The physics of a popsicle stick bomb

Jérémy Sautel, Andréane Bourges, Aude Caussarieu, Nicolas Plihon, and Nicolas Taberlet

Citation: American Journal of Physics **85**, 783 (2017); doi: 10.1119/1.5000797
View online: http://dx.doi.org/10.1119/1.5000797
View Table of Contents: http://aapt.scitation.org/toc/ajp/85/10
Published by the American Association of Physics Teachers

The physics of a popsicle stick bomb

Jérémy Sautel, Andréane Bourges, Aude Caussarieu, Nicolas Plihon, and Nicolas Taberlet[a)]
Univ Lyon, Ens de Lyon, Univ Claude Bernard, CNRS, Laboratoire de Physique, Département de Physique, F-69342 Lyon, France

(Received 3 June 2016; accepted 6 July 2017)

Popsicle sticks can be interlocked in the so-called "cobra weave" to form a chain under tension. When one end of the chain is released, the sticks rapidly disentangle, forming a traveling wave that propagates down the chain. In this paper, the properties of the traveling front are studied experimentally, and classical results from the theory of elasticity allow for a dimensional analysis of the height and speed of the traveling wave. The study presented here can help undergraduate students familiarize themselves with experimental techniques of image processing, and it also demonstrates the power of dimensional analysis and scaling laws.
[http://dx.doi.org/10.1119/1.5000797]

I. INTRODUCTION

A. What is a popsicle stick bomb?

Wooden popsicle sticks can be bent and interlocked into a pattern known as the "cobra weave" (due to its resemblance with the patterns formed by the scales of the snake of the same name), as shown in the top panel of Fig. 1. In the displayed setup, each individual stick is bent to rest alternately above and below four perpendicular sticks (see also the bottom panel of Fig. 2), but other geometrical configurations can also be woven. When weaving the chain, one realizes that a large potential energy is stored in the sticks, and the ends of the chain must be held still by using a weight or one's hand (not shown of Fig. 1). When one end of the chain is released, the so-called "stick bomb" detonates—the potential energy is released as the sticks pop up and a traveling wave is formed.

This system is reminiscent of similar mechanical chain reactions, such as travelling waves in a line of dominoes,[1–3] or in an assembly of mousetraps,[4,5] both systems displaying propagation fronts and conversion of the stored potential energy (whether gravitational or elastic) into kinetic energy. This stick bomb also involves a competition between elastic and gravitational energy, which also governs the bounce of a ball.[6,7]

In this article, the speed of the traveling wave and the height reached by individual sticks are studied through simple hands-on experiments.

B. What undergraduate students can learn from this problem

Such a dramatic physics demonstration has high pedagogical potential. This problem can indeed provide an amusing introduction to solid mechanics, giving a simple example of the conversion of stored elastic energy into kinetic and gravitational potential energy[8] in which students can measure an energetic yield and consider sources of dissipation. The problem also provides an interesting illustration of the theory of elasticity; the study of the statics and dynamics of a single bent stick can be modeled using the Euler–Bernoulli theory for beam deflection.[9,11–14] Moreover, students may familiarize themselves with experimental techniques, such as particle tracking, or with the interpretation of a space-time diagram.[15] Finally, this paper may help undergraduate students grasp the importance of dimensional analysis[16–18] as well as the predicting power of scaling laws.[19,20]

C. Outline

The aim of this study is to understand the dependence of the velocity and height of the traveling wave on the properties of the popsicle sticks and of the weave. In a nutshell, Sec. II examines the elasticity of a single stick, Sec. III focuses on the traveling wave obtained with a given set of identical sticks, and the properties of the sticks (dimensions and material) are investigated in Sec. IV.

To model the popsicle stick bomb and understand why a wave propagates, sticks are modeled by beams within the theory of elasticity framework whose important results are recalled in Sec. II. We will first focus on the static aspects by considering the mechanical equations, boundary conditions, and the energy stored in a beam. The dynamics of a stick will also be discussed for a clamped beam, by studying the frequency (in the linear approximation) of the first transverse vibration mode. We then explain how the properties of the sticks are measured and discuss the efficiency of the conversion of stored elastic energy into kinetic and gravitational energy.

Experimental data obtained for the traveling waves using a given set of wooden sticks will be explored in Sec. III, where the experimental methods used to measure the speed and height of the traveling wave are explained.

Finally, in Sec. IV, we will derive scaling laws allowing one to predict the speed and height of a cobra wave, given only a few intensive (Young's modulus, density) and extensive (geometrical dimensions) properties of a stick. The predicting power of the scaling law and the influence of dimensionless parameters are discussed.

II. ELASTICITY OF A SINGLE STICK

A. Euler–Bernoulli beam theory

This section analyzes the elastic properties of a single stick. Unless otherwise mentioned, all classical results are taken from Landau *et al.*[9] Note that Ref. 21 describes a simple and affordable device for the measurement of Young's modulus (designed so as to minimize the computations that in many instances prove to be a deterrent in the understanding of physical concepts), while Refs. 22 and 23 present several simple cantilever-based experiments using common

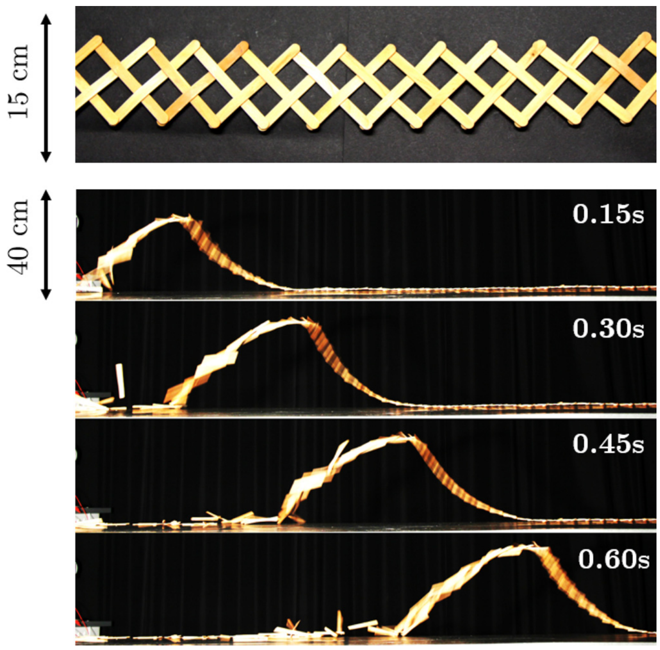


Fig. 1. Photographs of a typical experiment: (top) top-view of the popsicle sticks interlocked in the cobra weave; (bottom) side-view that shows the propagating wave.

household items (celery, carrot, and a plastic spoon) that are appropriate for introductory undergraduate laboratories or independent student projects.

1. General hypotheses

The Euler–Bernoulli beam theory is a simplification of the theory of elasticity, limited to small deformations and in which the shear stress is neglected. In this framework, there exists a neutral axis along the beam whose length remains constant and to which the cross section remains perpendicular. In the case where the beam is only constrained by (two) point supports (i.e., with no distributed load or torque), the position of the static beam $z(x)$ obeys[9]

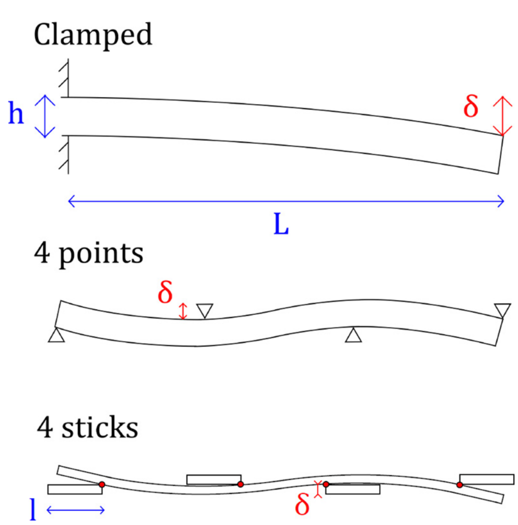


Fig. 2. Sketches of a single stick in three different configurations. For clarity, the thickness h and the deformation δ are largely exaggerated.

$$\frac{d^4z}{dx^4} = 0. \tag{1}$$

Here, z is the vertical position of the neutral axis, whose length remains constant, as a function of the x-coordinate along the beam. The result of Eq. (1) is that the shape of the beam is given by a third-degree polynomial whose coefficients are determined by the boundary conditions. Equation (1) can be derived analytically, but a hand-waving argument can help understand its fundamental physical meaning. The local curvature of the stick, $c = d^2z/dx^2$, is the first relevant derivative of the profile $z(x)$ that plays a crucial role. Indeed, gravity does not affect the shape of rigid beams and can therefore be neglected (except for the unrealistic case of extremely flexible sticks whose shape is affected by gravity). A stick can therefore be translated (affecting the absolute value of z) or rotated (changing dz/dx) without any changes to the physics of the bending. In terms of curvature, then, the content of Eq. (1) is that $d^2c/dx^2 = 0$, implying that the curvature varies linearly (or is constant) along the axis.

2. Elastic potential energy

The elastic potential energy stored in the bent beam is given by[9]

$$U_{\rm el} = \int_0^L \frac{1}{2} EI \left(\frac{d^2z}{dx^2}\right)^2 dx, \tag{2}$$

where E is Young's modulus and I the second moment of area, given by $I = lh^3/12$ for a rectangular beam of constant cross-section $l \times h$ and length L. As a simple example of the use of Eq. (2), consider the clamped beam with displaced end shown in Fig. 2. As explained above, between any two consecutive contact points, the equation of the beam is a third degree polynomial. In this case, the position of each end of the beam is known, which provides two of the four necessary boundary conditions. The final two boundary conditions are obtained by noting that the first derivative is continuous at the left end, giving $z'(0) = 0$, while the second derivative is zero at the right end because there is no net torque at this end. These boundary conditions suffice to determine the shape of the beam, which can then be substituted into Eq. (2) to obtain

$$U_{\rm el}^{\rm cl} = \frac{3}{2}\frac{EI}{L^3}\,\delta^2 \quad \text{(clamped)}. \tag{3}$$

A more complicated situation is the case of four contact points (see Fig. 2), which can be thought of as three separate segments for which 12 coefficients must be determined. The positions of each end of the three segments are known (providing six equations), and the continuity of the first and second derivatives provides another six equations. The equation of the beam can therefore be computed and the elastic energy can be computed to be

$$U_{\rm el}^{\rm 4p} = 432\frac{EI}{L^3}\,\delta^2 \quad \text{(4 points)}. \tag{4}$$

A similar computation can be carried out while taking into account the width l of the sticks, which shifts the position of the contact points as shown in Fig. 2. For sticks of width

$l = 10\,\text{mm}$ and length $L = 110\,\text{mm}$, the elastic energy can be computed numerically to be

$$U_{\text{el}}^{4\text{s}} \approx 872.4 \frac{EI}{L^3}\delta^2 \quad (4\,\text{sticks}). \tag{5}$$

3. Vibrations

While Sec. II A 2 describes beams at rest, understanding the dynamics of the beam deformation is clearly relevant for the study of the traveling wave. The equation of motion describing the dynamics of a uniform beam of mass m undergoing transverse deformations (forbidding any twisting) can be derived from the Lagrangian of a stick (see Ref. 9 for details) and is given by

$$EI\frac{\partial^4 z}{\partial x^4} + \frac{m}{L}\frac{\partial^2 z}{\partial t^2} = 0. \tag{6}$$

Solving Eq. (6) allows one to compute the eigenmodes of a vibrating beam and the corresponding frequencies. For instance, the angular frequencies of the eigenmodes of a beam clamped at one end (cantilever) are given by

$$\omega_n = \alpha_n \sqrt{\frac{EI}{mL^3}}, \tag{7}$$

with $\alpha_1 \approx 3.52$, $\alpha_2 \approx 22.0$, and $\alpha_3 \approx 61.7$. For a free beam (under no constraints), Eq. (7) still holds, with $\alpha_1 \approx 22.4$, $\alpha_2 \approx 61.7$, and $\alpha_3 \approx 121.0$. Note that in both cases the eigenfrequencies are proportional to a characteristic frequency

$$\omega^* \equiv \sqrt{\frac{EI}{mL^3}}, \tag{8}$$

with a geometrical prefactor that depends on the boundary conditions.

4. Equivalent spring-mass system

Obviously, the energy stored varies widely from one configuration to another, but it is important to note that the elastic potential energy is always proportional to an energy $(1/2)k\delta^2$, where $k \propto EI/L^3$, with a geometrical prefactor. This energy can be regarded as the potential energy stored in a (linear) spring (with spring constant k). Due to the large number of lengths in the problem (length, width, and thickness of the beam and pitch of the pattern), a direct dimensional analysis is not straightforward, and several dimensionless groups can be formed. However, the theory of elasticity provides one with an effective spring constant k, which enables dimensional analysis. For a stick in a cobra weave, the characteristic stored elastic energy is given by

$$U_{\text{el}}^* = 72.2\frac{Elh^5}{L^3}, \tag{9}$$

since, in this case, the deflection is given by $\delta = h$ (the factor $72.2 = 872.4/12$ comes from the prefactors in $U_{\text{el}}^{4\text{s}}$ and I).

Note also that the eigenfrequencies in Eq. (8) can be viewed as the angular frequency of a (linear) mass-spring system: $\omega^* \propto \sqrt{k/m}$, which defines a characteristic time

$$T^* \propto \sqrt{\frac{m}{k}}, \tag{10}$$

for a beam woven in the cobra pattern. In terms of the material properties, the characteristic time can be written as

$$T^* = \sqrt{\frac{L^4}{h^2E}\left(\frac{m}{lhL}\right)} = \frac{L^2}{h}\sqrt{\frac{\rho}{E}} = \frac{L^2}{h\,c_0}, \tag{11}$$

where ρ is the mass density and the ratio $c_0 = \sqrt{E/\rho}$ is the speed of sound in a material whose Poisson's ratio ν is zero.[9] For a more realistic value of $\nu = 0.3$ for hard wood,[24] the speed of sound (compression waves) is given by

$$\sqrt{\frac{E}{\rho}\frac{1-\nu}{(1+\nu)(1-2\nu)}} \approx 1.16\,c_0. \tag{12}$$

Interestingly, the characteristic time in Eq. (11) can therefore be seen as the product of the length-to-thickness aspect ratio of the sticks, L/h, and the time L/c_0 it takes a sound wave to propagate along the stick.

B. Measurement of the mechanical parameters

The mass and dimensions of the sticks can be easily measured using a scale and a caliper. Meanwhile, Young's modulus E (or, equivalently, the effective spring constant k) was measured using two methods on a beam clamped at one end. The first method consists of simultaneously measuring the force F applied at the free end of the stick (using a dynamometer) and the resulting deflection δ (using a caliper). The relation between the two is given by the Euler–Bernoulli theory to be

$$F = \frac{3EI\delta}{L^3}. \tag{13}$$

We have checked that the relation between the force and the deflection remains linear for small deflections ($\delta < L/10$), and the slope gives a direct measurement of the effective spring constant, which leads to the value of the Young's modulus.

The second method consists of studying the first mode of vibration of a clamped stick and is far more reproducible than the first method. The frequency of this mode is given by Eq. (7), which allows the determination of Young's modulus (or the effective spring constant). Figure 3 shows the position of the free end of a wooden stick of length $L = 85\,\text{mm}$ after it was given an initial deflection of $\delta = 5\,\text{mm}$ and then released. The data were obtained by tracking the position of the end of the stick using the `Analyze Particle` tool in the video analysis program ImageJ,[10] from a video taken at 1000 fps with a resolution of 10×1500 pixels. It is worth noting that the vibrations of the stick are rapidly damped due to internal dissipation within the stick.

C. Energy release and efficiency

In this section, we discuss the efficiency of the conversion of elastic potential energy into gravitational energy. To measure the ratio of these energies, we performed a simple experiment consisting of a stick resting between two nails (as shown in the inset of Fig. 4). A force F is applied (and

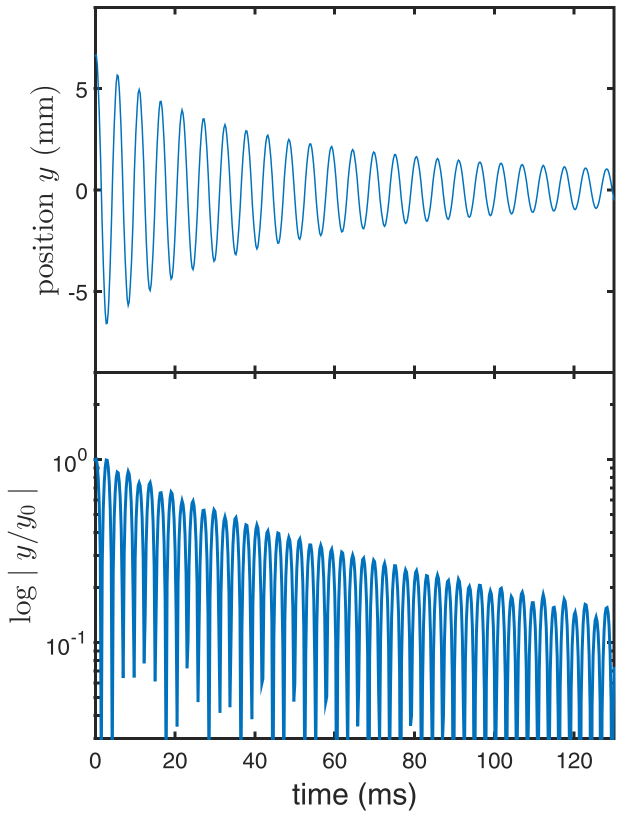


Fig. 3. Free oscillations of the free end of a clamped wooden stick (Wood 2 in Table I) shown on linear (top) and logarithmic (bottom) scales. Measuring the oscillation frequency leads to the value of Young's modulus of the stick. The damping is caused by internal dissipation within the stick.

measured) at the free end of the stick using a dynamometer. The energy stored in the stick is not computed from theory, which relies on a number of assumptions, but is instead directly measured as the work provided by the dynamometer. Moreover, we checked that for small deformations (less than 10% of the length of the stick), the force is a linear function of the deflection: $F(x) \propto x$. The stored energy is therefore given by

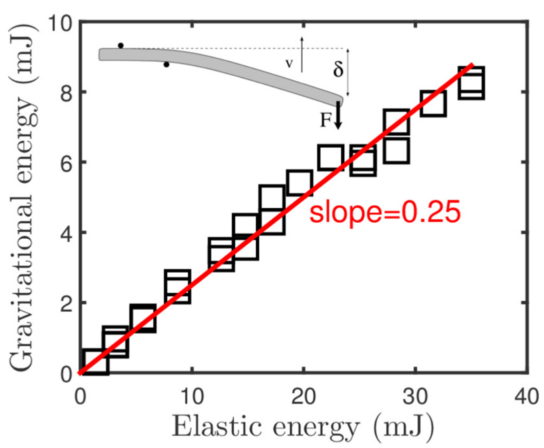


Fig. 4. Gravitational energy of a stick as a function of the elastic energy stored, in the case of the simple setup shown in the inset. A stick is placed between two nails and bent at its free end. When the force is released, it moves vertically upward to a maximum displacement. The data show a remarkably linear behavior, which defines a constant conversion efficiency of roughly 25%.

$$U_{\mathrm{el}} = \int_0^{\delta} F(x)\, dx = \frac{1}{2} F(\delta)\, \delta. \tag{14}$$

When the stick is released, it first rises up to some maximum displacement before continuing its oscillatory motion. We measure the maximum height H reached by the center of mass of the stick, allowing us to compute the maximum gravitational potential energy $E_g = mgH$ (where g is the gravitational field strength). Repeating this experiment for several values of F, one is able to plot E_g as a function of U_{el} in order to determine the efficiency of the conversion of elastic energy into gravitational energy. As can be seen in Fig. 4, only a fraction of the available energy is converted into gravitational potential energy. In fact, this plot shows a (reasonably) constant conversion efficiency of 0.25, meaning that 75% of the elastic energy is either converted into vibrations (mostly transverse but possibly in twisting modes as well) or dissipated (either internally or due to air resistance).

This empirical efficiency of 25% is specific to the setup used here, and one should therefore expect a different efficiency for a stick in their own cobra weave. The simple geometry presented in this section optimizes the height reached by a stick whereas a more symmetrical shape, such as the cobra weave (see Fig. 2) would favor vibrations at the expenses of height. Moreover, in the cobra weave, friction between the sticks can be very high and is another source of energy dissipation, as is collisions between the sticks. Hence, the efficiency of the conversion of elastic energy into gravitational energy for a popsicle stick bomb is expected to be even lower than 25% (as will be discussed in Sec. III B).

III. SPEED AND HEIGHT OF THE WAVE FOR ONE SET OF STICKS

The height and velocity of the wave may depend on numerous parameters. The material properties and the dimensions of the sticks will be discussed in Sec. IV. Here, we focus on results obtained using a given set of wooden sticks, varying only the pitch of the cobra pattern.

A. Experimental methods

A 3 m-long chain of sticks is woven following the pattern of Fig. 1, while keeping the extremities clamped using large weights. When the weight is removed at one end, the stick bomb detonates and the propagation of the wave is filmed using a camera (Ximea, xiQ MQ013MG-ON) at rates up to 2000 fps with a resolution of 1500×10 pixels. In order to perform a spatiotemporal analysis, the videos were processed using the `Orthogonal View` function in ImageJ. Choosing a fixed horizontal line located at approximately half the height of the wave, a threshold is then applied to make the picture black and white. The profile along this line is then plotted for all frames, yielding the space-time diagram shown in Fig. 5.

The chain reaction triggered at one end of the chain gives rise to a traveling wave, and Fig. 5 shows the evolution of the front and rear positions of the wave as a function of time. A steady state, in which the width of the wave (at half-height) remains constant and the trajectory is a straight line, is reached after the front has traveled for no less than 1.5 m (dashed line). It is therefore important that the length of the chain be longer than this transient length (which may depend on the type of sticks used). The velocity of the wave is

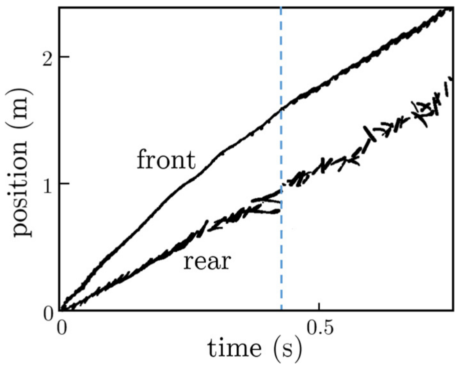


Fig. 5. Space-time diagram of the wave using wet wooden sticks. The two lines represent the horizontal position of the ascending (front) and descending (rear) portions of the "hump" seen in Fig. 1, measured at half the height. A steady state is reached after the front has traveled 1.5 m (dashed line), corresponding to roughly 100 sticks.

measured as the slope of the trajectory in the steady-state regime. Meanwhile, the height of the wave can be measured directly (see Fig. 1) by finding the height reached by the center of mass of the highest stick. The results are then averaged over the steady state.

We note that the sticks used in Figs. 1 and 5 were made of remarkably flexible wood and were soaked in water for several days to produce a low wave speed (1.7 m/s), which results in sharper images and clearer space-time plots. However, in the rest of the paper, more rigid sticks are used (see Table I for details).

B. Results and discussion

The results presented in this section were obtained using one set of rigid sticks (Wood 2 in Table I) that were soaked in water for three weeks (thickness $h = 2.3$ mm, width $l = 10$ mm, length $L = 114$ mm, density $\rho = 970$ kg m^{-3}, and Young's modulus $E = 5.0$ GPa, measured using the method presented in Sec. II B).

This section is devoted to the study of the influence of the pitch p of the pattern (see Fig. 6) on the speed and height of the wave. The pitch appears to be a crucial parameter and can be easily changed by varying the angles between the sticks when building the chain. Geometrically, the pitch is given by

$$p = \frac{2L}{3}\sin(\alpha/2)\,, \tag{15}$$

where α is the angle between the sticks (see Fig. 6). The same set of sticks was used to build several chains with

Table I. Parameters for the different sticks used.

Name	h (mm) $\pm$ 0.1	l (mm) $\pm$ 0.1	L (mm) $\pm$ 1	c_0 (m/s) $\pm$ 100
Wood 1	2.2	9.5	113	4600
Wood 2	2.4	10.0	114	2000
Wood 3	2.3	10.0	114	2300
Wood 4	1.1	5.3	136	6500
PVC 1	2.0	9.7	100	1400
PVC 2	2.0	9.7	120	1400
PVC 3	2.0	9.7	140	1400
PVC 4	2.0	9.7	133	1400

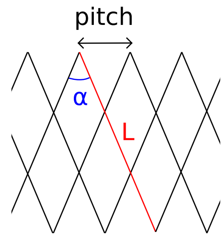


Fig. 6. Sketch of the cobra weave; the black lines correspond to the sticks of length L. The pitch p is defined as the distance between the extremities of two consecutive sticks.

different values of p, and the results of this study are shown in Fig. 7.

The first conclusion that can be drawn is that the wave height is unaffected when the pitch is varied, whereas the speed of the wave clearly increases with increasing pitch. As discussed in Sec. II, for the simpler case of a single stick, the height reached by the wave finds its origin in the elastic energy stored when the chain is built. In a steady state, each individual stick can on average only hope to recover its own elastic energy. When neglecting both the width l of the bent stick (i.e., considering 2D cross sections as in Fig. 2) and the twisting of the sticks, the elastic energy stored in the bending is independent of the pitch, which leads to a constant height when the pitch is varied. The data presented in Fig. 7 also enable one to estimate the efficiency of the energy conversion. On average, the efficiency is roughly 5%, considerably

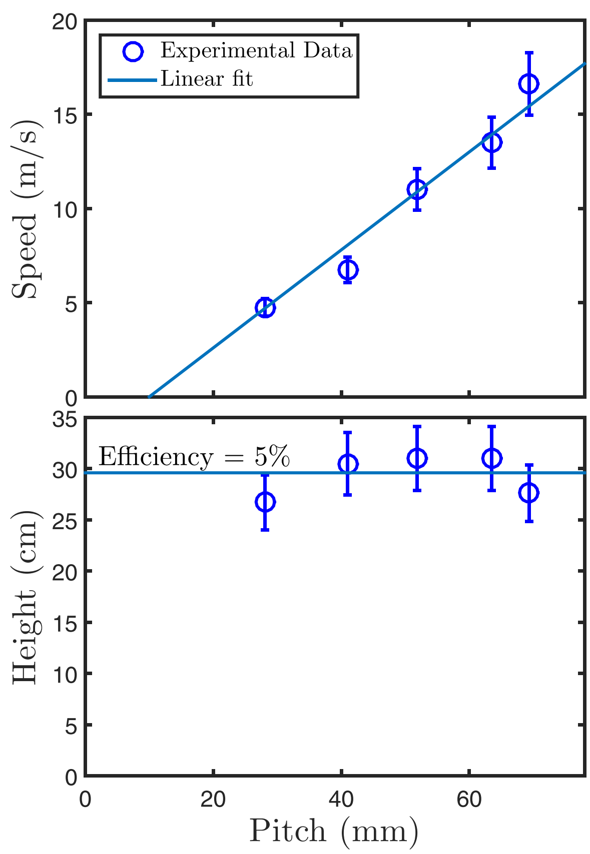


Fig. 7. Dependence of the speed (top) and height (bottom) of the wave on the pitch of the pattern. The speed V depends linearly on the pitch, with a forced offset corresponding to the width l of the sticks; the height H is independent of the pitch. The horizontal line indicates an efficiency of 5%.

less that the 25% obtained with the more favorable setup of Fig. 4.

As discussed in Sec. II, the motion of a stick occurs over a characteristic time T^*, specific to each set of sticks and with numerical prefactors that depend on the exact configuration. For a given set of sticks (for a fixed characteristic time T^*), when the pitch is varied, a simple dimensional analysis provides one with a characteristic velocity p/T^*. If the velocity is relevant to the phenomenon, the speed of the wave should scale as the ratio p/T^*. This predicted linear dependence of the velocity as a function of pitch is experimentally supported (solid line on top of Fig. 7). For the sake of clarity, the width l of the sticks is not shown in Fig. 6, but clearly the pitch must exceed this value. Therefore, the linear fit is obtained while forcing an offset of $l = 10$ mm. While the velocity can be accurately computed from space-time diagrams such as in Fig. 5, reproducing the experiments leads to a typical variability of $\sim$10% in the speed. Similarly, the maximum height is measured using video snapshots (see Fig. 1), which again leads to a typical uncertainty of $\sim$10%.

In conclusion, a dimensional argument allows one to predict both the linear dependence of the speed and the constant height as a function of the pitch.

IV. SCALING LAWS

A. Various materials and dimensions

The previous section presented data obtained with the same wooden sticks as the pitch of the cobra weave was varied. In addition to these experiments, the dimensions and material of the sticks were varied (Table I gives a summary of the sticks used) while the pitch of the cobra weave was kept constant (with $\alpha = 90°$). In order to vary Young's modulus, the wooden sticks were soaked in water, with durations ranging from a few days to a few weeks. This process succeeded in making the wooden sticks less rigid, but it also affected their density (up to a factor 2) and size (with an increase of up to 10%). For all sets of sticks, both Young's modulus E and the density ρ were independently measured prior to any experiment and the corresponding (so-called) speed of sound $c_0 = \sqrt{E/\rho}$ is reported in Table I. Plastic sticks of various sizes, cut from a sheet of PVC (polyvinyl chloride), were also used.

Figure 8 shows the experimental results obtained using these various sticks plotting both the height H and velocity V of the wave. The relation between H and V is not expected to be simple, and the data are not expected to fall onto a master curve; the figure is only intended as a synthetic way to present the experimental results. However, a general trend can be seen as the stiffer and lighter sticks tend to travel faster and reach a greater height.

B. Characteristic velocity and height

As explained in Sec. III B, the velocity of the wave is proportional to the pitch of the pattern divided by the characteristic time of vibration of a stick. Therefore, a relevant characteristic velocity can be defined as $p/T^* \propto L\sqrt{k/m} \propto \sqrt{E/\rho}(h/L) \equiv V^* = c_0\, h/L$. While the linear dependence on the pitch was confirmed in Sec. III, the more complex dependence on the material parameters through the speed of sound c_0 and the aspect ratio h/L will now be examined. It is

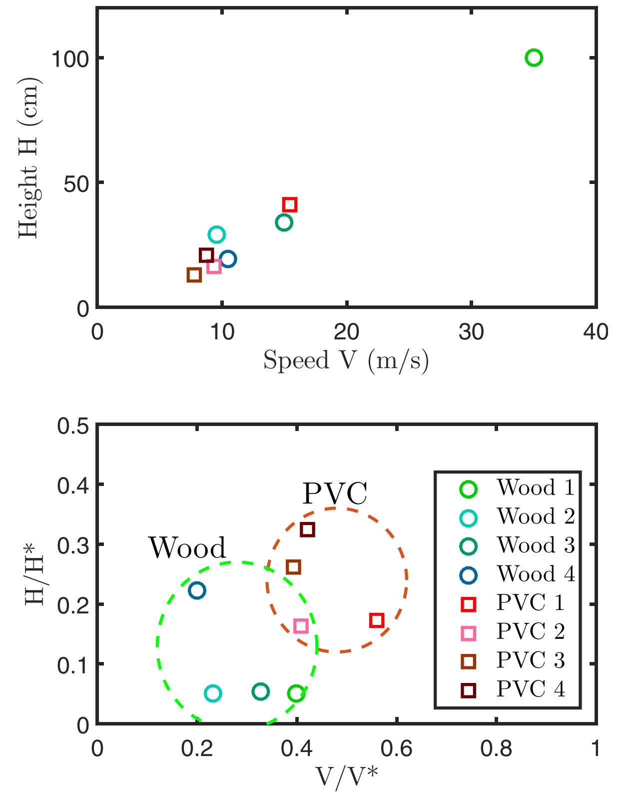


Fig. 8. Top: The speed V and height H of the wave for each type of sticks tested. The squares represent the four types of PVC sticks, and the circles the four types of wooden sticks. The points are spread out and no simple relationship between the two quantities is expected. Bottom: Plot of the rescaled data, V/V^* and H/H^*. One can see an apparent collapse of the data, depending on the material used.

important to emphasize that the characteristic speed V^* is not a theoretical prediction of the actual speed since unknown numerical prefactors have been left out.

The gravitational potential energy $E_g = mgH$ gained by a single stick finds its origin in the elastic energy stored during the weaving. Comparing these two energies is equivalent to comparing the height H with a characteristic height H^* defined as $H^* \equiv 72.2U^*_{\text{el}}/mg$ [see Eq. (9)]. This characteristic height can then be seen as the maximum average height reached by the sticks if all the potential energy were converted into gravitational energy. Again, the predicted dependence of the height on the material parameters and dimensions of the sticks requires an experimental validation.

To summarize, we have identified both a characteristic horizontal velocity (V^*) and a characteristic height (H^*) of the traveling wave to be

$$V^* = c_0 \frac{h}{L} \tag{16}$$

and

$$H^* = 72.2 \frac{{c_0}^2}{g} \left(\frac{h}{L}\right)^4. \tag{17}$$

The characteristic velocity and height are specific to each set of sticks and depend solely on the speed of sound and the aspect ratio. For dry wood, the speed of sound is on the order of 4000 m/s, and the aspect ratio of the wooden popsicle sticks is $h/L = 2/110$, leading to a characteristic velocity of $V^* \simeq 70$ m/s. While the data in Fig. 8 are somewhat smaller than this value, given that we have neglected numerical prefactors, it is at least the right order of magnitude. Meanwhile, the characteristic height has a value $H^* \simeq 18$ cm, which again gives the right order of magnitude.

C. Collapse

As one can see in Fig. 8, the values of V and H vary broadly for the different types of sticks used; the speed ranges from 7 to 36 m/s and the height from 15 to 100 cm. Another way to view these data is to plot the same data after rescaling by the characteristic quantities (which are specific to each type of sticks), as shown in the bottom panel of Fig. 8. When viewed in this way, the data appear to collapse into two separate groups, according to which material the sticks are made from (wood or PVC).

The existence of these two sub-sets might have been expected, as the dimensional analysis used to derive the characteristic scales inherently excludes some phenomena that might play a crucial role. Two possibilities are friction between the sticks (either static or dynamic) and internal dissipation. These phenomena can be characterized, respectively, by the friction coefficients (μ_s and μ_d) and the quality factor Q (or damping) of the oscillations (see Fig. 3). It is clear that increasing either the friction or the damping can only lead to lower values of the height and the velocity. Performing accurate and reproducible measurements of friction coefficients can be a difficult endeavor, but in our experiments, it is evident that the wooden popsicle sticks have greater friction coefficients than the smooth PVC sticks. Moreover, measurements of the decay rate of the free vibrations of a clamped beam clearly indicate that the internal dissipation of wood is greater than that of PVC. These observations allow us to understand, at least qualitatively, the separation of the data into the two groups seen in the bottom panel of Fig. 8.

V. CONCLUSION

In summary, the linear theory of elasticity allows us to derive scaling laws for the height and speed of the traveling wave. These predictions are then compared to a series of experiments that vary the pitch of the weave as well as the dimensions and materials of the sticks. This problem is a good example of how dimensional analysis can shed new light on a complex phenomenon.

Several aspects of the popsicle stick bomb are not addressed in this paper and might deserve further attention. For example, it can be seen in Fig. 1 that the shape of the wave is not symmetric. Indeed, the wave shape differs depending on which end of the chain is released, i.e., on the direction of propagation. Equivalently, if the chain is flipped over before release, the wave properties are affected. Note that the scaling laws should still apply although the numerical prefactors might differ. In particular, it would be interesting to study whether the rescaled velocity and height of this so-called "reverse" wave fall in the same region as those of the "regular" wave.

Another factor that has not been investigated is the influence of the base. Indeed, the speed and height depend on whether the chain is set on a hard surface (such as a laboratory table) or on a softer surface (such as a rug or carpet). Numerous home videos found online[25] also show that the wave can propagate without a direct support. It would therefore be interesting to study the case of a chain hanging vertically.

Finally, the pattern of the weave can be varied. A cobra weave can be created by interlocking one stick to three or five (instead of four) perpendicular sticks and a square pattern can also be built. The elastic energy and characteristic time would clearly vary but once more it would be interesting to check if the scaling laws hold.

ACKNOWLEDGMENTS

The authors acknowledge support from the PALSE program of the University of Lyon Saint-Etienne, the University Lyon Claude Bernard, the Société Française de Physique, and the École Normale Supérieure de Lyon and its Physics Department. The authors are grateful to P. Besserve, A. Chemin, A. Guittonneau, and G. Panel for fruitful discussions. The work presented here was done in preparation for the International Physicists Tournament (http://iptnet.info/), a world-wide competition for undergraduate students. The authors would recommend participation in the IPT as a unique learning opportunity for undergraduate students.

[a]Electronic mail: nicolas.taberlet@ens-lyon.fr

[1]J. M. J. van Leeuwen, "The domino effect," Am. J. Phys. **78**, 721–727 (2010).

[2]D. E. Shaw, "Mechanics of a chain of dominoes," Am. J. Phys. **46**, 640–642 (1978).

[3]L. A. Whitehead, "Domino 'chain reaction'," Am. J. Phys. **51**, 182 (1983).

[4]H. D. Rathgeber, "Mousetrap model of chain reactions," Am. J. Phys. **31**, 62–62 (1963).

[5]J. Higbie, "The better mousetrap: A nuclear chain reaction demonstration," Am. J. Phys. **48**, 86–87 (1980).

[6]J.-Y. Chastaing, E. Bertin, and J.-C. Géminard, "Dynamics of a bouncing ball," Am. J. Phys. **83**, 518–524 (2015).

[7]J.-Y. Chastaing, G. Pillet, N. Taberlet, and J.-C. Géminard, "Transversal stability of the bouncing ball on a concave surface," Phys. Rev. E **91**, 052918–1–5 (2015).

[8]B. A. Lindsey, P. R. L. Heron, and P. S. Shaffer, "Student understanding of energy: Difficulties related to systems," Am. J. Phys. **80**, 154–163 (2012).

[9]L. D. Landau, L. P. Pitaevskii, A. M. Kosevich, and E. M. Lifshitz, *Theory of Elasticity: Course of Theoretical Physics*, 3rd ed. (Butterworth-Heinemann, Oxford, UK, 1986), Vol. 7.

[10]ImageJ is a free image processing and analysis program can be determined at <https://imagej.nih.gov/ij>.

[11]R. D. Edge, "Elasticity," Phys. Teach. **21**, 608 (1983).

[12]J. Casey, "The elasticity of wood," Phys. Teach. **31**, 286–288 (1993).

[13]K. Turvey, "An undergraduate experiment on the vibration of a cantilever and its application to the determination of Young's modulus," Am. J. Phys. **58**, 483–487 (1990).

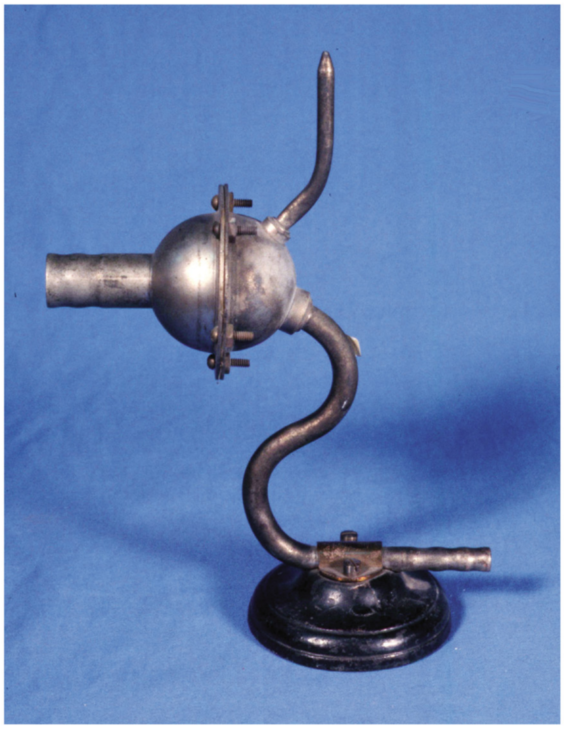
[14]T. Hpfl, D. Sander, and J. Kirschner, "Demonstration of different bending profiles of a cantilever caused by a torque or a force," Am. J. Phys. **69**, 1113–1115 (2001).

[15]A. B. Kaiser, "Space-time diagrams," Am. J. Phys. **40**, 1701–1702 (1972).

[16]H. E. Huntley and J. V. Kline, "Dimensional analysis," Am. J. Phys. **24**, 534–535 (1956).

[17]J. Palacios and H. L. Armstrong, "Dimensional analysis," Am. J. Phys. **33**, 513–514 (1965).

[18]W. J. Remillard, “Applying dimensional analysis,” Am. J. Phys. **51**, 137–140 (1983).
[19]T. R. Tretter, “Godzilla versus scaling laws of physics,” Phys. Teach. **43**, 530–532 (2005).
[20]H. D. Sheets and J. C. Lauffenburger, “Looking for scaling laws, or physics with nuts and shells,” Phys. Teach. **37**, 376–378 (1999).
[21]A. Niculescu and R. Shumaker, “Apparatus for measuring Young’s modulus,” Phys. Teach. **41**, 364–367 (2003).
[22]K. A. Pestka, “Young’s modulus of a marshmallow,” Phys. Teach. **46**, 140–141 (2008).
[23]K. A. Pestka and C. Warren, “Hooke’s law and the stiffness of a plastic spoon,” Phys. Teach. **50**, 470–471 (2012).
[24]D. E. Kretschmann, *Wood Handbook*, General Technical Report FPL-GTR-190 (U.S. Department of Agriculture, Washington, 2010).
[25]As one example, a roller coaster popsicle stick bomb can determined at <https://youtu.be/G9-y0i8NT3U>.



Manometric Flame Apparatus

This device was developed in 1862 by Rudolph Koenig, the Parisian maker of fine acoustical apparatus. It was used into the early years of the twentieth century to examine the wave-shapes of sounds. The heart of the apparatus is the manometric flame capsule. In this apparatus, sound enters the capsule from the left-hand side (probably conveyed there via a funnel and a length of rubber hose), and impinges on a rubber membrane placed between the two halves of the capsule. Illuminating gas enters at the lower right-hand side and burns in a small flame at the orifice in the upper right-hand corner. The oscillations of the membrane modulate the gas supply, and the height of the gas flame varies accordingly. The oscillating gas flame is viewed in the rotating mirror, which supplies the necessary time base to make the wave shape visible. The device is at Grinnell College. (Picture and Notes by Thomas B. Greenslade, Jr., Kenyon College)