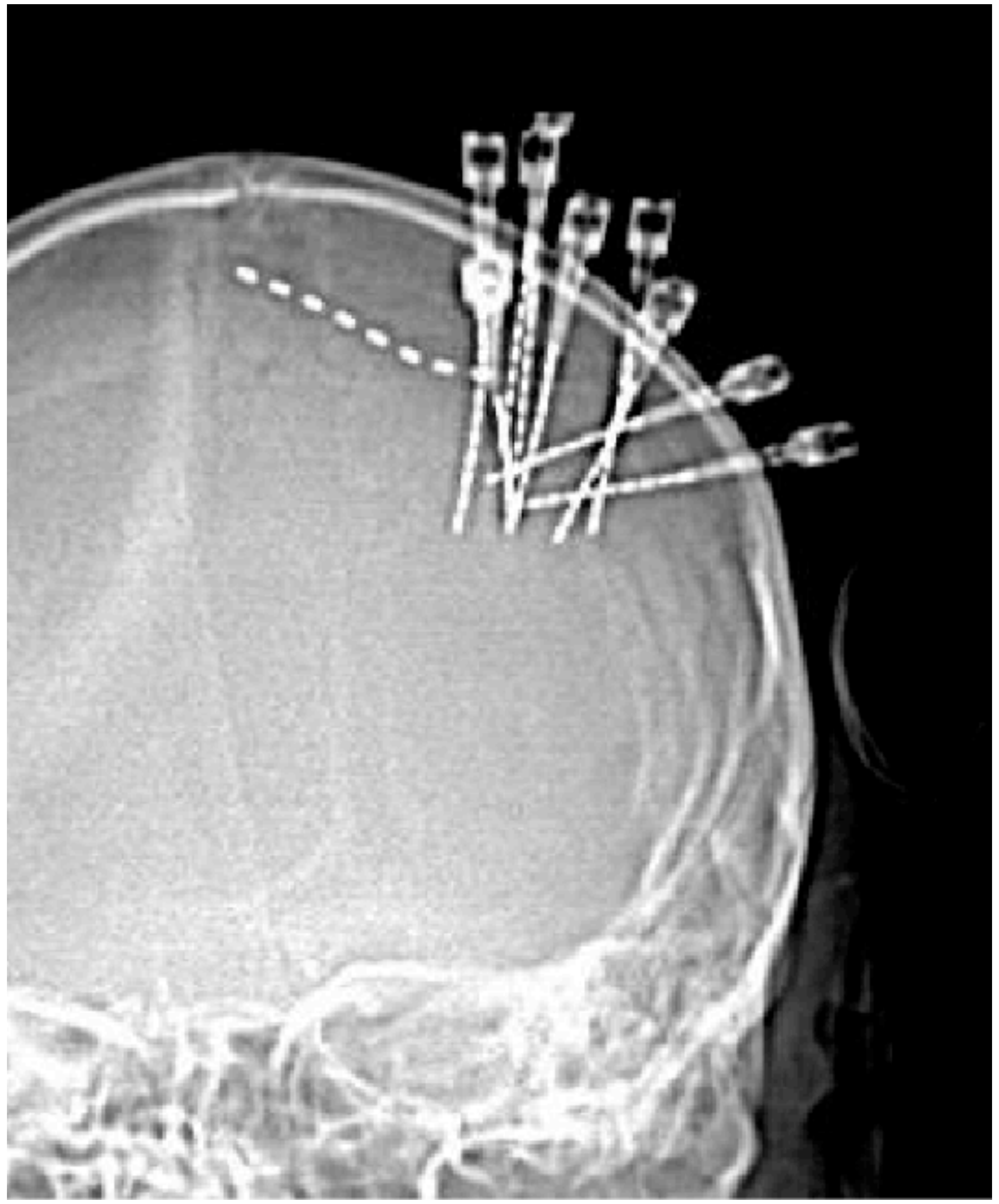
Fast Ripple (250-500Hz)

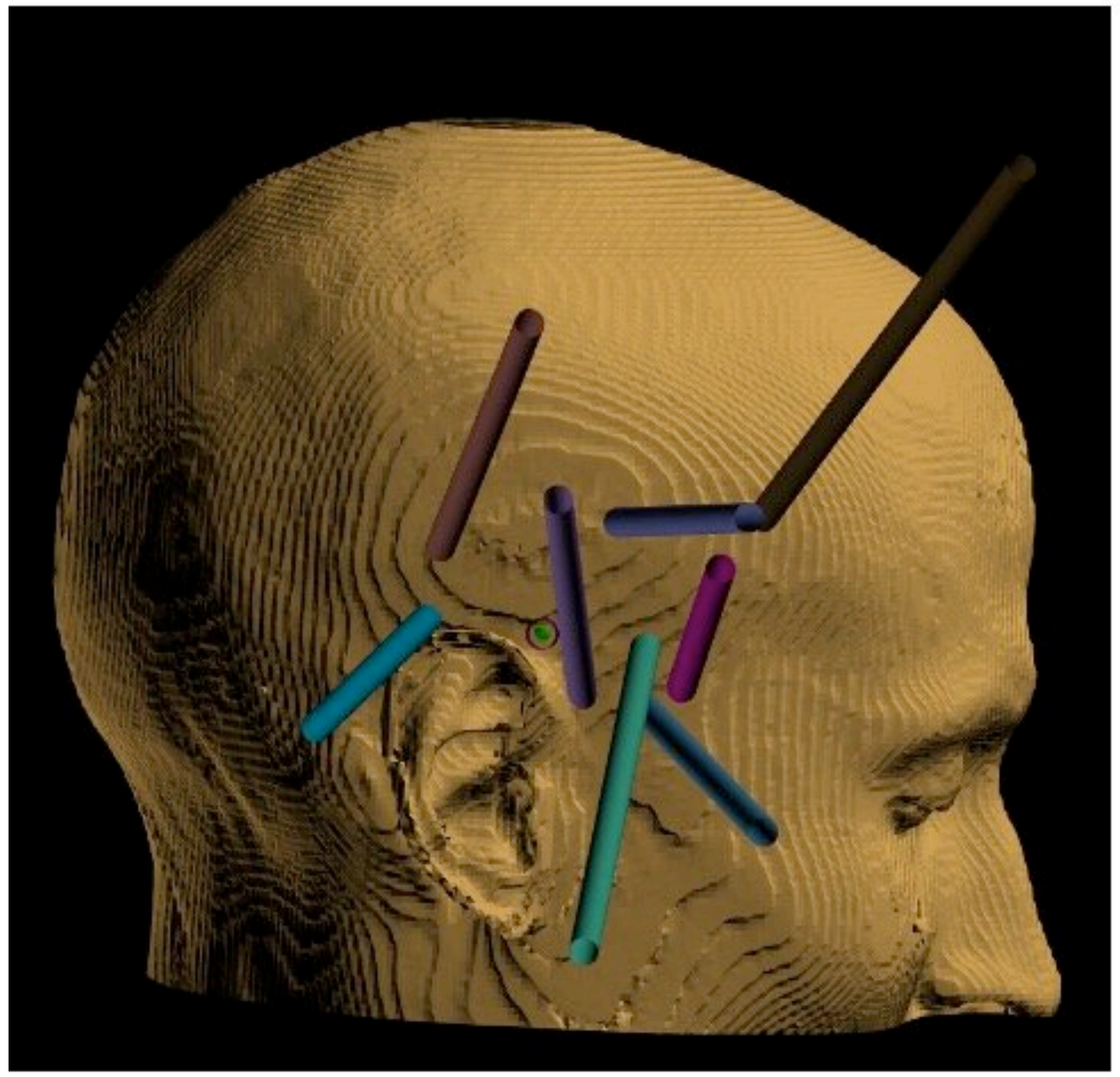
HFO (especially fast-ripples) may provide information on the **epileptogenic brain areas** (2018 state of the art). Therefore, we want to look at their signature in sEEG to extract relevant information for epileptic patients.

Engel Jr, J., Bragin, A., Staba, R., & Mody, I. (2009). High-frequency oscillations: What is normal and what is not? Epilepsia, 50(4), 598–604. doi:10.1111/j.1528-1167.2008.01917.x



Primer on sEEG

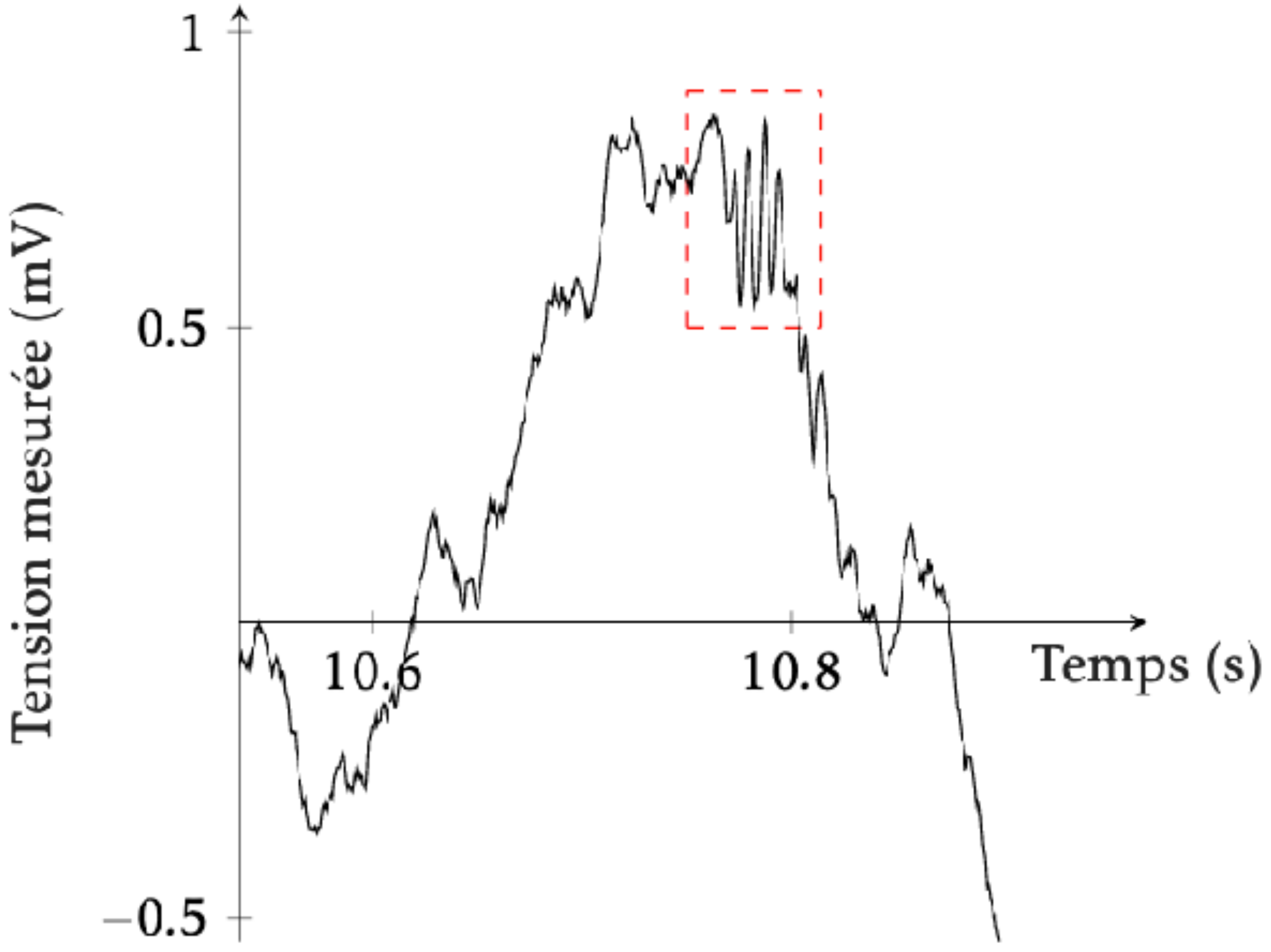




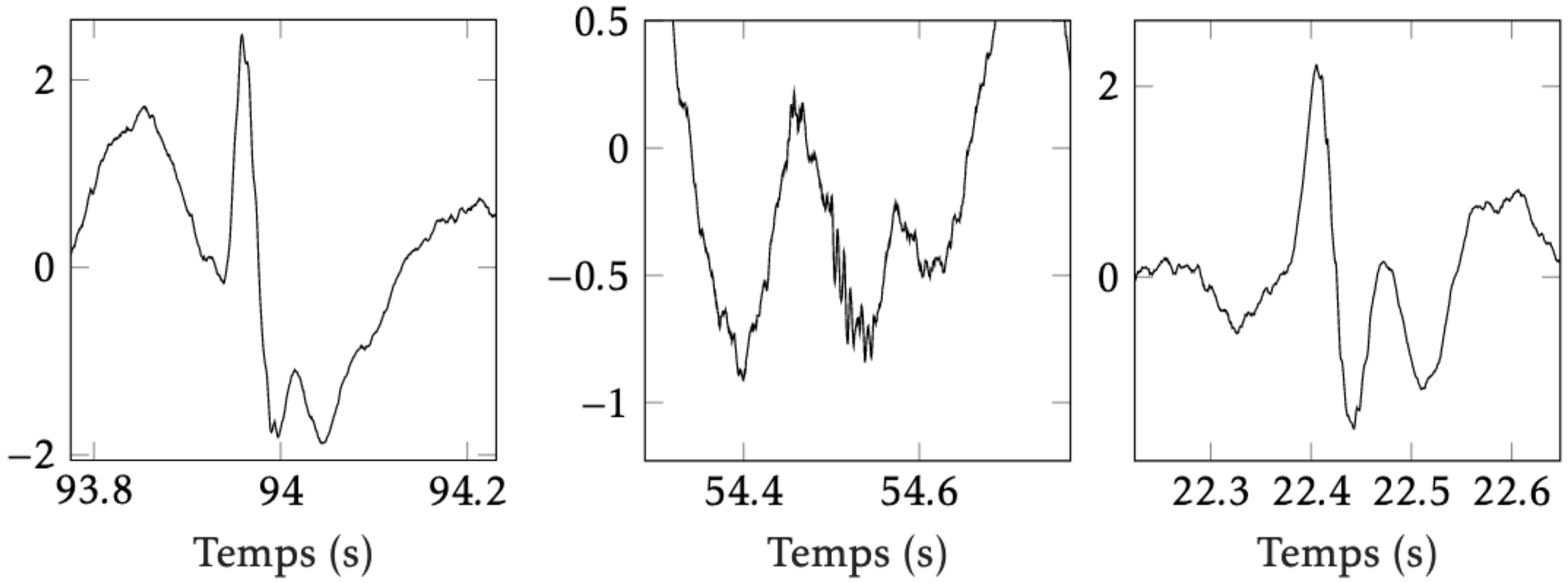
Stereotactic EEG. Invasive electrodes are implanted into the patient's brain, and provide spatially localized measures, often sampled in the kHz range thus allowing to study high frequency events. *Image from Matthew Mian (https://mian-neurosurgery.com/seeg)*

Challenges in Detection

HFO are only one of the many physiological phenomenons that can be observed in (s)EEG signals.



Easy case. Some events are clearly visible and easily isolated, even to the inexperienced eye.

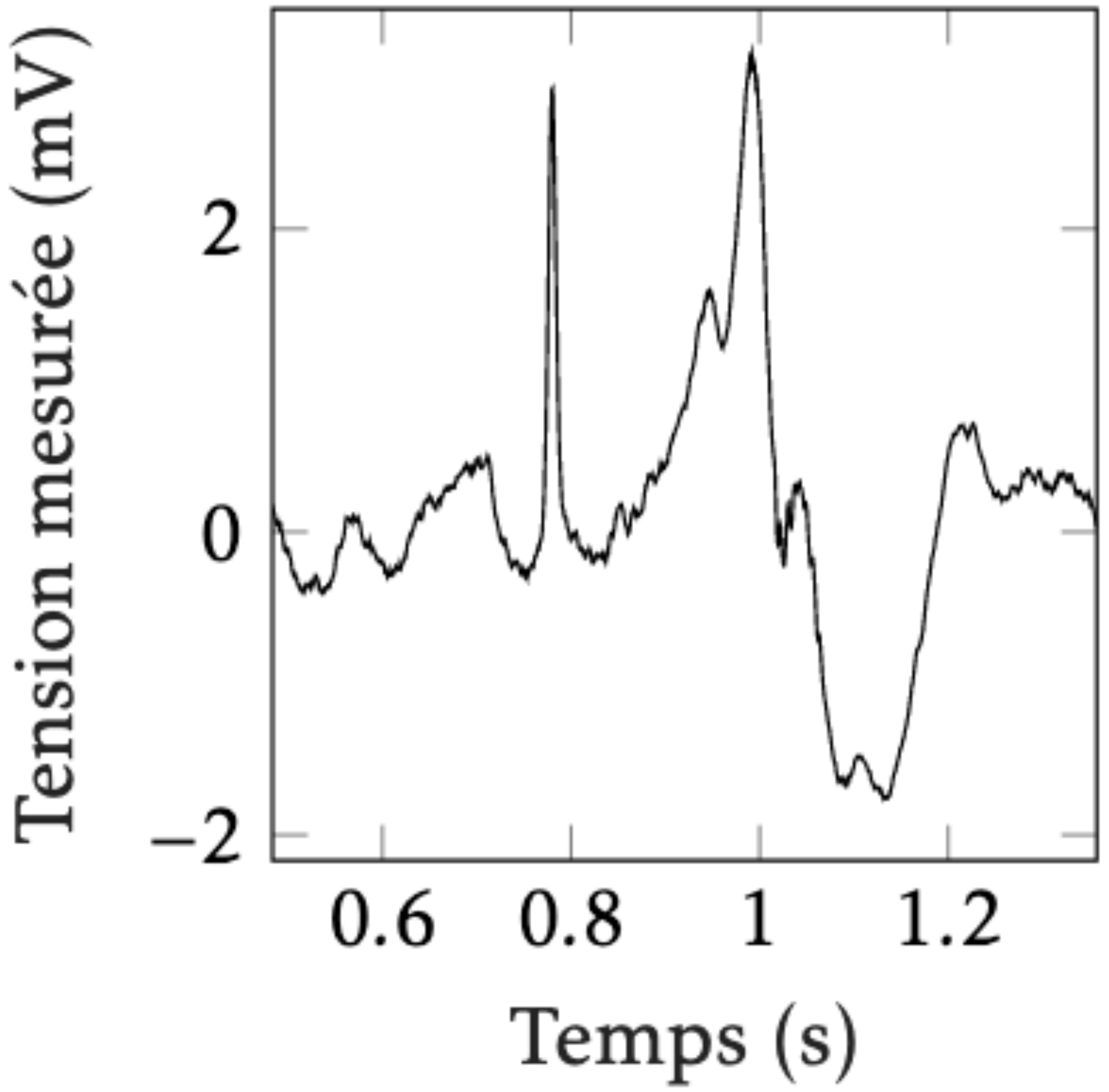


Harder case. HFO can happen simultaneously with other physiological events. From left to right:

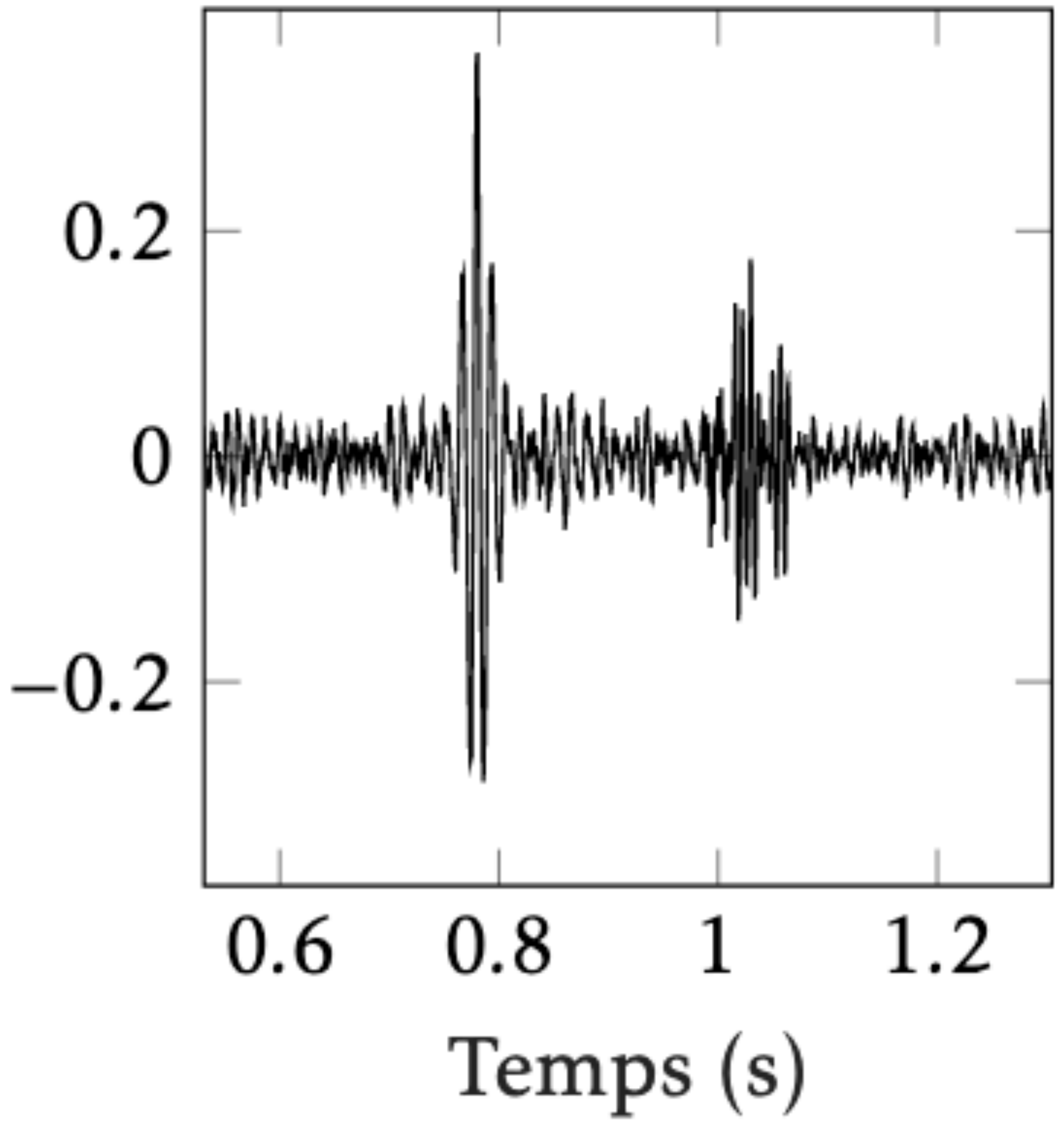
1. spike + ripple
2. ripple + fast-ripple
3. spike + ripple + fast-ripple

Challenges in Detection

Artefacts make the events harder to detect using standard filtering techniques.



Left. An artefact at t≈0,8s followed by two simultaneous HFO (ripple and fast-ripple) at t≈1s.

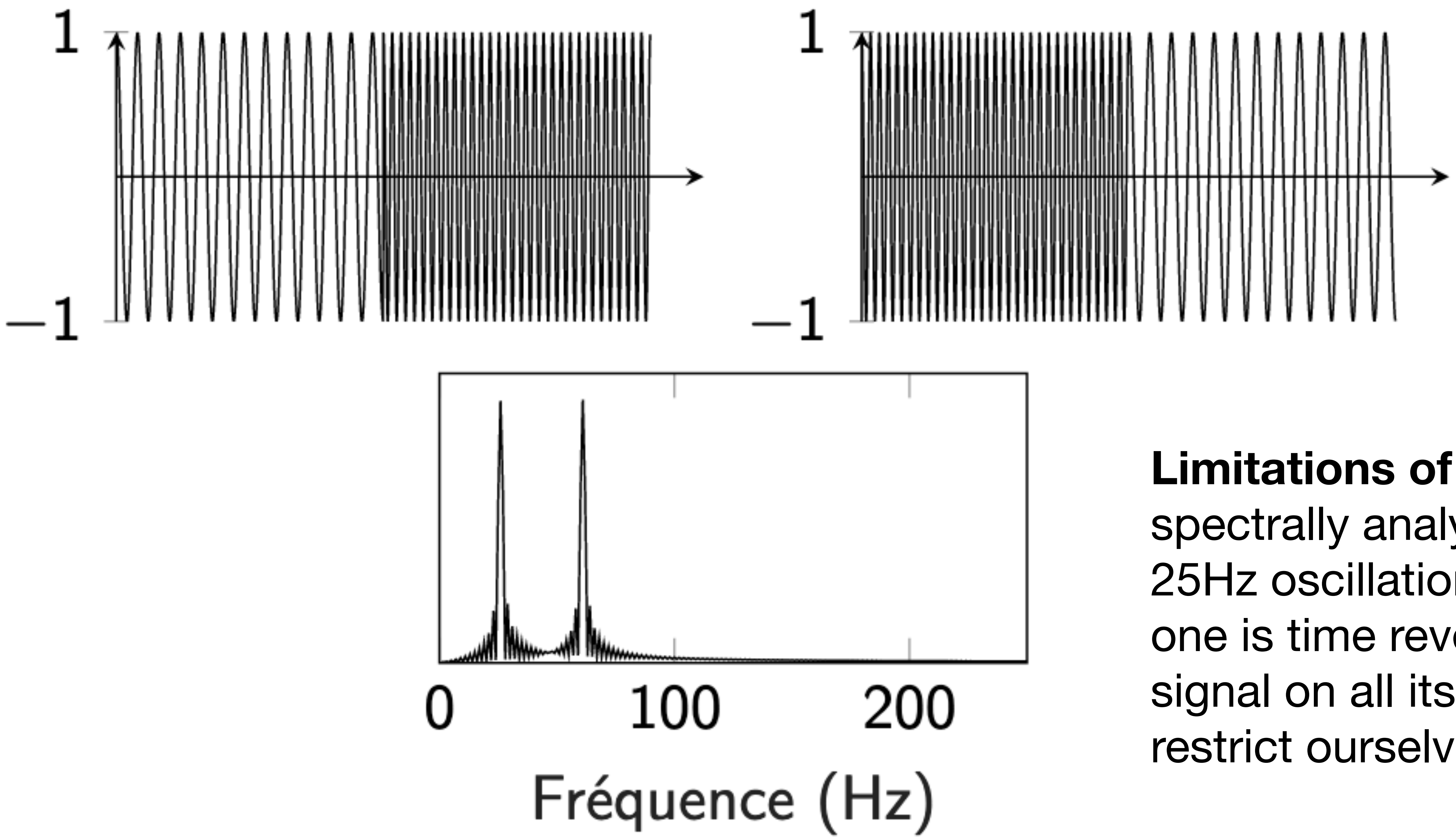


Right. Filtered signal, in the ripple band (80-250Hz)

The Gibbs phenomenon makes filters hard to use in order to single-handedly detect HFOs: their extended frequency range (from 80 to 500Hz) is prone to be polluted by transient events and artefacts.

Why Time-Frequency Analysis?

Explicit the link between time **and** frequency. Try to forget about the pure temporal (observed signal) and pure frequency (Fourier transform) and link the two of them.



Limitations of spectral analysis. Here two signals are spectrally analyzed. The first one is a succession of 0,25s of 25Hz oscillations and 0,25s of 60Hz oscillations. The second one is time reversed. The Fourier analysis, when analyzing this signal on all its duration is unable to distinguish the two (if we restrict ourselves to the amplitude of its coefficients).

Why Time-Frequency Analysis?

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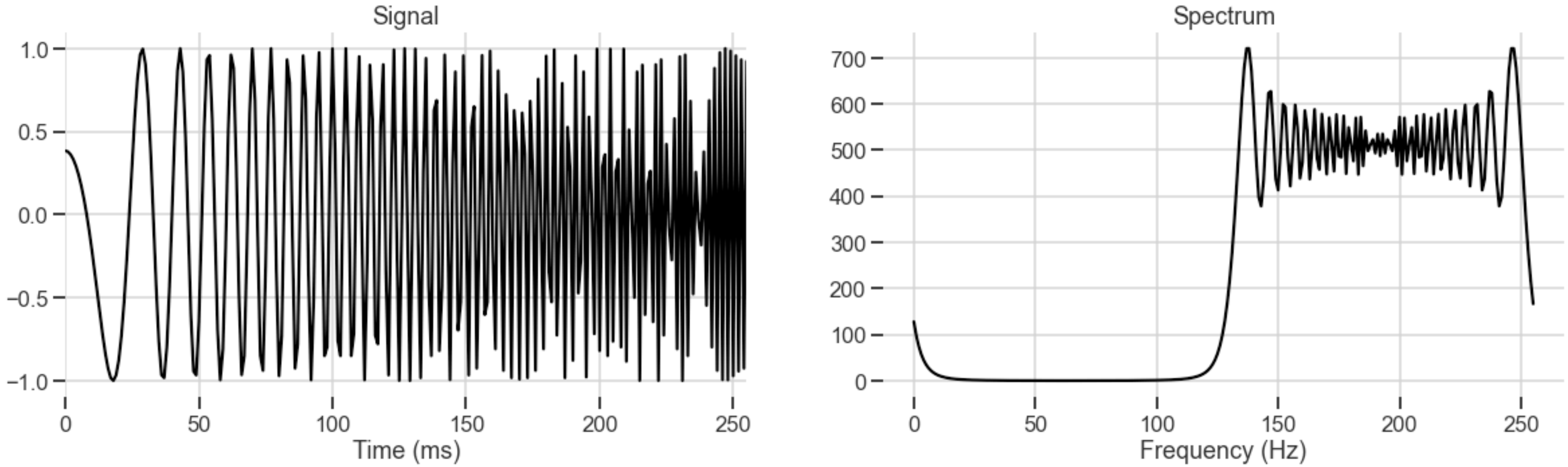
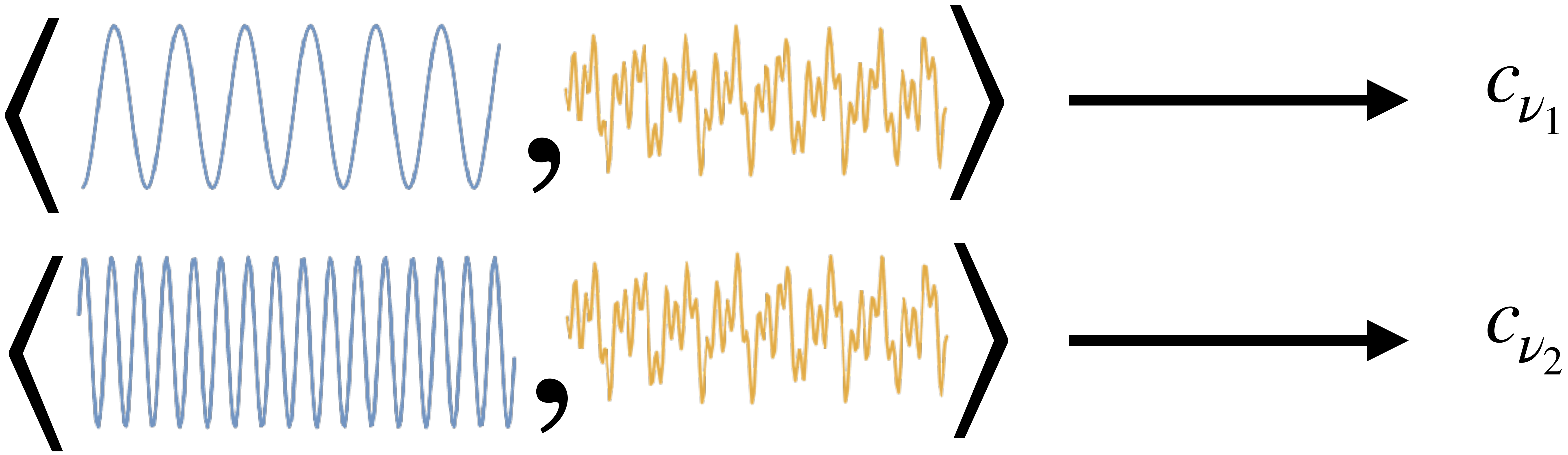


Limitations of spectral analysis. Here we perform a spectral analysis of a *chirp* (left), which is a signal whose frequency is linearly modulated over time. Think to a *slide-whistle* or $\cos(\omega t \times t)$. The resulting spectrum (right) is hard to analyze and does not really capture the underlying notion of *instantaneous frequency*.

Time-Frequency Analysis

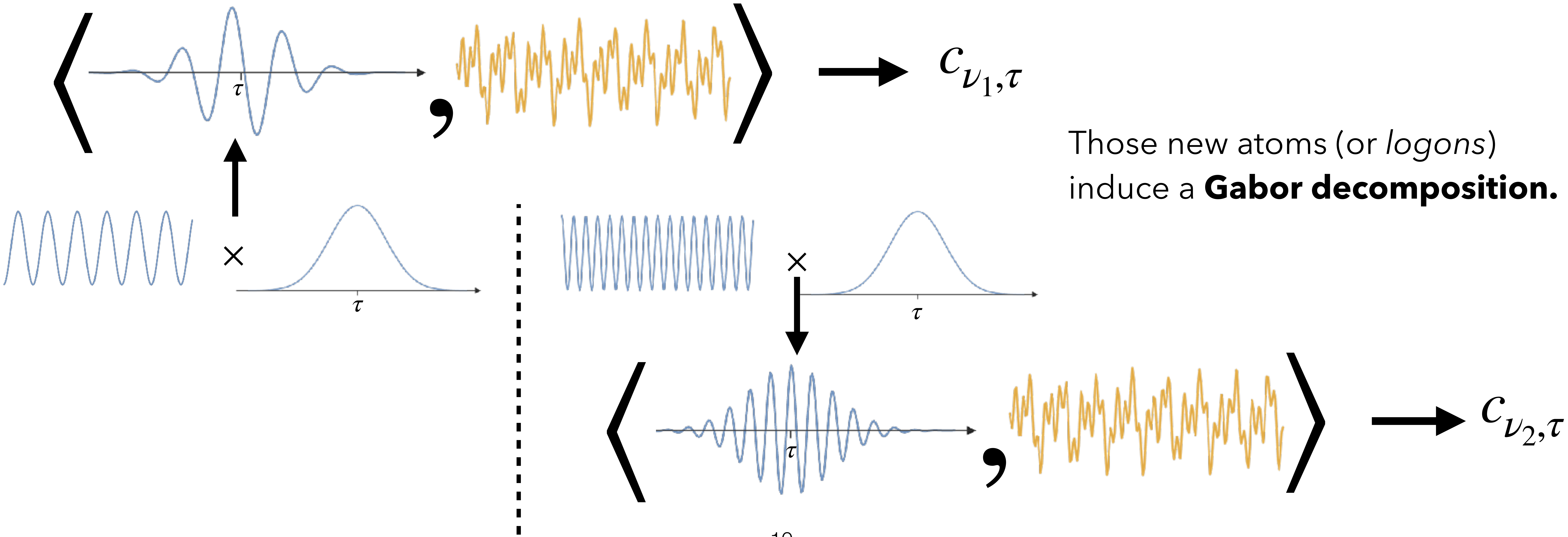
Back to Fourier

Regular Fourier analysis is about decomposing a **signal** on a **basis of atoms** whose frequencies are perfectly localized. For a frequency ν, one can compute the coefficient c_ν of this decomposition.



Time-Frequency Analysis

In time-frequency analysis, we seek to decompose a **signal** on a **basis of atoms** which are now localized **both in time and in frequency**. For a frequency ω and a time τ, one can compute the coefficient $c_{\omega,\tau}$ of this decomposition.



Those new atoms (or *logons*) induce a **Gabor decomposition.**

Time-Frequency Analysis

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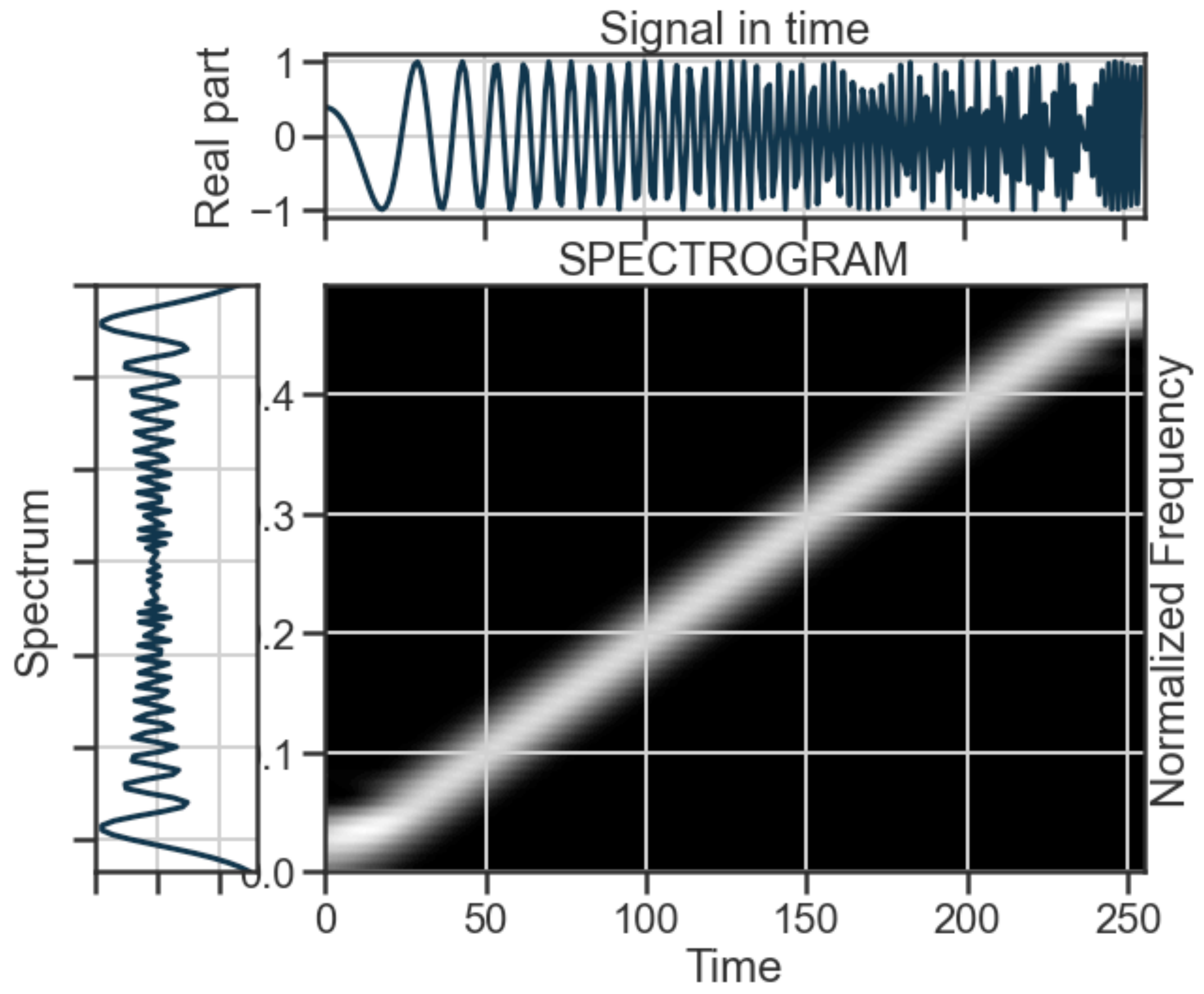
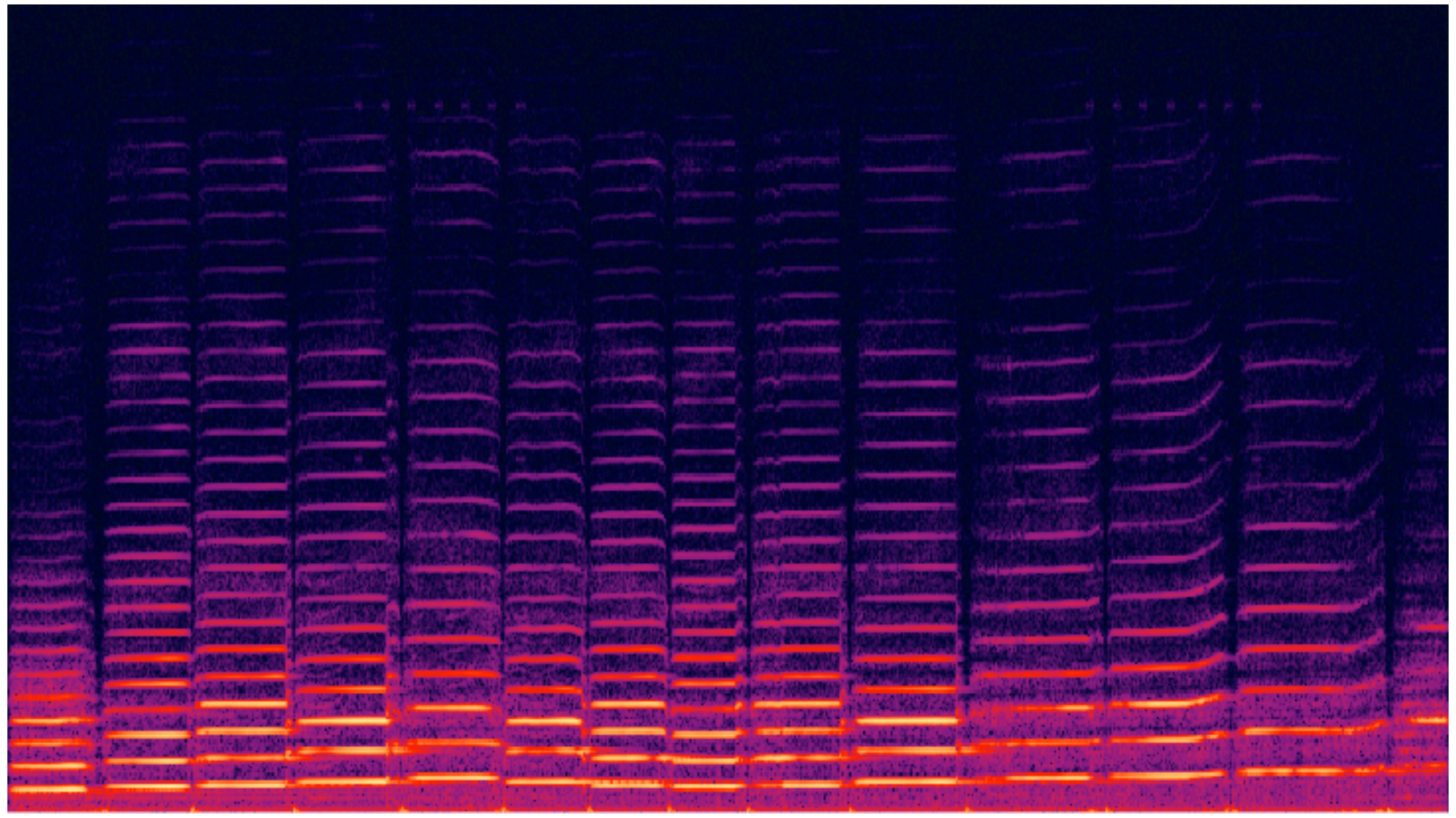
$c_{\nu,\tau}$

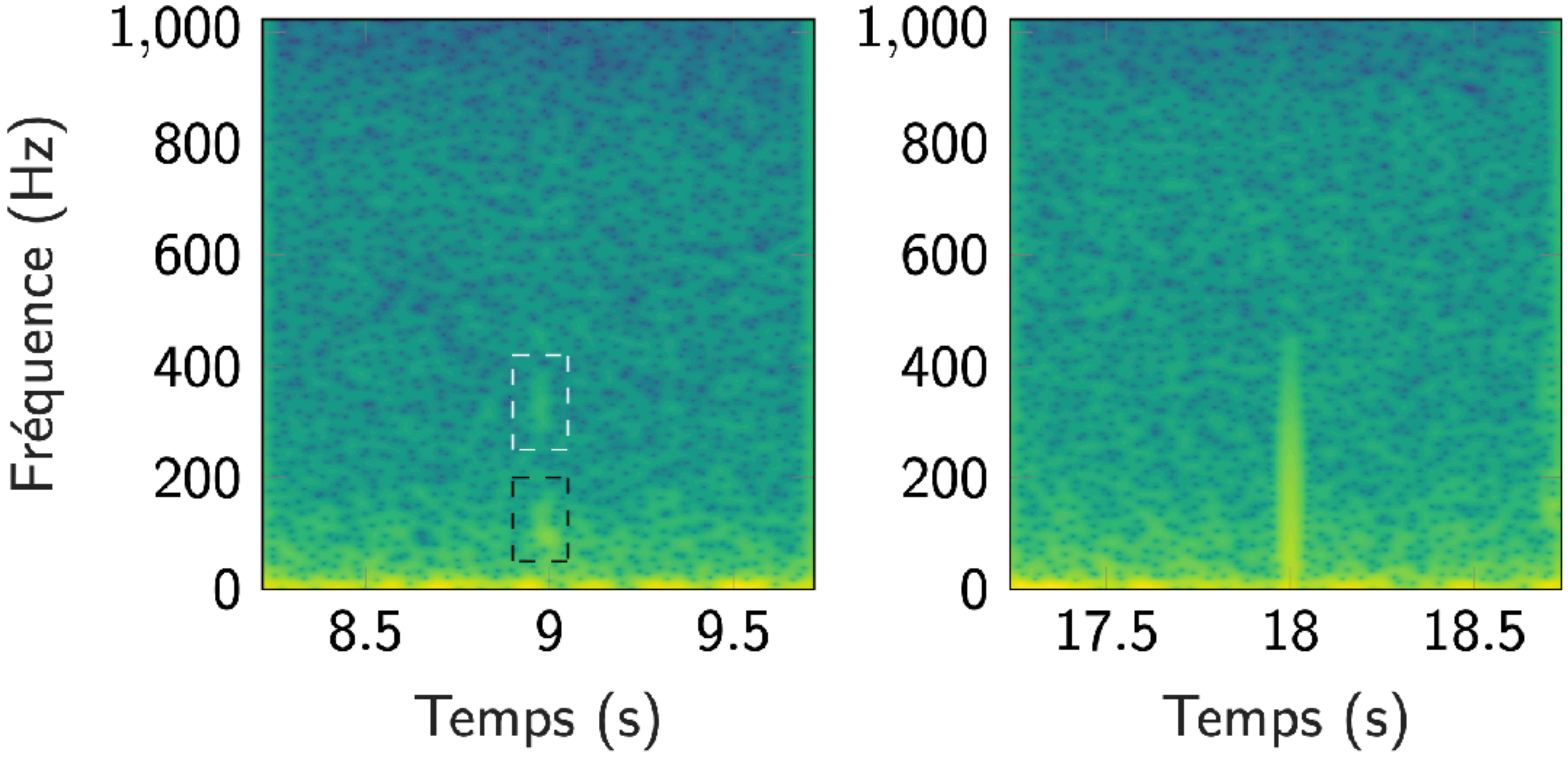




A spectrogram of a violin waveform. Created by User:Omegatron for WikiMedia.

Time-Frequency Analysis

Let's try to see how this applies to High-Frequency Oscillations! We can compute right-away some spectrogram of the data, and observe some signature of those events.



Time-Frequency Signature of sEEG data. Left: a simultaneous occurence of ripples and fast-ripples. Right: an artefact.

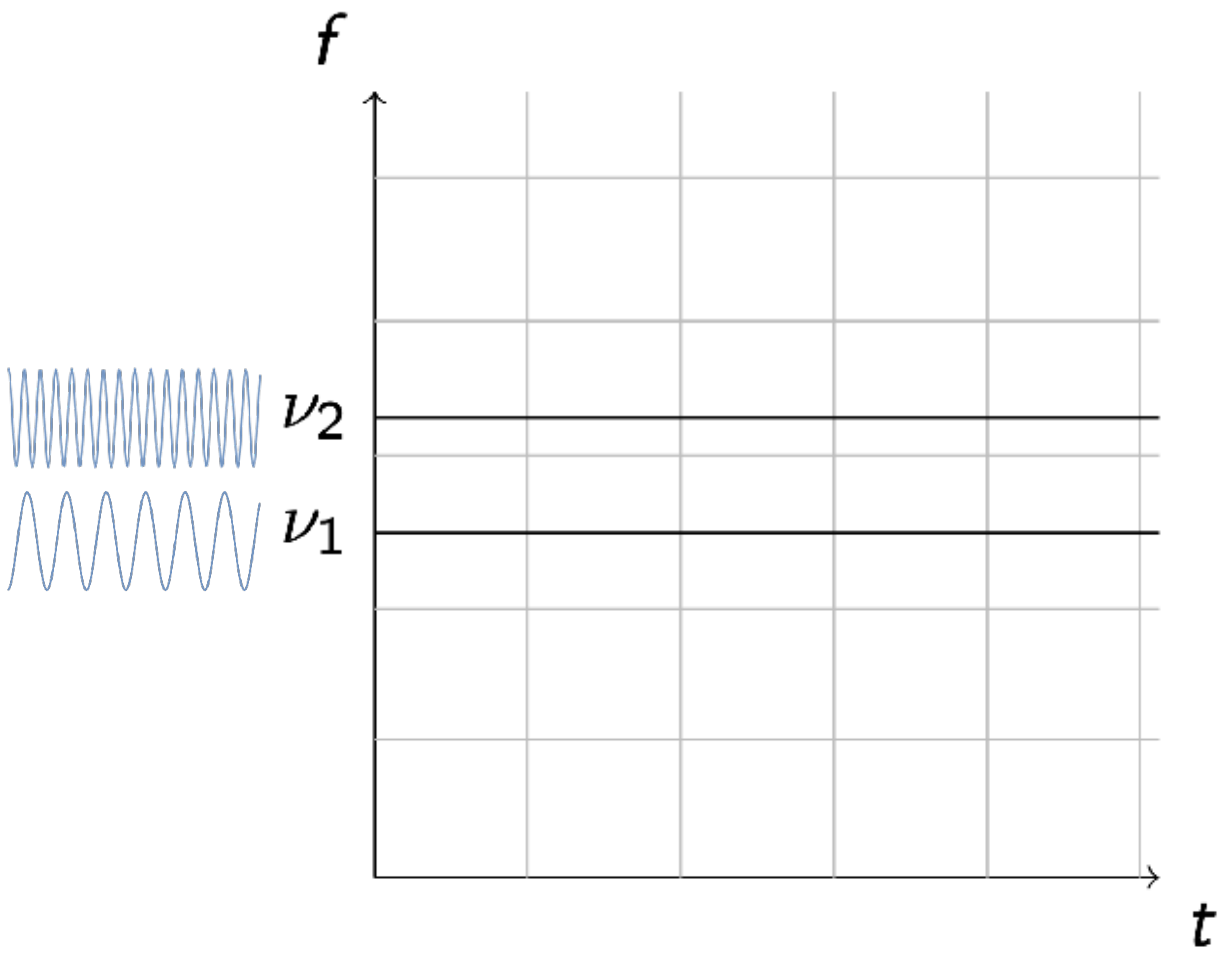
High-Frequency Oscillations often have present a peculiar **"island-like" signature** in the time-frequency plane.

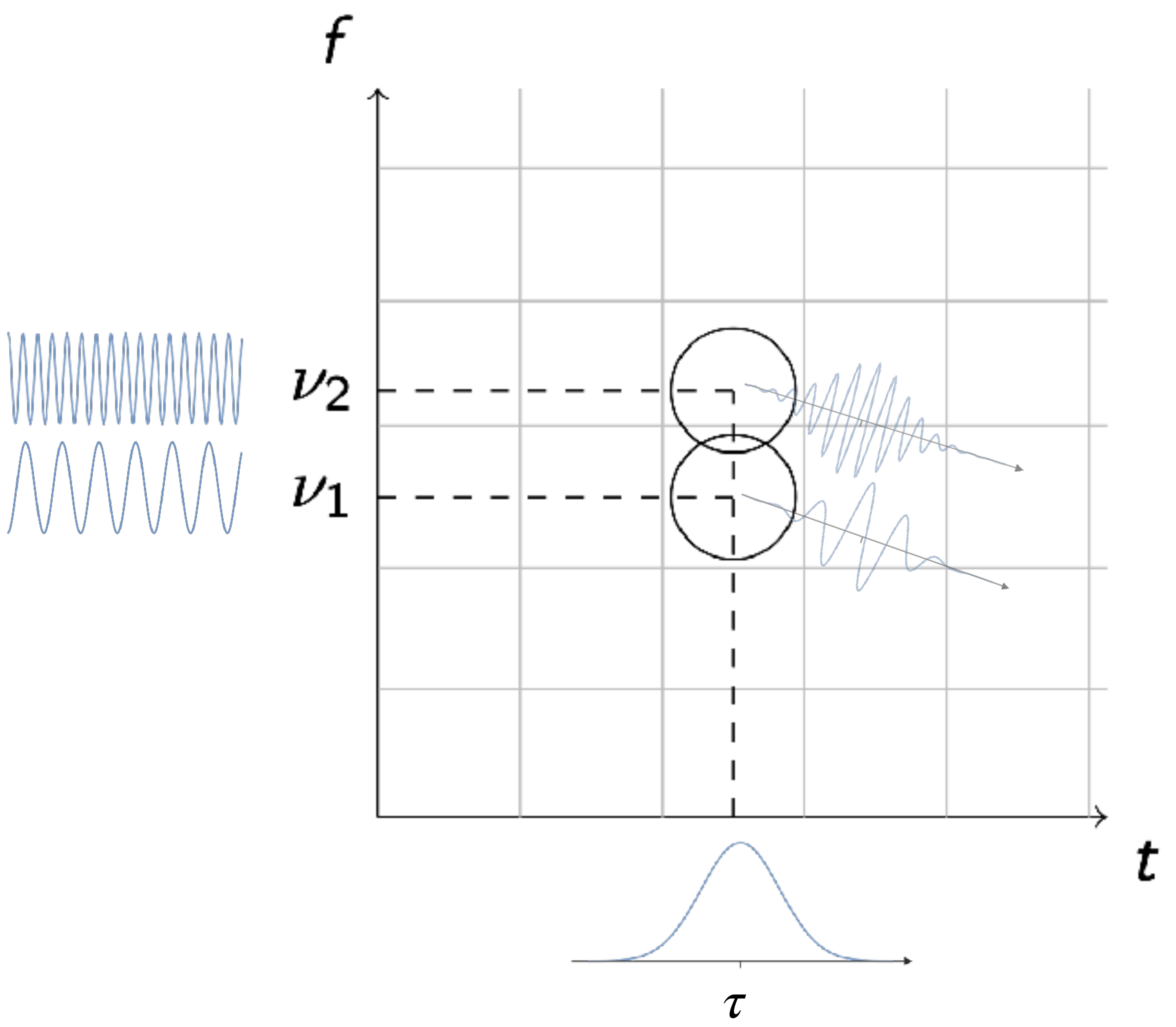
A number of methods have been proposed to detect HFO using this "characterization". Most of them were based on very ad-hoc schemes, with very specific heuristics, even though a precise definition was not agreed upon.

We proposed a new approach to extract precise descriptors of candidate events.

Time-Frequency Analysis

Introducing a localisation both in time and in frequency induces **uncertainty**.





This uncertainty (as in "spread of our atoms in the time-frequency plane") cannot be avoided. See the Heisenberg-Gabor inequality:

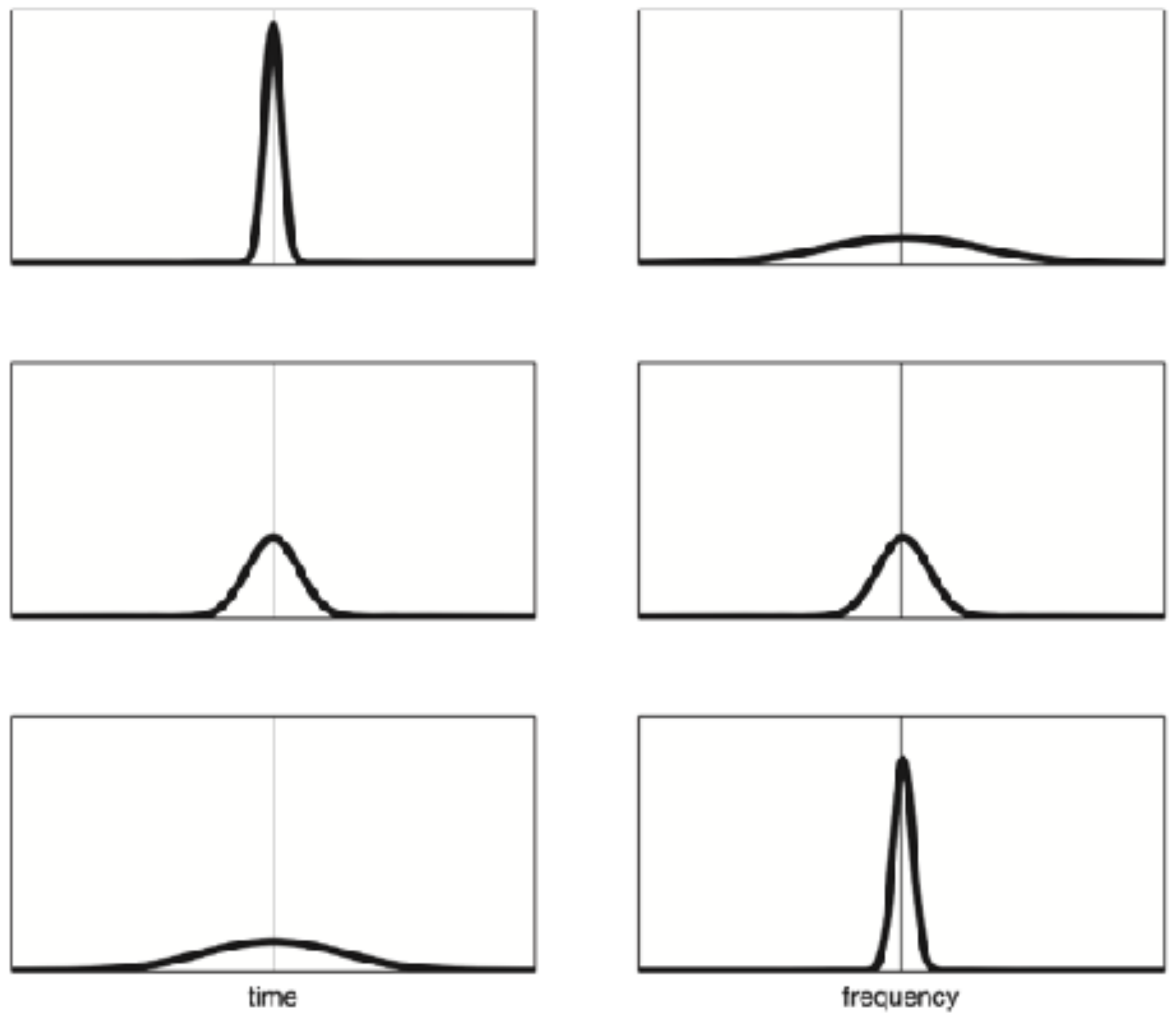
$$\Delta t \times \Delta \nu \geq \frac{1}{4\pi}$$

Fourier Transform. The atoms used are perfectly localized in frequency, but not at all in time.

Gabor Decomposition. The atoms used are now localized both in time and in frequency, but are spread on the time-frequency plane.

Time-Frequency Analysis

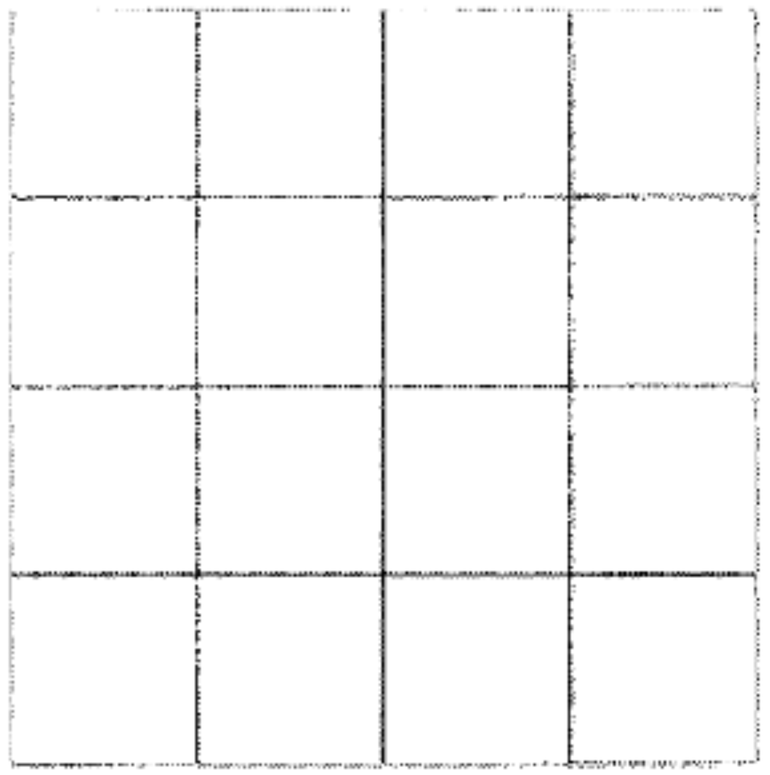
Why have we used gaussian windows and not just small sections of our signal (i.e. a rectangular window) and performed standard spectral analysis on those segments?

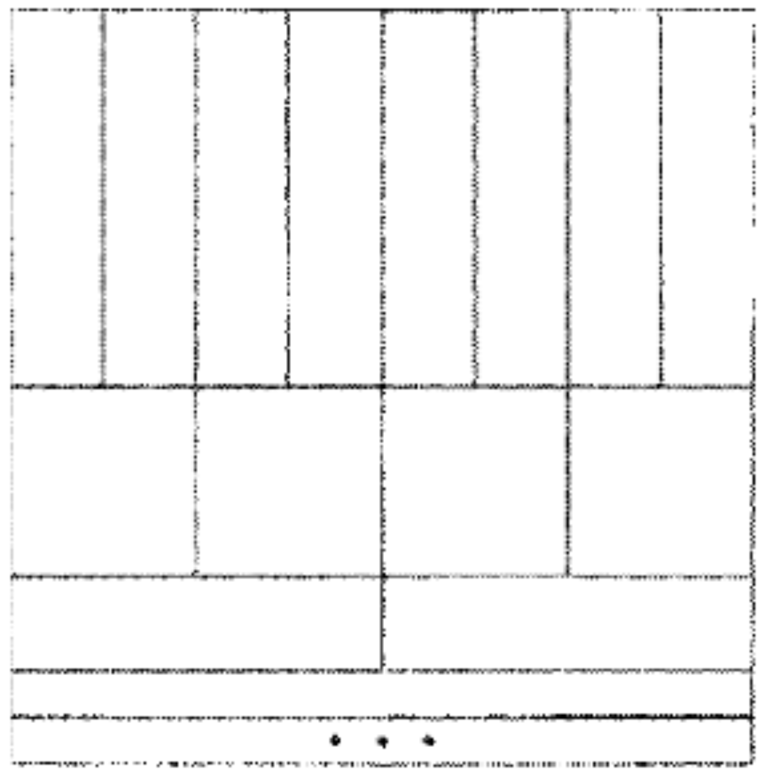


Gaussian atoms saturate the Heisenberg-Gabor inequality! Hence those are the ”most precise” atoms to span the time-frequency plane!

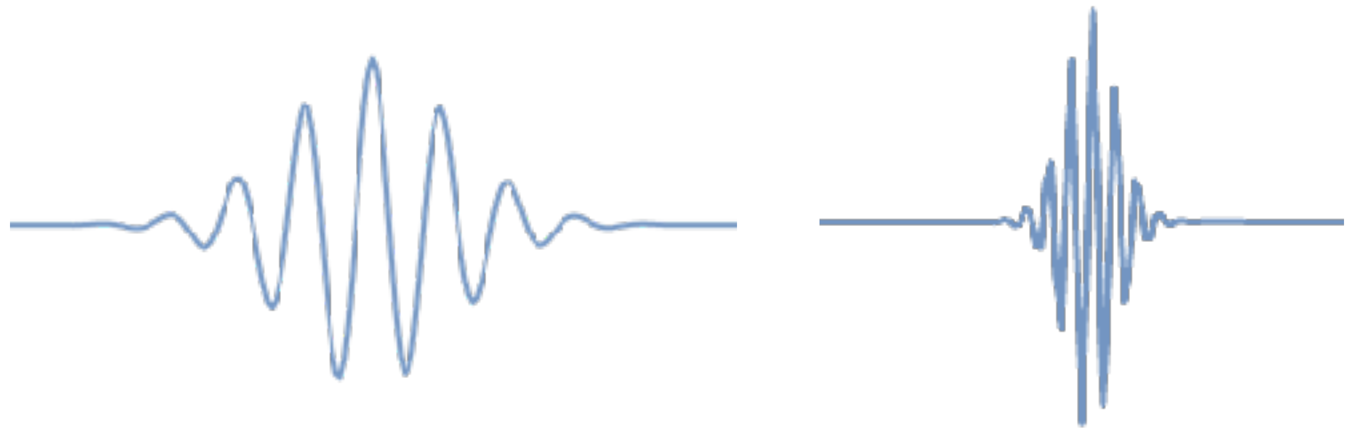
More windows (and procedures to construct atoms can be devised).

Gabor Decomposition. Atoms share the same “temporal scale” and are only modulated in frequency (+ time shifted).

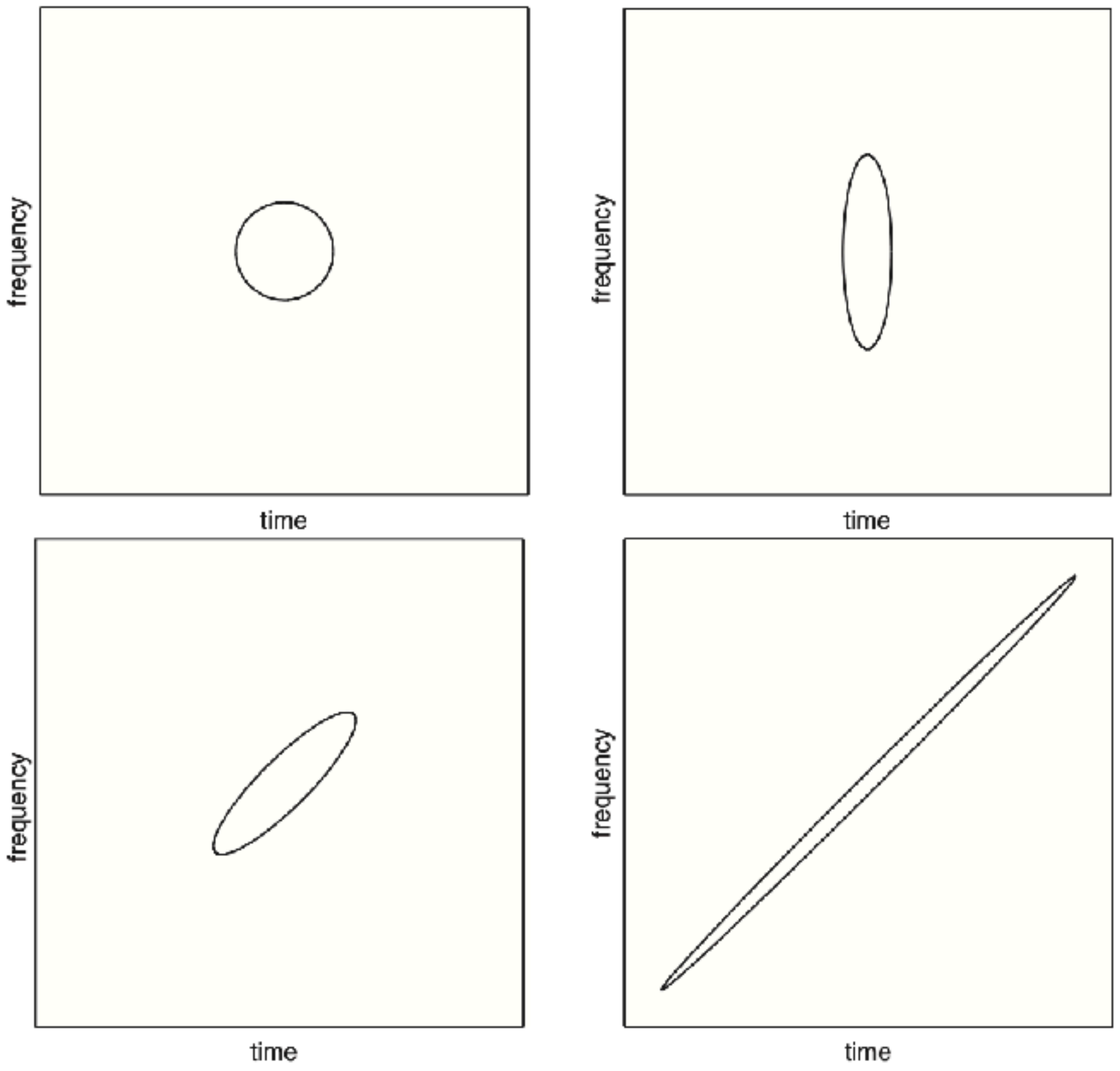




Wavelet Decomposition. Atoms are affine dilation of a common oscillatory element (+ time shifted).

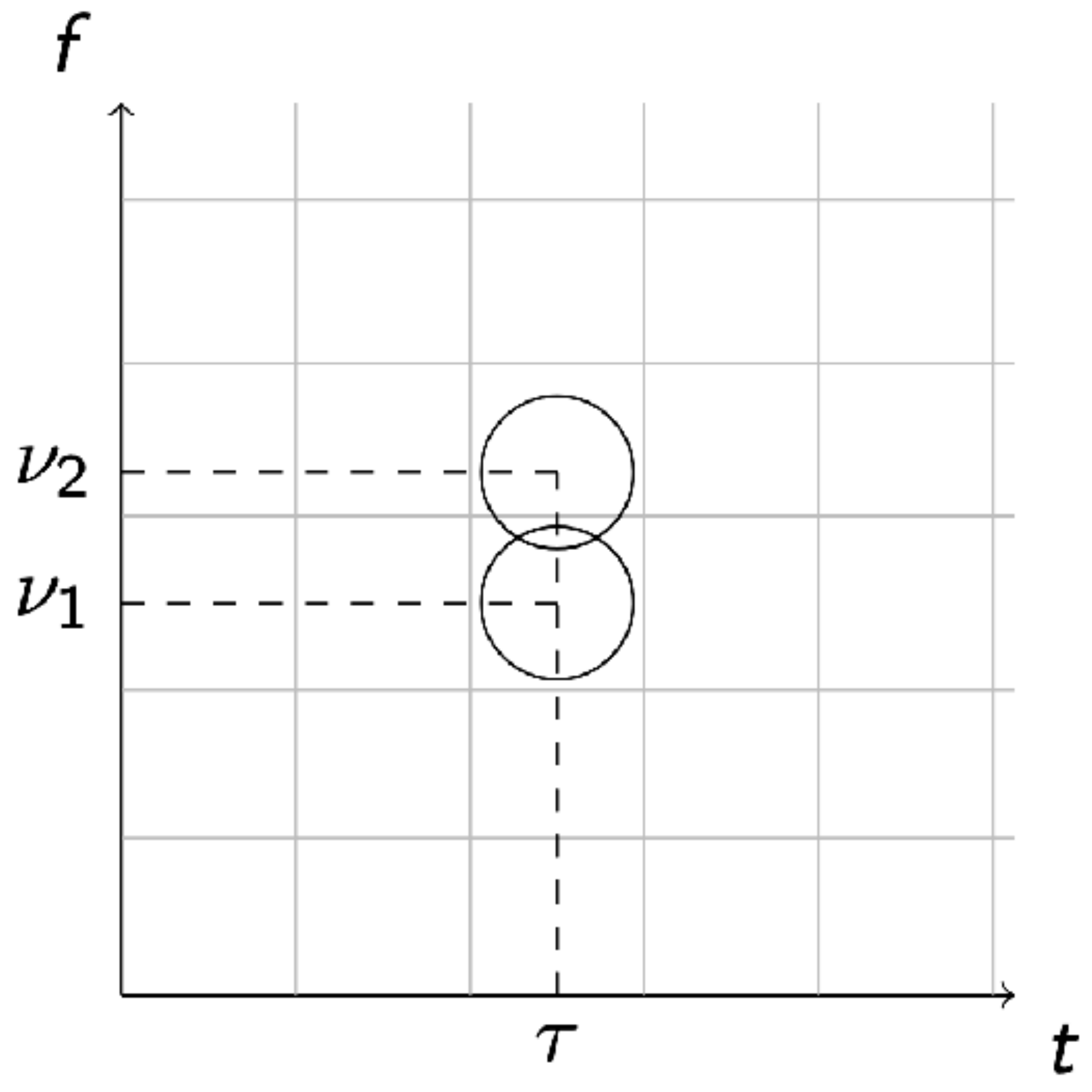


Time-Frequency Analysis



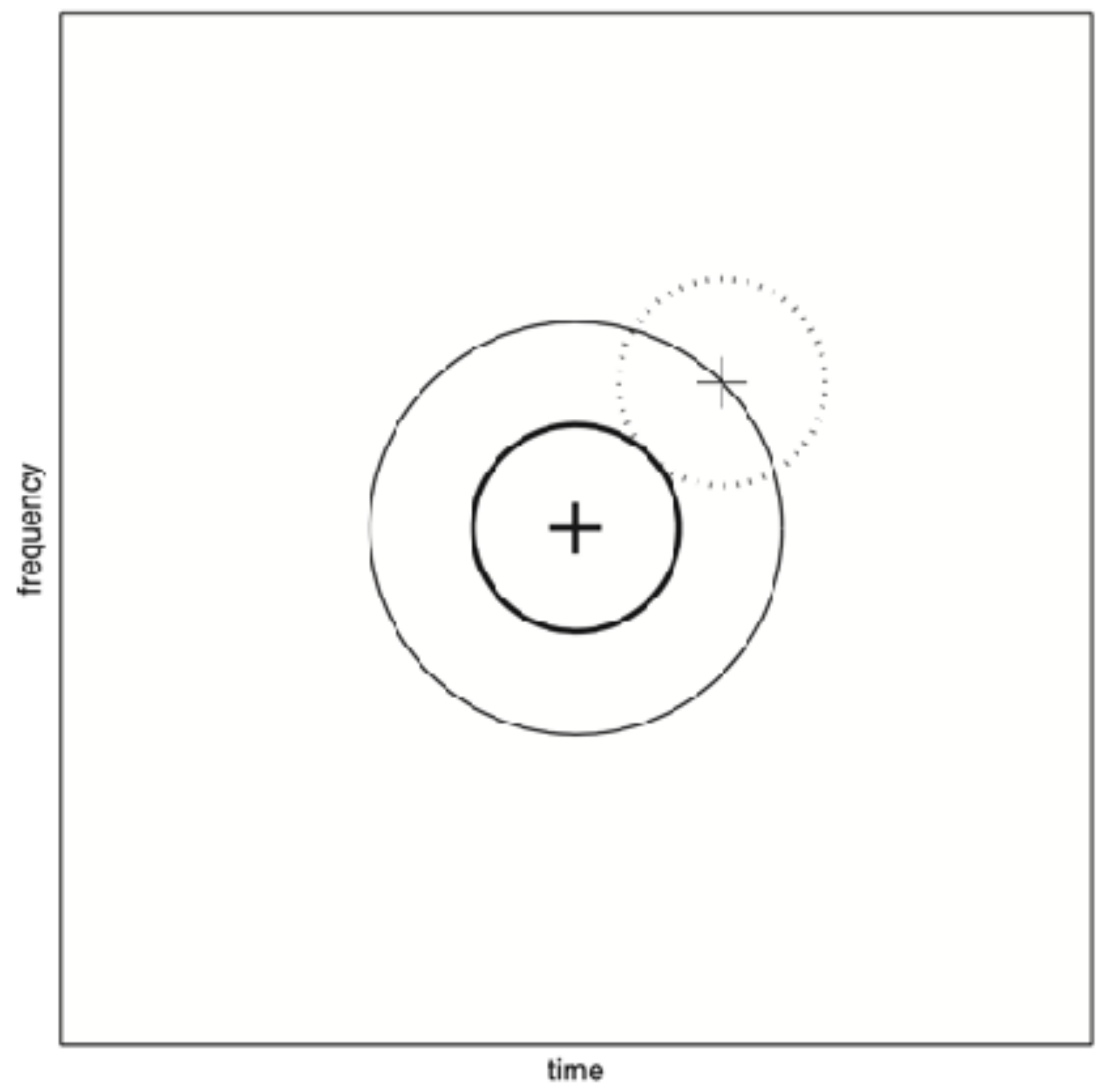
We can design other atoms that are anisotropic, and even be perfectly localized along a direction (but still not both in time and frequency).

Time-Frequency Analysis



Also note that due to the uncertainty, such decomposition are highly redundant: atoms overlap. Therefore all coefficients of the time-frequency plane are somehow linked. Formally, we exhibit a **reproducing kernel** K such that

$$c_{\tau,\nu} = \iint_{t,f} K(\tau, \nu, t, f)\ c_{t,f}\ \mathrm{d}t\mathrm{d}f$$



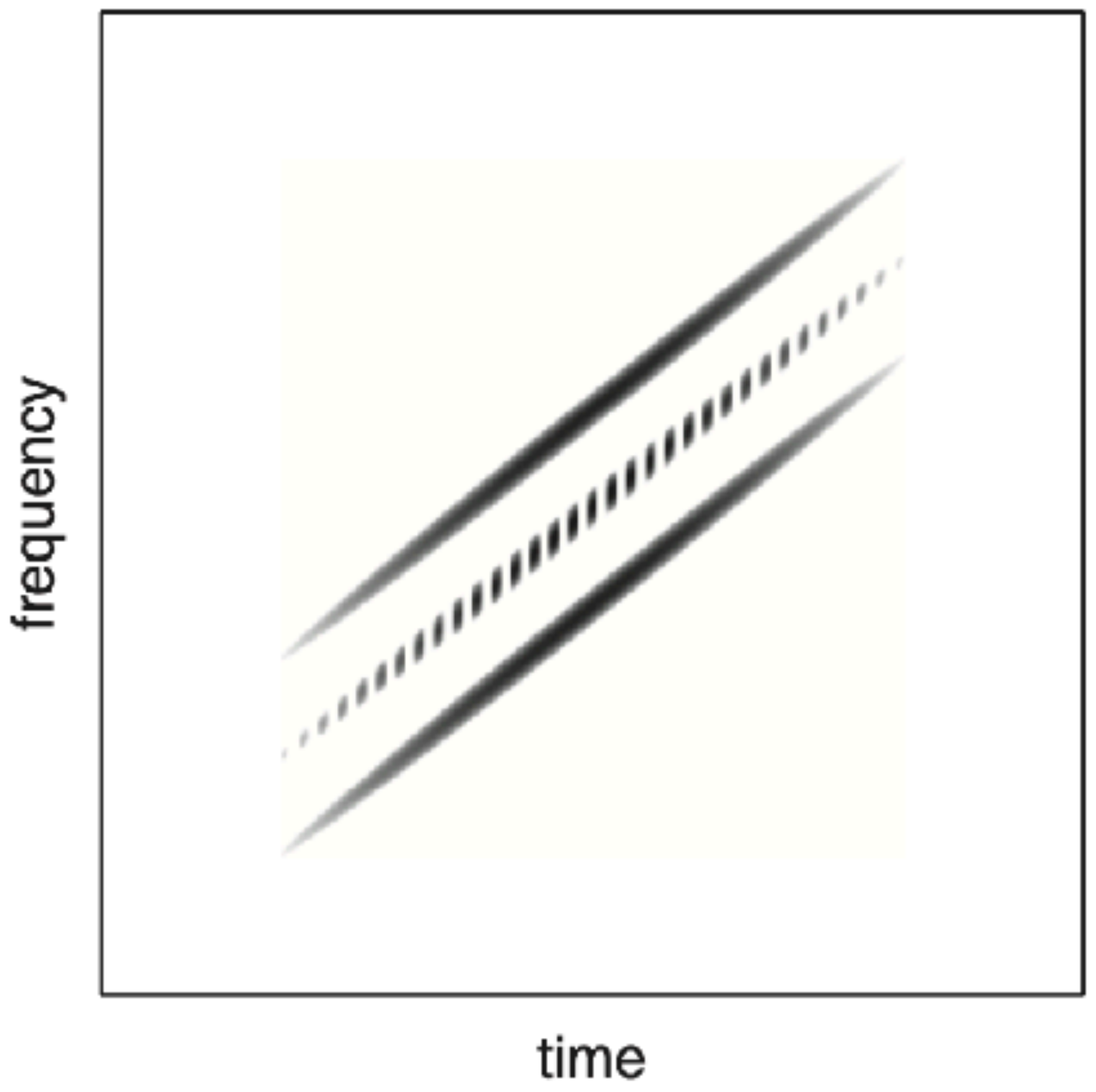
Reproducing Kernel. All coefficients in the time-frequency plane are linked together. (figure from *Explorations in Time-Frequency*, Flandrin.)

There is a strong connection between the chosen window and the actual kernel.

Spectrogram Sharpening

Exploiting Uncertainty

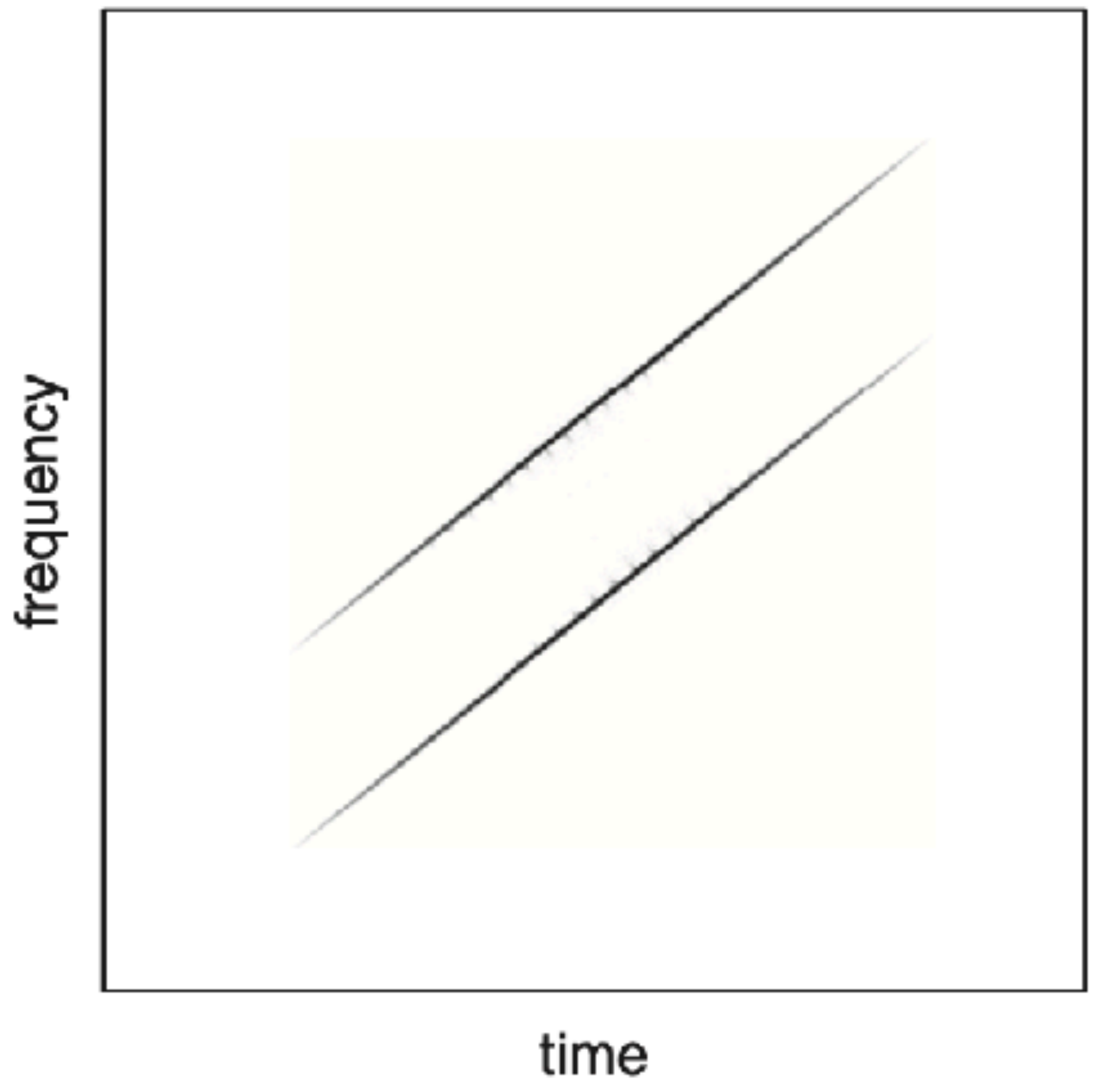
(a)



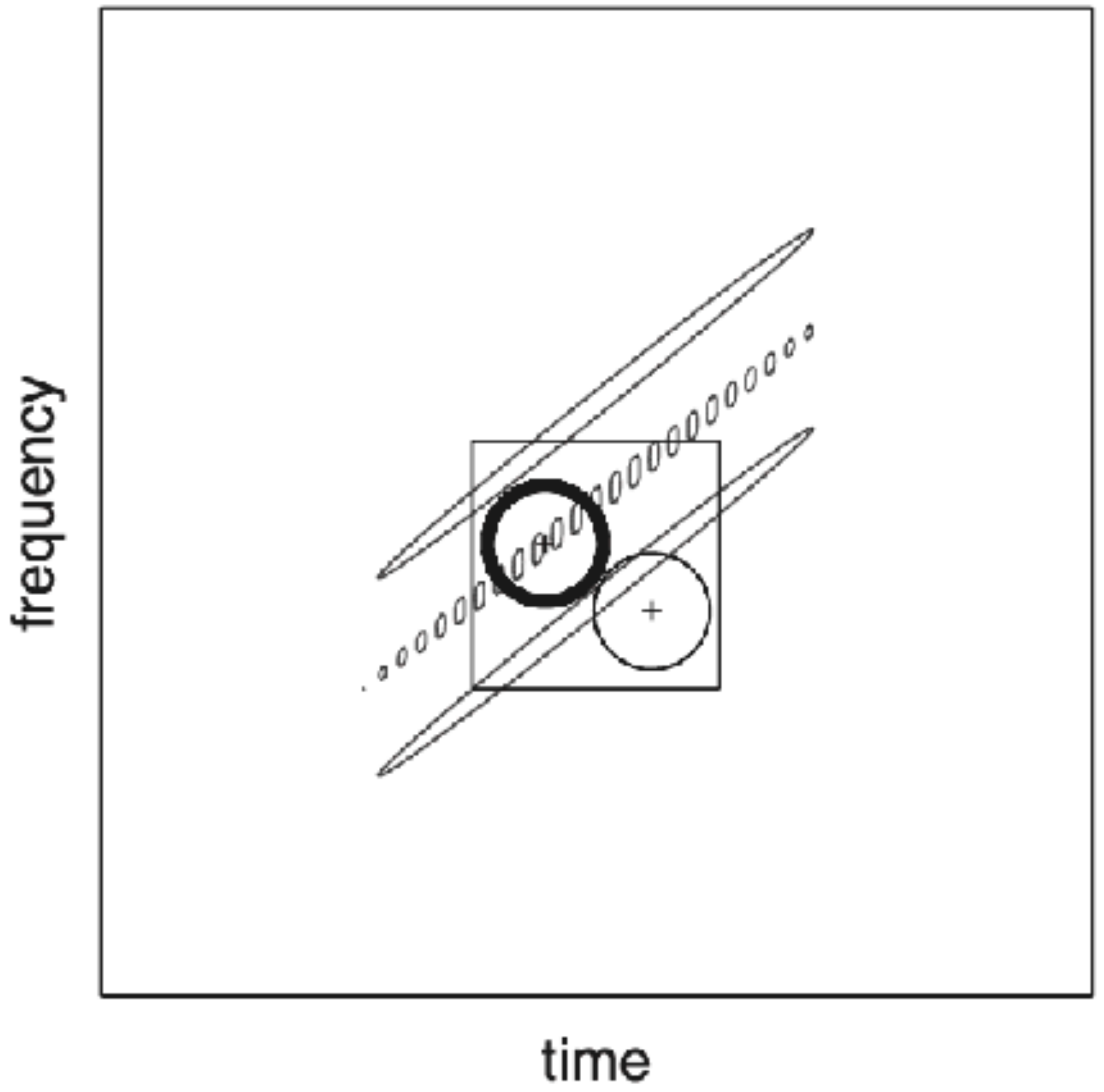
A common operation is to perform what is called **spectrogram reassignment**.

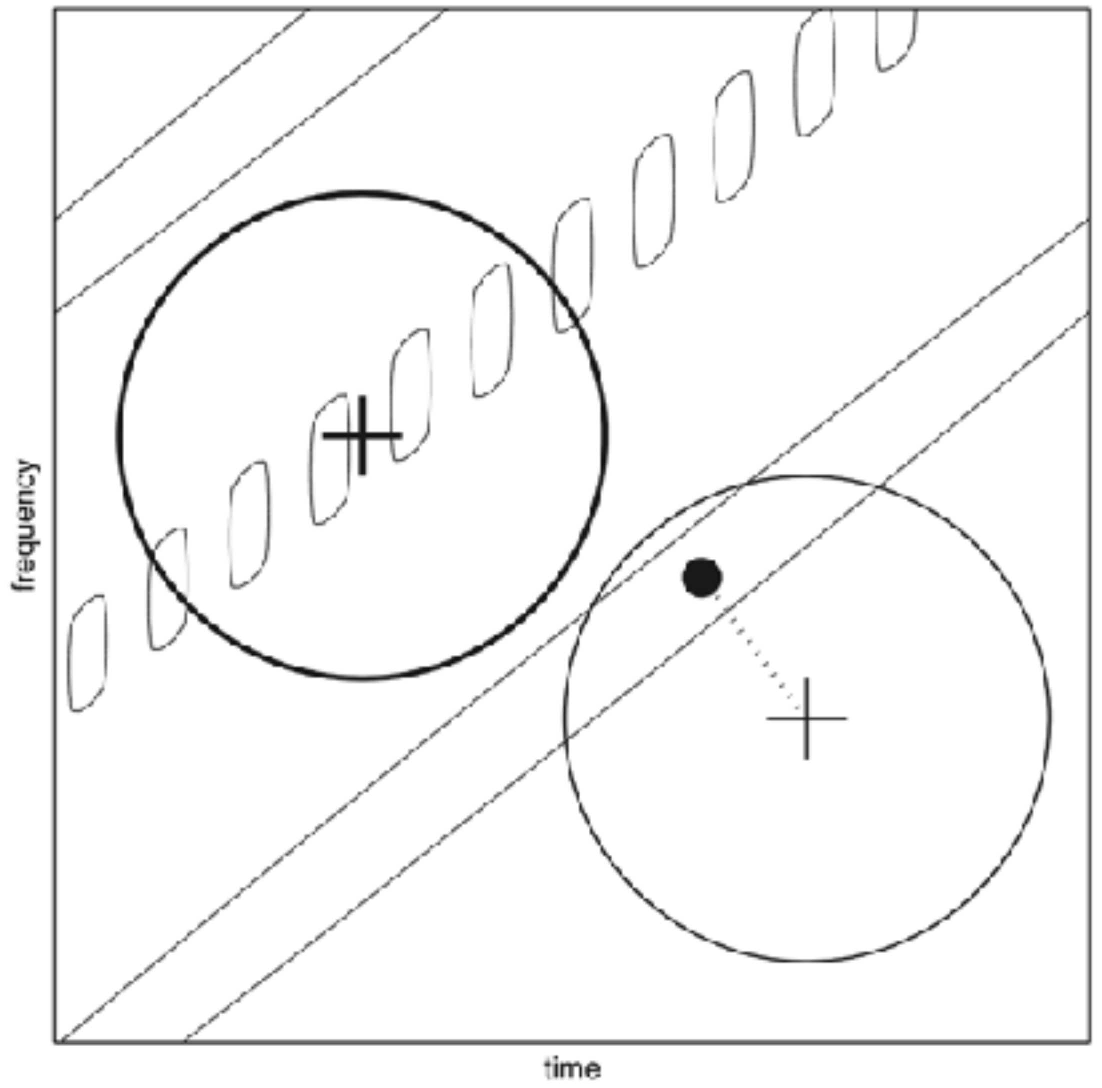
The idea is to move each "mass" of the spectrogram toward the centroid reachable in the support of the reproducing kernel.

(d)



(c)

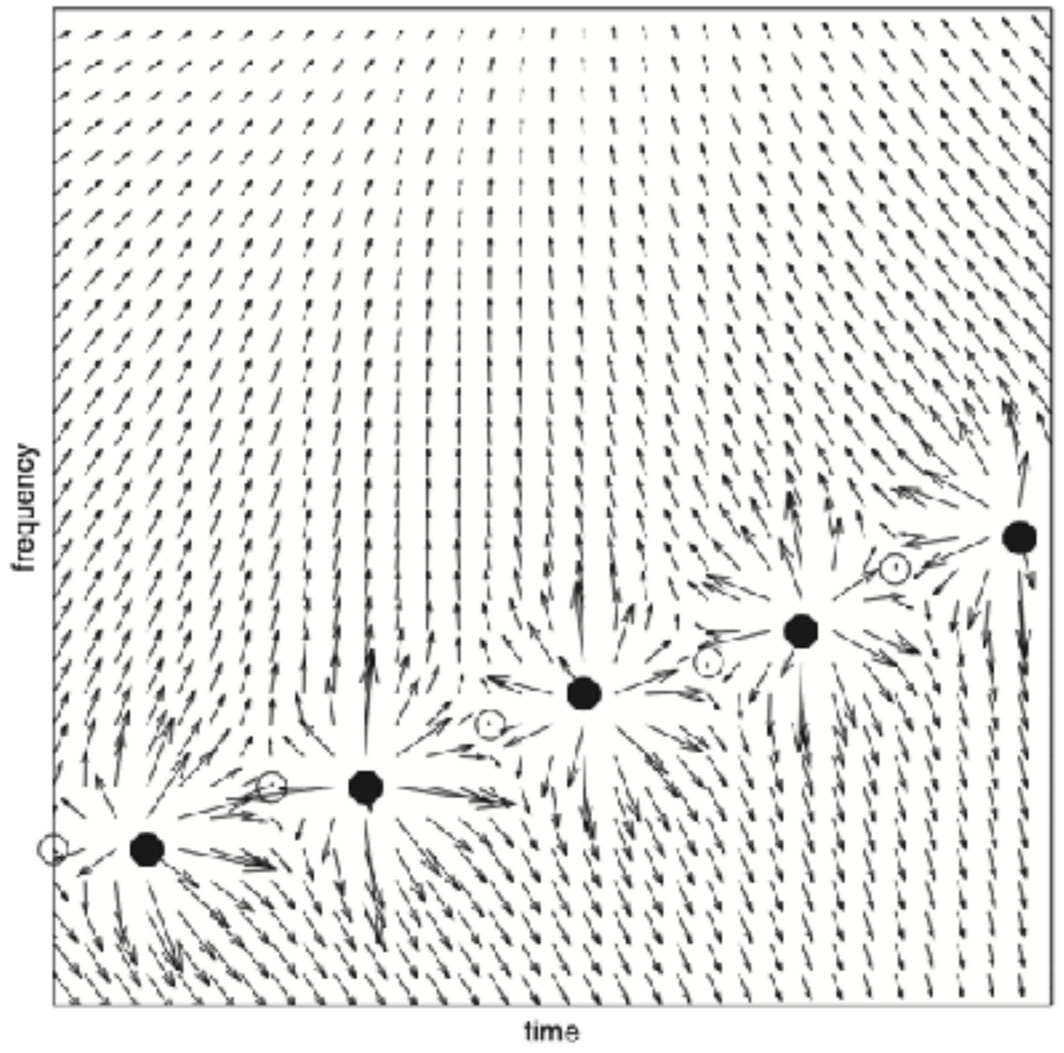


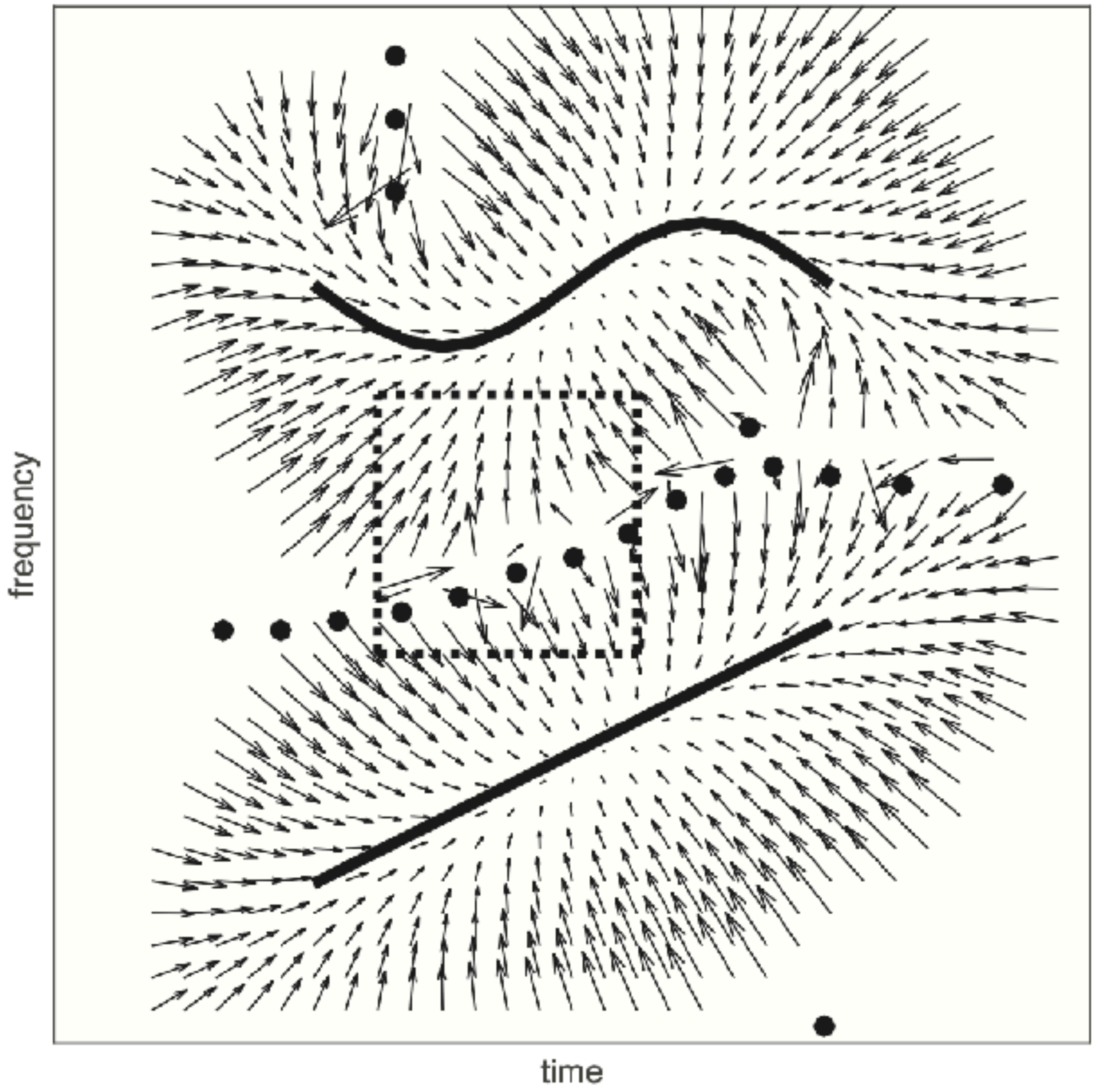


Spectrogram Sharpening

Said differently, having the energy distributed in the time-frequency plane following the reassignment field allows to recover precise structures underlying the signals (of course with some separability conditions, etc).

If we study this field more in detail, we observe a precise role of both zeros and maxima of the spectrogram: **maxima are attractors** of the field, while **zeros are repellers**.





Renseignement Field. Figure from *Explorations in Time-Frequency, Flandrin.*

Spectrogram Zeros

There is an even stronger result that emphasizes the role of zeros in the structure of the spectrogram. If we see the time-frequency plane as the complex plane (denoting $z = \tau + i\nu$), we have that

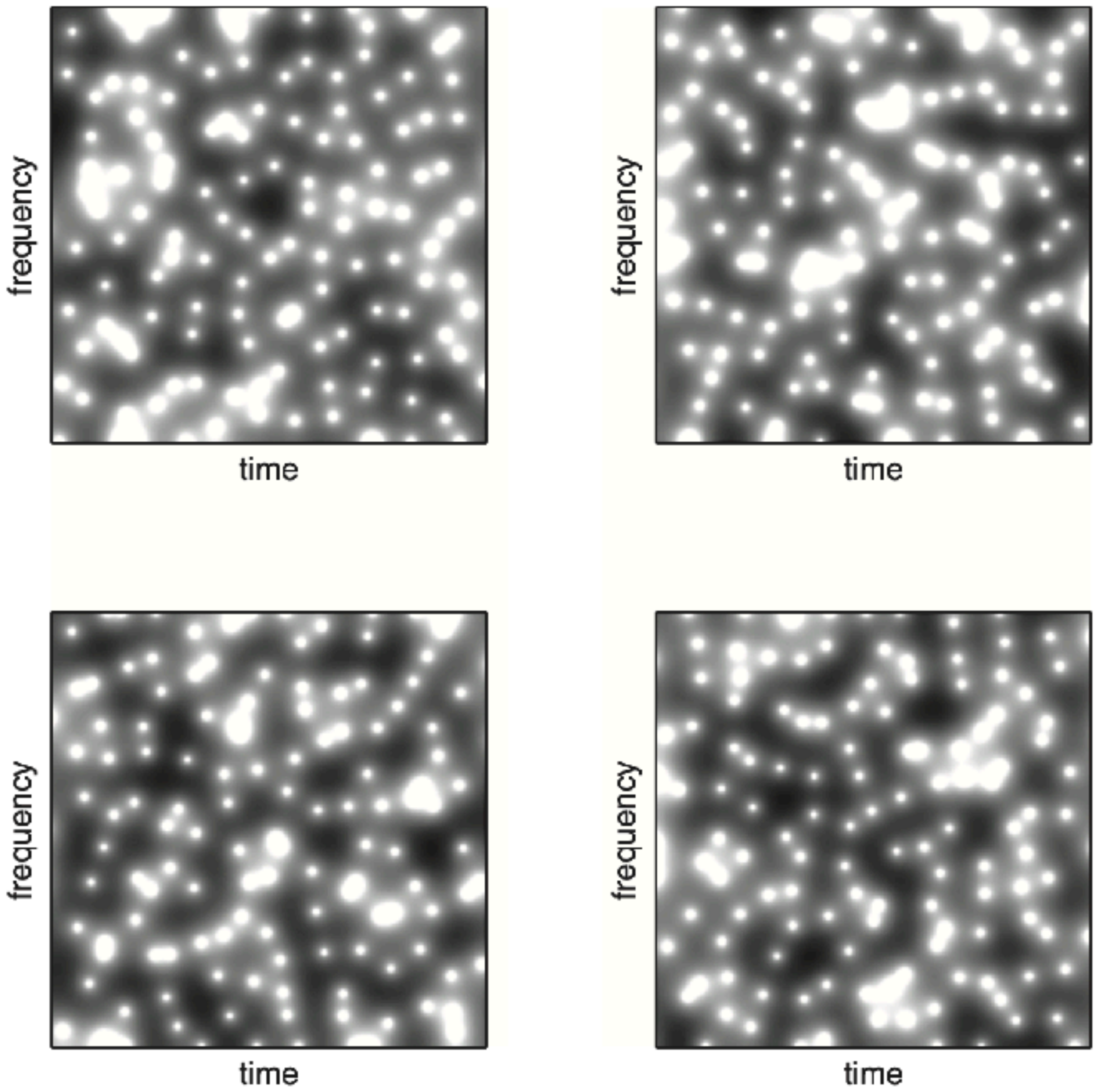
$$c(z) = f(z)\prod_n \left(1 - \frac{z}{z_n}\right) \exp\left(\frac{z}{z_n} + \frac{1}{2}\left(\frac{z}{z_n}\right)^2\right)$$

This result (a Weierstrass-Hadamard factorization) allows to express the value of the spectrogram at any (τ, ν) point using only the positions of the zeros (z_n).

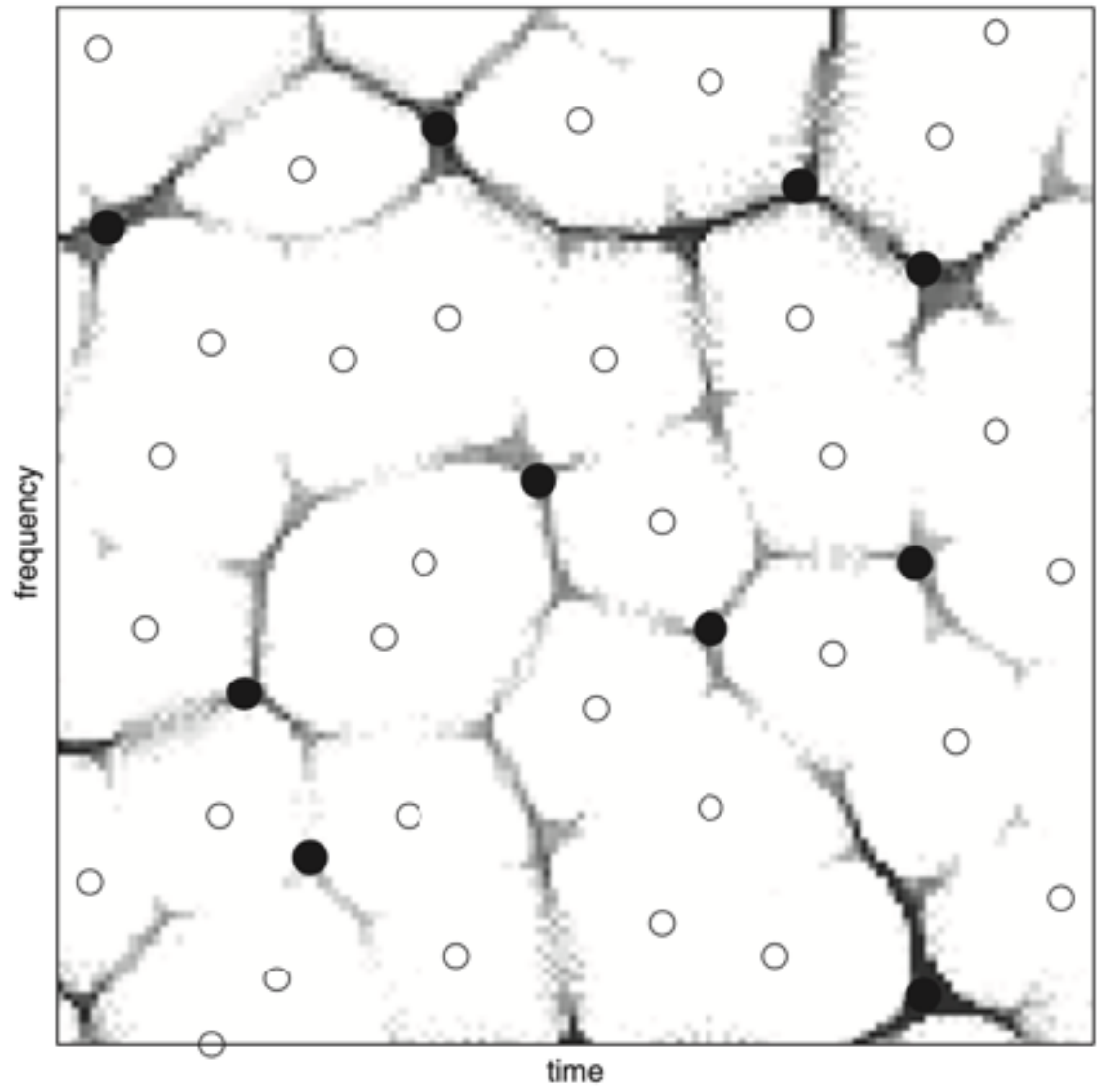
The zeros are thus a sparse descriptor which fully characterizes the spectrogram!

Zeros and Noise

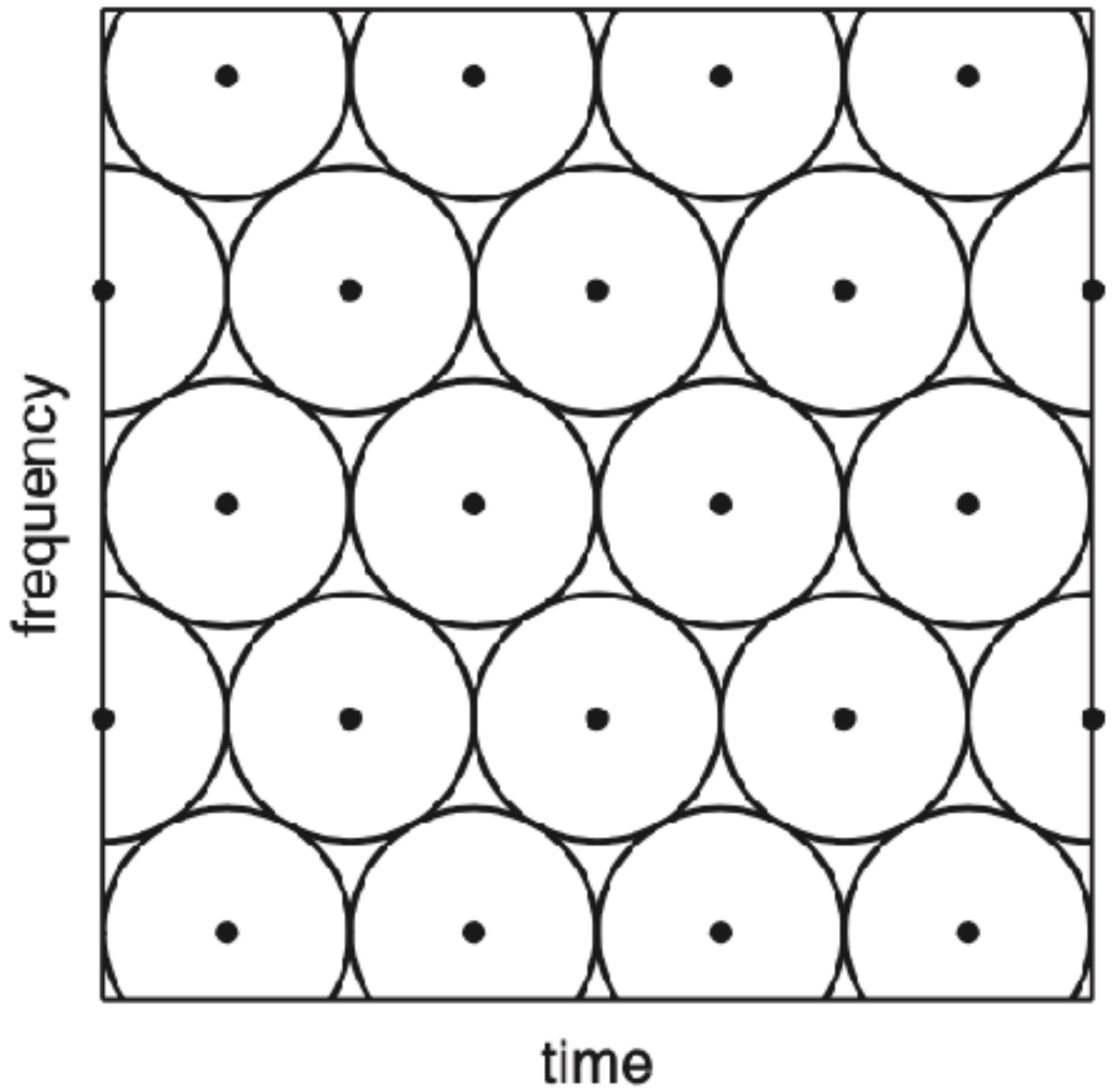
The Remarkable Structure Behind Random Noise



Spectrogram of White Noise. Zeros are represented in white while dark values represent high energy sites.



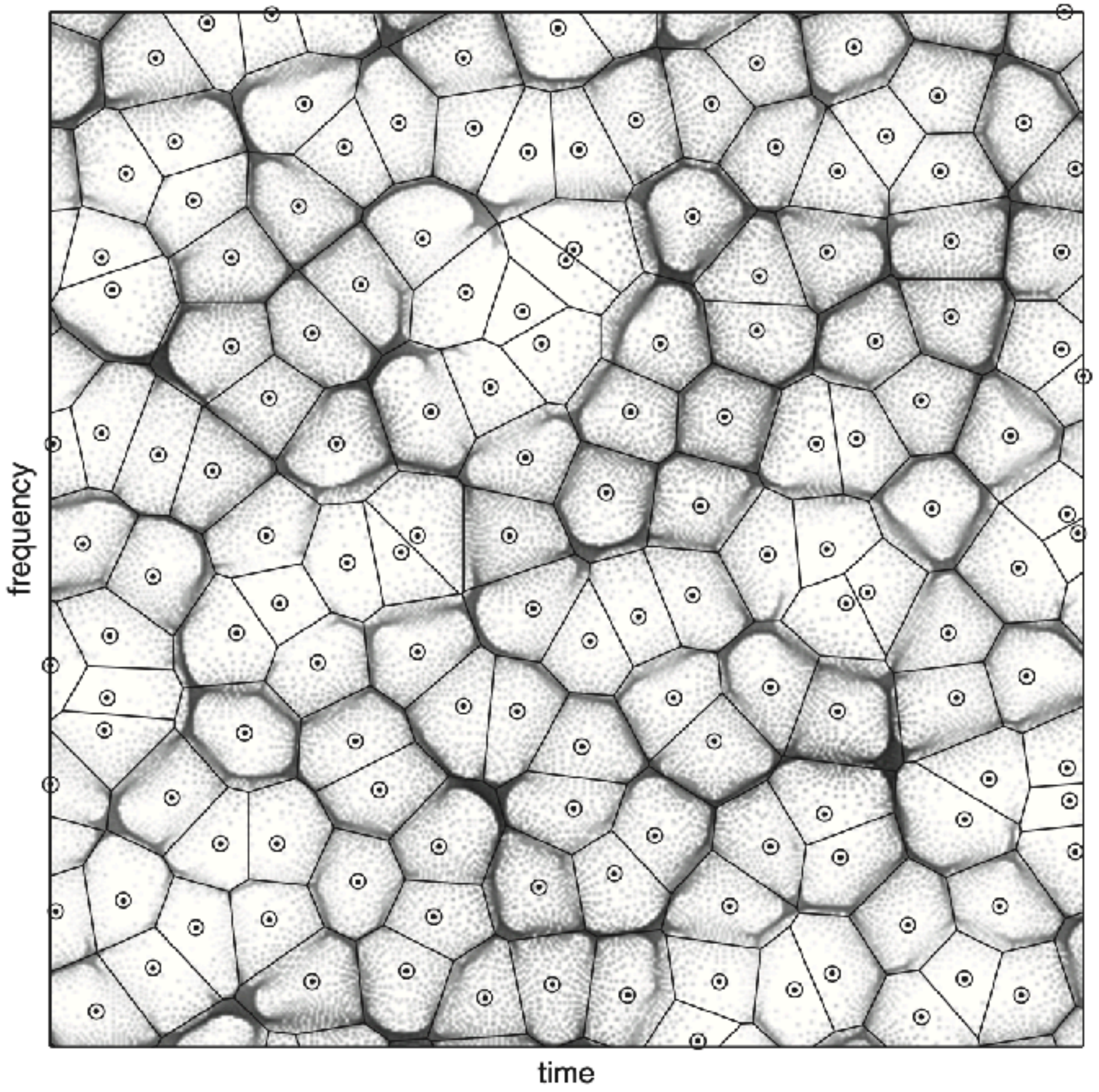
Reassigned Spectrogram of White Noise. Voronoi-diagram like structure are exhibited when following the reassignement field. The zeros are represented as the white circles.



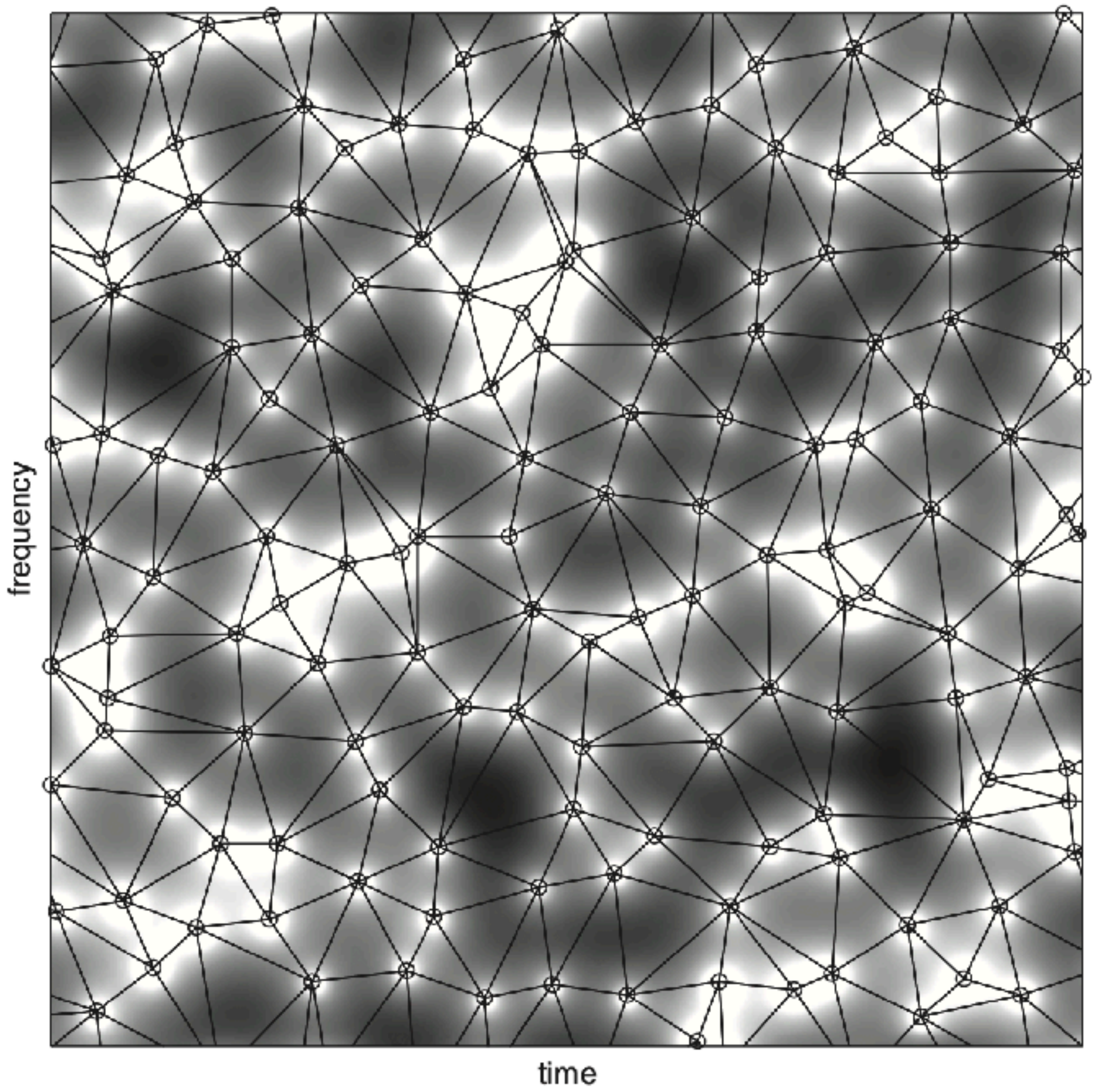
Probabilistic Analysis. Spectrograms of white noise might be seen as realizations of a “mean” model which consist in a “crystallographic”-like packing of gaussian atoms.

Figures from *Explorations in Time-Frequency, Flandrin*

Zeros and Noise



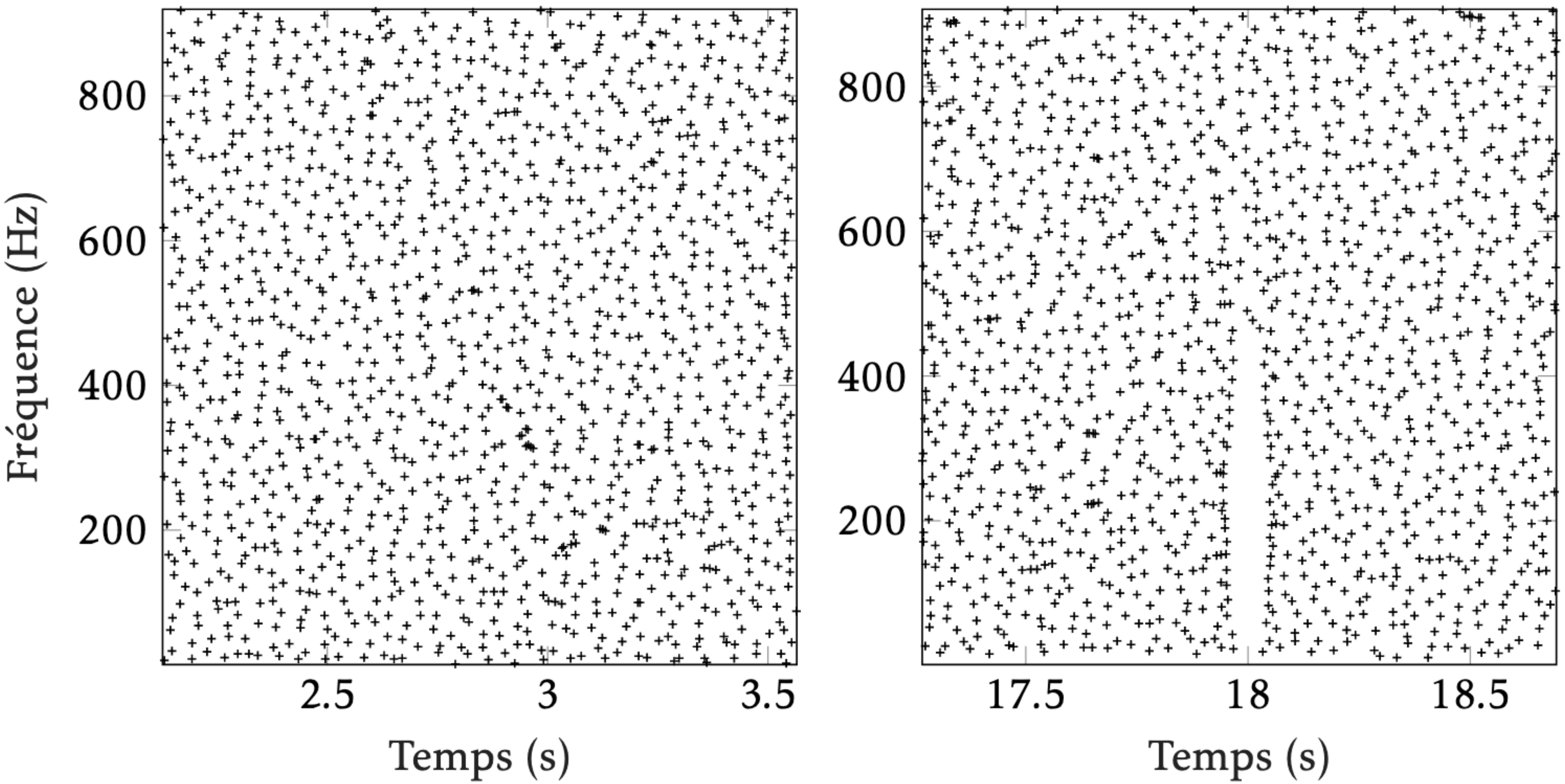
Reassigned Spectrogram of white Gaussian noise. The Voronoi tessellation of the zeros loci is represented on top of the reassigned spectrogram.



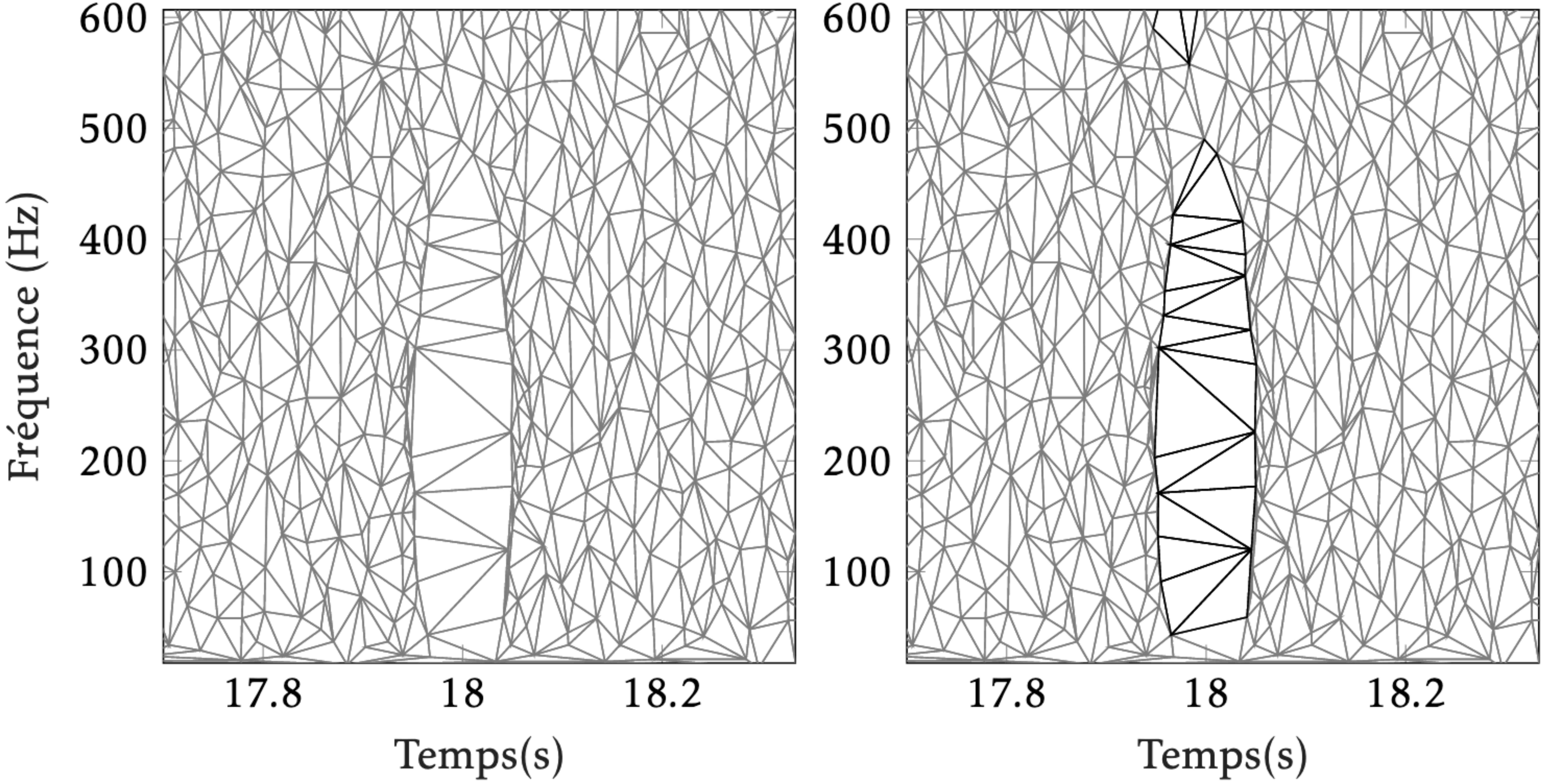
Delaunay Triangulation. The Delaunay triangulation is a dual structure of the Voronoi tessellation. The Delaunay triangulation of packed logons is expected to be composed of quasi-equilateral triangles.

Figures from Flandrin.

Application to HFO



Application to HFO



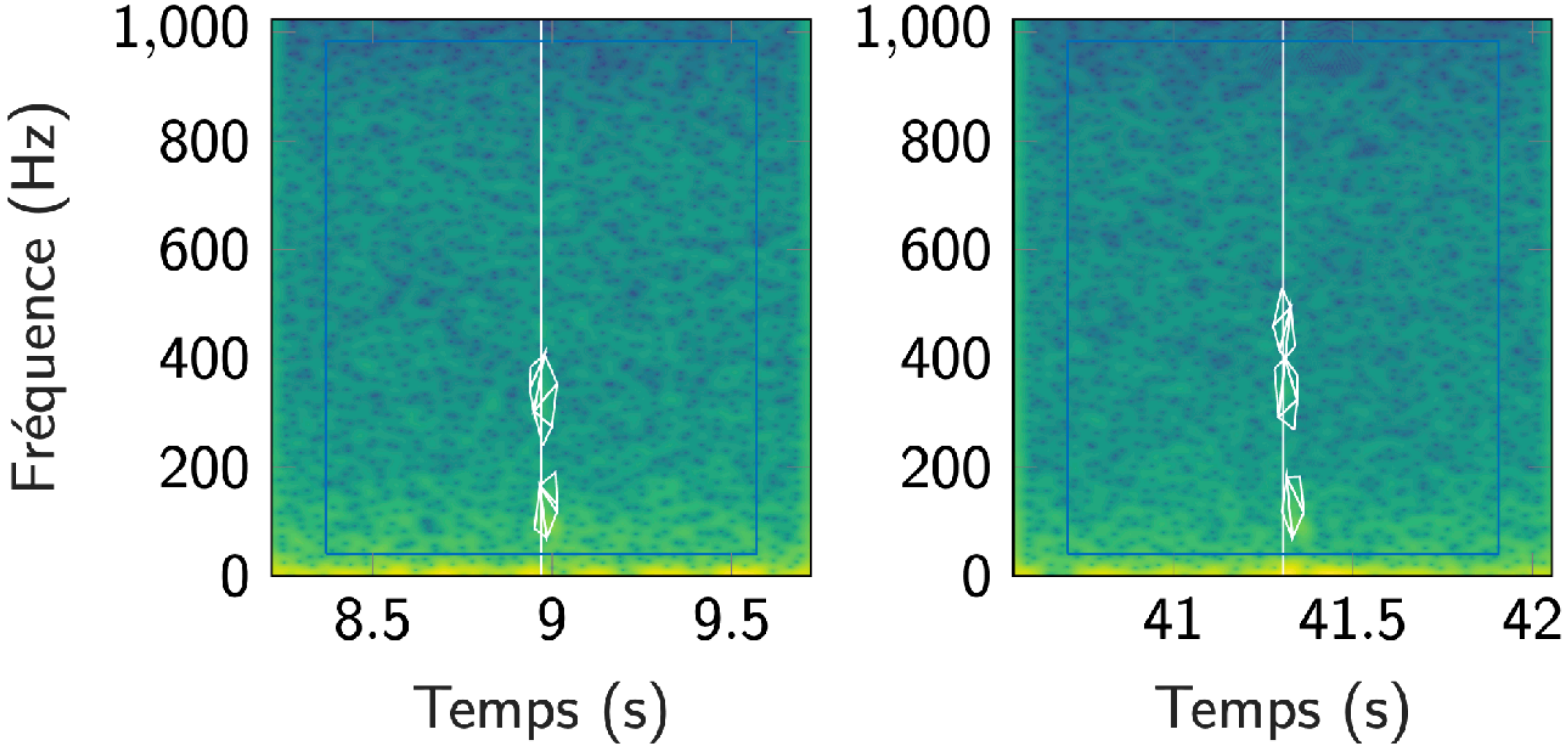
Application to HFO

Twist in the initial approach where we looked at the energy: now the signal of interest is defined as what lies between the absence of energy (i.e. the "silence"!)

An Algorithm to extract candidate HFO signature.

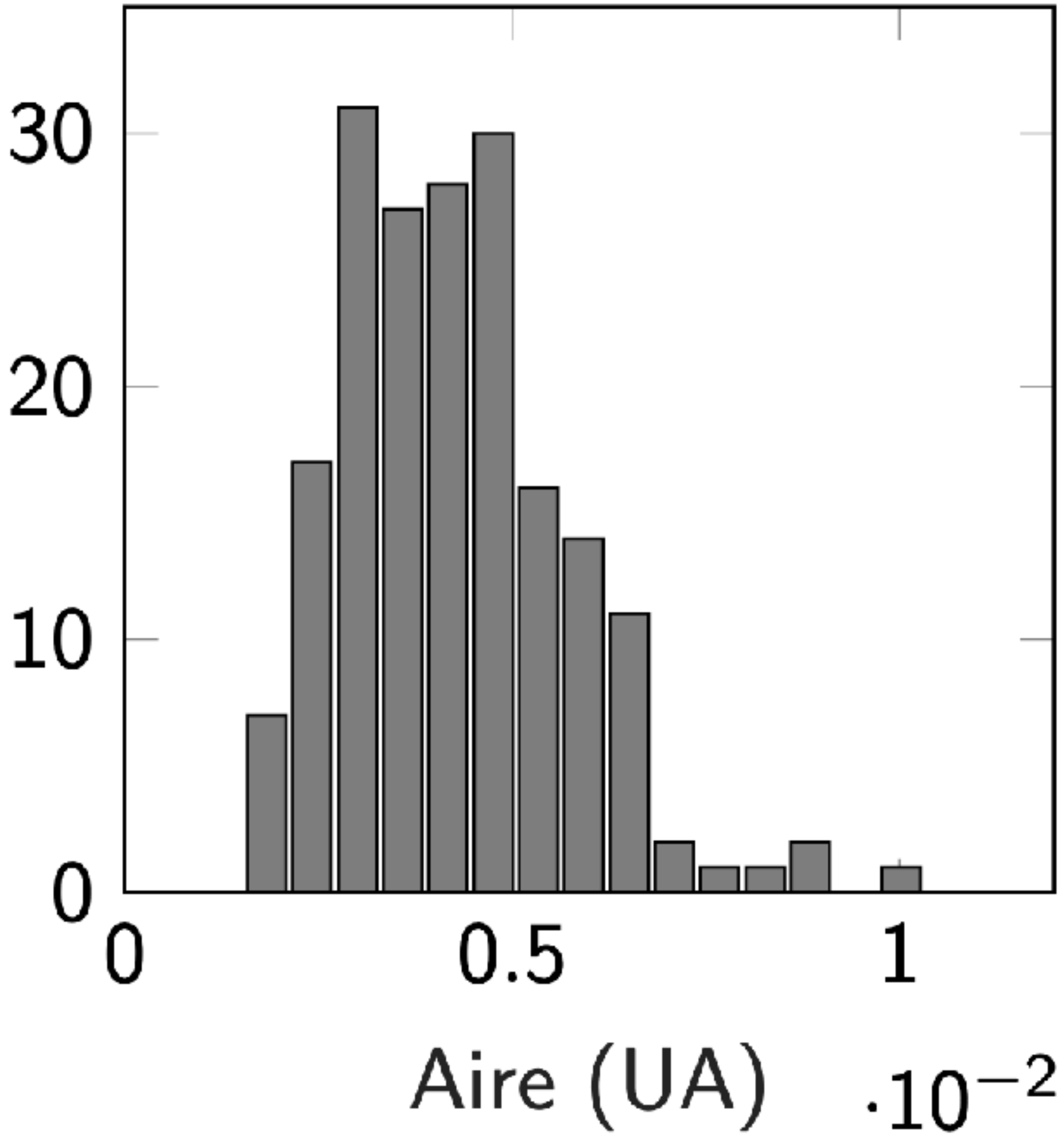
1. Select candidate events timepoints (t_i) by filtering in the relevant band
2. For each event $\tau \in (t_i)$
   1. Compute the spectrogram of the signal around τ
   2. Extract the positions of the zeros (z_n) in this spectrogram
   3. Compute the Delaunay triangulation of those (z_n)
   4. Keep only the triangles having one length greater than 1% of all the lengths in the triangulation
   5. Compute the connected components of the remaining triangles
   6. Remove those that do not overlap $t = \tau$

Application to HFO



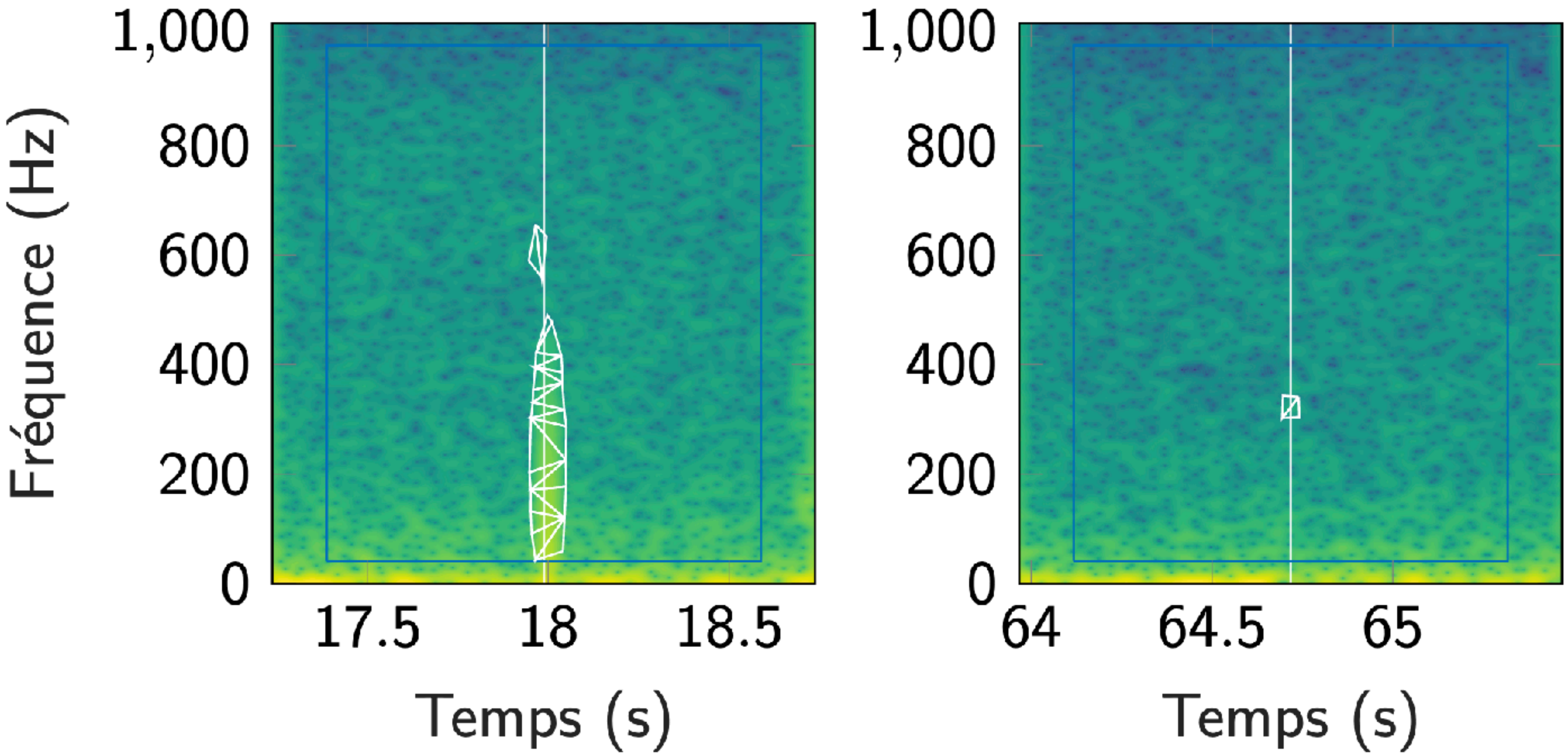
HFO detected by our method. Those examples are events which were marked as HFO and that were correctly detected using our method. The sparse geometric description of the events allows to extract relevant features.

Application to HFO



Statistic Analysis of HFO. This method allows to very easily construct quantitative statistics describing the geometric signature of those events. Here for instance is the distribution of the “areas” of the signatures of the events that were qualified as HFO.

Application to HFO



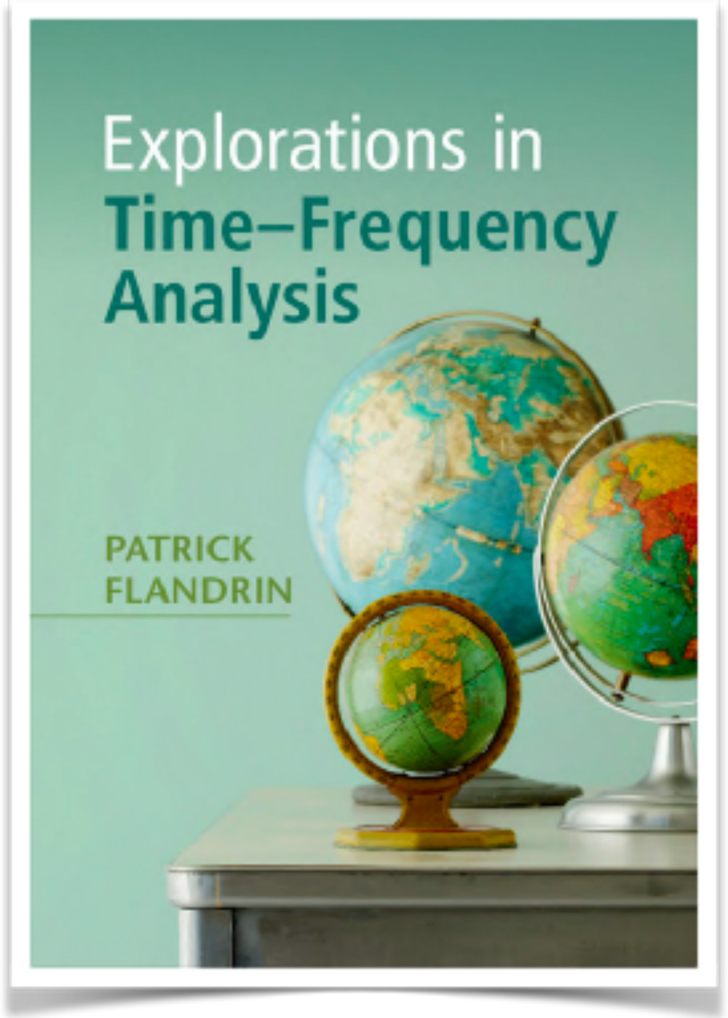
False Positives. Those examples are events for which energy was observed in the frequency band of interest, even though they were not qualified as HFO. Extracting geometric features using our method allowed to very easily reject them based on the area criteria.

Conclusion

Results were promising but could have used a bit more evaluation (actually computing performance metrics, test on a broader ranger of data, etc).

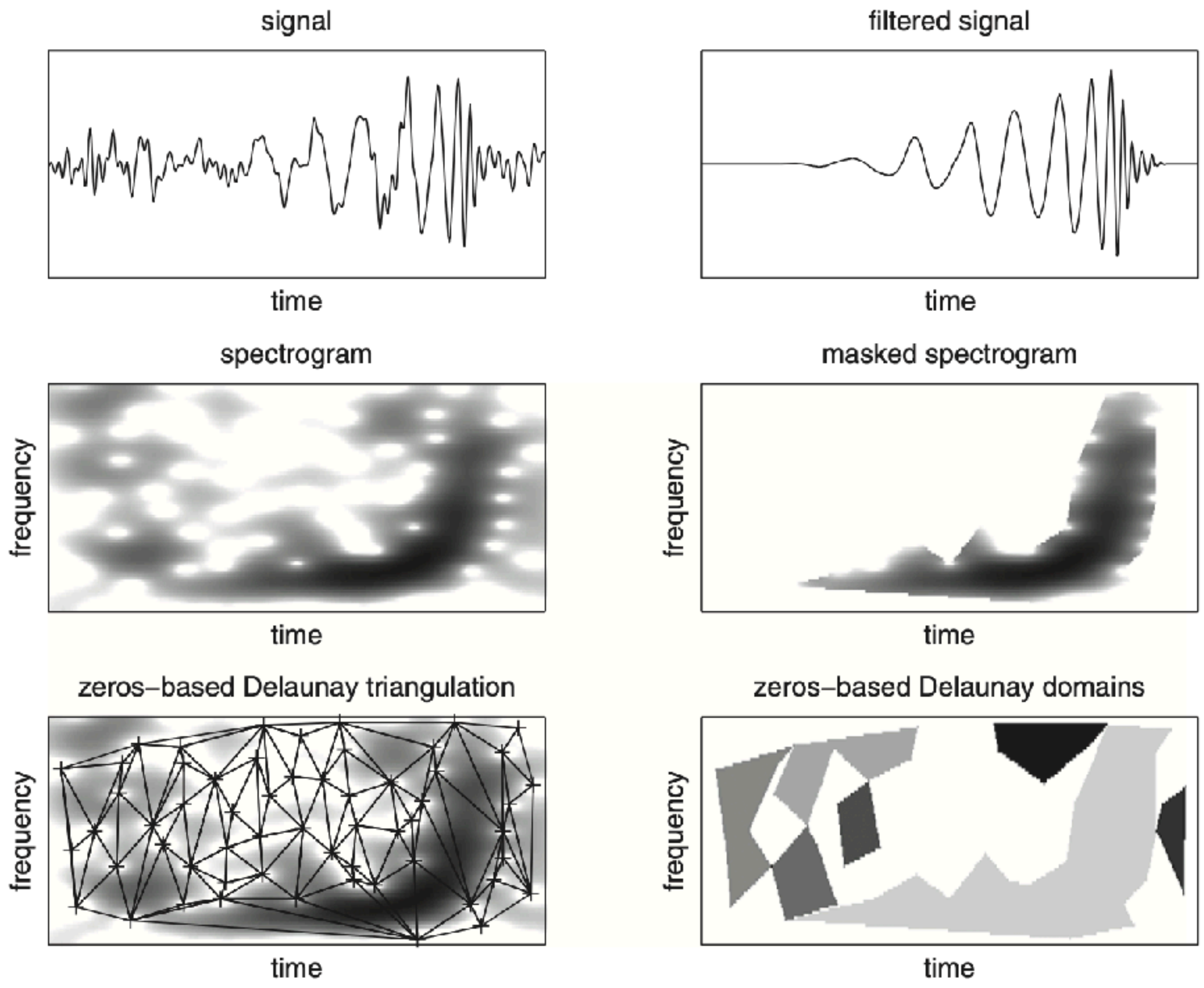
Our method was a non-parametric and almost prior-free way to extract relevant features describing the geometry of HFO in spectrograms.

Such method is even generalizable to many contexts and tasks.



Flandrin, P. (2018). Explorations in Time-Frequency Analysis. Cambridge: Cambridge University Press. doi:10.1017/9781108363181

Example of Other Domain of Application



Zero-based filtering of the GW150914 event (gravitational waves)