How tall can gelatin towers be? An introduction to elasticity and buckling

Nicolas Taberlet,[a)] Jérémy Ferrand, Élise Camus, Léa Lachaud, and Nicolas Plihon
CNRS, Laboratoire de Physique, Univ Lyon, Ens de Lyon, Univ Claude Bernard, F-69342 Lyon, France

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The stability of elastic towers is studied through simple hands-on experiments. Using gelatin-based stackable bricks, one can investigate the maximum height a simple structure can reach before collapsing. We show through experiments and by using the classical linear elastic theory that the main limitation to the height of such towers is the buckling of the elastic structures under their own weight. Moreover, the design and architecture of the towers can be optimized to greatly improve their resistance to self-buckling. To this aim, the maximum height of hollow and tapered towers is investigated. The experimental and theoretical developments presented in this paper can help students grasp the fundamental concepts in elasticity and mechanical stability.

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I. INTRODUCTION

The stability of freestanding structures is of great importance in architecture and relies on simple physical principles.[1–3] The main potential causes of collapse include misalignments of the structure and failure of the building material itself. However, when a tower is submitted to a compressive normal force, it can undergo sudden lateral deflections, a phenomenon known as buckling.[4] In particular, the weight of the structure leads to non-uniform normal loads (more intense at the base of the tower than at its top), a phenomenon that will be referred to as self-buckling in the remainder of the text. In this article, we propose to study this instability at the laboratory scale in simple, affordable, hands-on experiments using gelatin-based materials. The rigidity (or Young's modulus) of the material can be easily varied by changing the gelatin concentration and its consequences on the maximum height of a tower prior to buckling are spectacular. We show that the linear continuous theory of elasticity allows for precise prediction of the maximum height of long gelatin bars (of square or circular cross section). We introduce a useful trick to repeat over and over the experiments using individual gelatin-based stackable bricks produced using a homemade silicone mold of classic Lego® bricks, which allows them to firmly interlock as displayed in Fig. 1. The usefulness of this method relies on the failure mode of the tower when it exceeds the maximum stable height (see the rightmost photograph of Fig. 1): while irreversible fractures appear within a solid block, individual bricks simply detach in the brick tower (and may be used again for further studies). Moreover, the versatility of the gelatin bricks allows one to investigate the increase in stability of smartly designed freestanding structures.

The experimental protocol described in this article provides an interesting hands-on illustration of the theory of elasticity, more specifically to the Euler-Bernoulli theory for beam deflection,[5–9] and to the phenomenon of buckling. The results presented in this article may also be used as an illustration of the importance of dimensional analysis.[10–12] Moreover, we show that the predicted frequency of free oscillations (obtained through a hand-waving argument rather than a rigorous and unnecessarily complex derivation) shows excellent agreement with the experimental measurements.

The article is organized as follows. In Sec. II, the framework of the linear theory of elasticity is reviewed, a dimensional analysis of self-buckling is proposed, theoretical predictions for the critical self-buckling height are derived, and the frequency of oscillations of stable towers are theoretically investigated. Section III presents experimental results on the mechanical properties of the gelatin-based gels as well as the maximum height of stable towers and the frequency of free oscillations. Finally, the increase in stability of hollow or tapered towers is discussed in Sec. IV.

II. THEORY OF SELF-BUCKLING FROM ELASTICA

This section reviews and derives theoretical predictions of the critical self-buckling height and of the oscillation frequency of elastic beams using a continuum approach. The relevance of this approach is shown in Sec. III B: the critical self-buckling height measured of long gelatin-based blocks is identical to that of towers made from stacked bricks (and having the same geometry).

A. Euler-Bernoulli beam theory

The bending and buckling of an elastic tower is well described by the Euler-Bernoulli beam theory, which is a simplification of the linear elasticity theory applied to the case of small deflections.[13] This theory postulates the existence of a neutral fiber (at the center of the beam), whose length remains unchanged and which remains locally perpendicular to the cross section of a bent beam. The essential result of this theory is that there exists a linear relation between the bending moment M within the beam and its local curvature κ

$$M = EI\kappa, \tag{1}$$

where E is the Young's modulus and $I = \int\int r^2\, dA$ is the second moment of area, known as the area moment of inertia in engineering (r being the distance to the neutral fiber and dA the surface element in the cross section). In the case of a rectangular beam[14] of width w and depth d (where by convention $w < d$), the neutral fiber is a central plane along the beam and $I = w^3 d/12$ and the surface area of the cross section is $A = wd$.

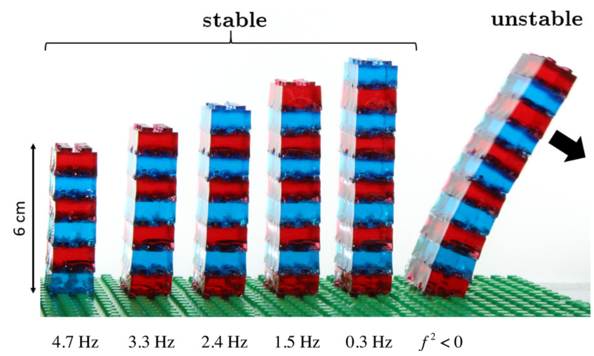

Fig. 1. Stability of gelatin-based Lego brick towers. Below a critical height (10.5 ± 0.5 cm for a gelatin concentration of 16%), when perturbed, a tower will oscillate around its stable vertical equilibrium position (at the indicated frequency). Above the critical height, any small-amplitude perturbation causes the tower to collapse (right-most tower).

B. Dimensional analysis

Let us first consider the case of a narrow strip of paper (or a thin metal or plastic sheet) representing the beam and held between two fingers at a 45° angle, as in Fig. 2(a). If the strip is short enough (typically less than 2 cm) it will remain perfectly straight while a longer strip (say, 20 cm) will be bent, perhaps significantly, under its own weight. There is, therefore, a clear competition between the flexural rigidity (EI) and the weight (per unit length) $\rho g A$, where ρ is the density of the material and g is the gravitational field strength, the ratio of which has the dimensions of a length cubed. One can therefore define, from dimensional considerations, a characteristic length

$$L_c = \left(\frac{EI}{\rho g A}\right)^{1/3} \tag{2}$$

for this situation. For typical paper ($w = 0.1$ mm, $\rho = 800$ kg/m^3, $E = 3$ GPa), this equation gives $L_c \simeq 7$ cm, which corresponds well to the simple experiment described above. Therefore, L_c can be viewed as a critical length above which an oblique beam noticeably bends under its own weight.

C. Buckling and self-buckling

The case of a vertical beam is more subtle. If the elastic tower is initially perfectly vertical (however tall), its own weight exerts no moment and cannot cause the beam to bend or buckle. However, a straight beam may become unstable to lateral deflections above a critical load, known as Euler's critical load (in the case of a tower, corresponding to a critical height).[4] Buckling is thus an instability with a well-defined threshold (or bifurcation).

When the total load is located at the tip of a vertical tower (therefore neglecting the weight of the tower itself), the critical load above which a tower will buckle when submitted to any perturbation depends on the boundary conditions. For a beam of height H clamped at its base and free at its top, the critical load F_c is given by[4]

$$F_c = \left(\frac{\pi}{2}\right)^2 \frac{EI}{H^2}. \tag{3}$$

The case of self-buckling—where the load is uniformly distributed along its height (and with no load at the top)—is mathematically more complex but physically very similar to Euler's critical load, and was solved as early as the XVIIIth century.[15] The critical height in this case is given by[16]

$$H_c = \left[\left(\frac{3j_{1/3}}{2}\right)^2 \frac{EI}{\rho g A}\right]^{1/3} \simeq 1.986\left(\frac{EI}{\rho g A}\right)^{1/3}, \tag{4}$$

where $j_{1/3} \simeq 1.866$ is the first zero of the Bessel function $J_{1/3}(x)$. Below this height, when disturbed, a tower simply oscillates around the stable vertical position, whereas above this height any small perturbation will cause the structure to fail. Note that the height H_c only differs from the one obtained by dimensional analysis, Eq. (2), by a numerical factor.

As a side note, from Eqs. (3) and (4) one can see that the critical height of a tower whose load is uniformly distributed is $3j_{1/3}/\pi \simeq 1.78$ times that of the same beam if its entire weight $\rho g A H_c$ were placed at the top.

D. Oscillations of a vertical beam clamped at its base

An elastic beam hanging from the ceiling is somewhat reminiscent of a pendulum pushed back to the vertical position by a linear spring; it can oscillate around its vertical equilibrium position due to elasticity (with an angular frequency ω_e, if the weight is neglected) and due to gravity (with an angular frequency ω_g if the elasticity is neglected). These two independent frequencies are given by[14,17]

$$\omega_e^2 = \left(\frac{\beta_1}{H}\right)^4 \frac{EI}{\rho A} \tag{5}$$

and

$$\omega_g^2 = \left(\frac{j_0}{2}\right)^2 \frac{g}{H} \tag{6}$$

where $\beta_1 \simeq 1.875$ is the first zero of the function $1 + \cos(x)\cosh(x)$ and $j_0 \simeq 2.405$ the first zero of the Bessel function $J_0(x)$. In the case of the spring-pendulum, for small-amplitude oscillations the system is linear and the resulting angular frequency is simply given by $\omega^2 = \omega_e^2 + \omega_g^2$ (since forces are additive in Newton's second law of motion). As a crude first-order assumption, one can consider that the

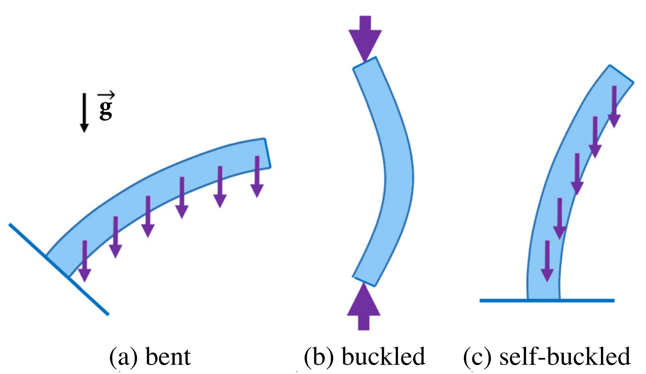

Fig. 2. Schematics of elastic beams: (a) oblique beam bent under its own weight, (b) buckled tip-loaded beam, and (c) clamped beam self-buckled under its own weight. The first case causes a continuous deformation of the beam while the two others display a well-defined threshold and lead to sudden large-amplitude deformations.

previous linear calculation still holds for the hanging elastic beam. In the case of a vibrating tower whose height is less than the critical self-buckling height, the weight has a destabilizing effect and the angular frequency of the small-amplitude oscillations is now given by

$$\omega^2 = \omega_e^2 - \omega_g^2. \tag{7}$$

From Eqs. (5) and (6), the height of the tower when the frequency vanishes (i.e., $\omega_e = \omega_g$) is thus found to be

$$H_c = \left[\left(\frac{2\beta_1^2}{j_0}\right)^2 \frac{EI}{\rho g A}\right]^{1/3} \simeq 2.045\left(\frac{EI}{\rho g A}\right)^{1/3}. \tag{8}$$

Although this equation is not strictly identical to Eq. (4), it is still proportional to the characteristic length and differs only by the numerical prefactor. Nevertheless, the numerical values of the two prefactors are very close (within 3%), which indicates that the crude model proposed for the oscillation frequency is relevant. As shown below, experimental measurements of the frequency are discussed in Sec. III C and are in good accordance with Eq. (8).

III. EXPERIMENTS USING GELATIN-BASED TOWERS

A. Mechanical properties of the gelatin gel

The experimental investigation reported below was carried out using blocks or bricks made of gelatin, an affordable and safe visco-elastic material. Gelatin is a gelling agent made of hydrolyzed collagen and obtained from skin, bones, and connective tissues of pigs, chickens, cows, and fish.[18] An elastic gel is formed when dissolved in hot water and left to cool. The mechanical properties of the gel depend on the gelatin mass concentration C, on the preparation protocol (most importantly on the duration and temperature at which the gel sets), and on the initial gelation strength of the dry gelatin (characterized by the standardized Bloom number test).[19] The gelatin used in our experiments has a Bloom number ranging from 200 to 225. The mechanical properties of gelatin gels are very sensitive to temperature changes; typically, a variation of 2 °C in the ambient temperature can cause a 20% change in the Young's modulus.[20] When exposed to air, gelatin samples quickly dry out and must therefore be kept moist during experiments. Moreover, they may slowly swell when stored in water over several days. All gelatin blocks or bricks were produced using homemade molds and were left to set in a refrigerator for 24 h prior to the experiments (which were conducted at 20 °C). The density of the gels (for concentrations up to 30%) does not significantly differ from that of pure water. We note that agar-agar gels set within minutes and represent a good alternative to gelatin-based gels.

The Young's modulus E of the gelatin gels and their compressive strength σ_c (the pressure at which they fail, forming irreversible fractures) were measured using an Anton Paar AR1000 rheometer in which cylindrical samples (20 mm in height and diameter) were tested. This device can simultaneously measure the applied normal force (with an accuracy of 10^{-4} N) and the resulting deformation (with an accuracy of less than 1 μm). The Young's modulus is measured as the initial slope of the stress-strain curve while the compressive

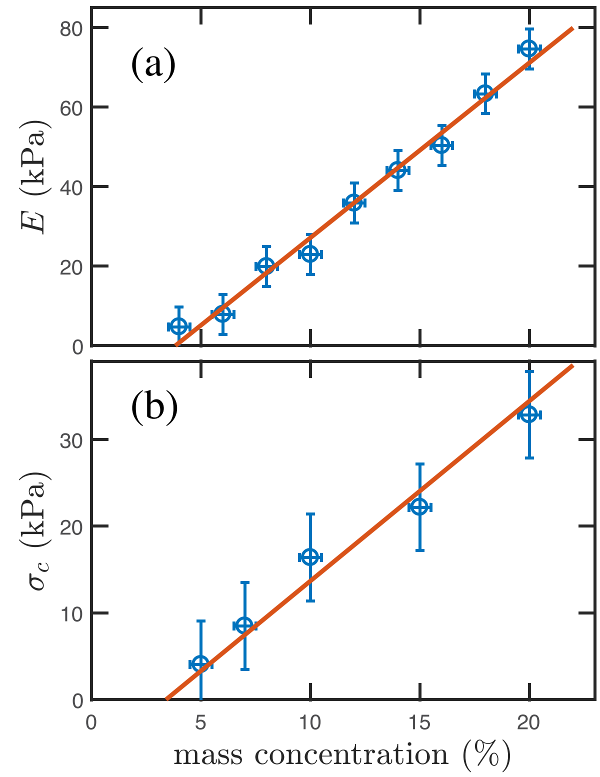

Fig. 3. (a) Young's modulus E and (b) compressive strength σ_c of gelatin gels at 20 °C as a function of the mass concentration in gelatin. Both quantities display a linear dependence on the concentration, and vanish at $C = 3.5\% \pm 0.5\%$, indicating that below this critical concentration the medium can be considered liquid. The solid lines are linear fits.

strength is computed from the maximum stress sustained before the sample fractures.

Figure 3 shows the results as a function of the concentration at a temperature of 20 °C. The Young's modulus ranges from 0 for $C \simeq 3.5\%$ (meaning that the solution is simply liquid at low concentrations) to 75 kPa for $C = 20\%$. Note that the values of E and σ_c are of the same order of magnitude, which indicates that the material can undergo very large deformations (more than 50%) before failing.

We want to emphasize that the high precision of a commercial rheometer is an unnecessary luxury since, as mentioned above, the mechanical properties of the gel are very sensitive to the concentration, the preparation protocol, and to small temperature variations. The measurements of E and σ_c can be easily performed by compressing a sample directly on a scale while the deformation is measured with a caliper.

B. Maximum height of simple towers before self-buckling

Before introducing the use of stackable bricks made of gelatin, we first report the maximum height of a continuous beam made of gelatin. Two blocks (16×16 mm and 32×32 mm) of concentration 14% were used and their height prior to buckling is reported in Fig. 4 (open circles). Clearly, these two points align well with the other data sets obtained with stacked bricks. This confirms that the behavior of towers made of individual bricks is identical (when it comes to the critical self-buckling height) to that of solid towers of gelatin gels, and it validates the continuum approach used in Sec. II. Note, however, that for large deformations (far above the critical height) the solid blocks may

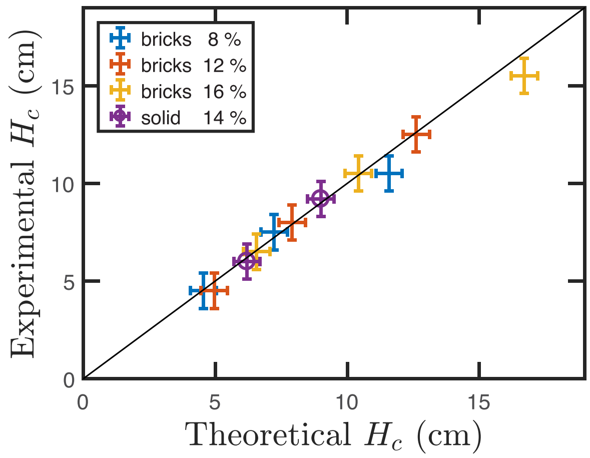

Fig. 4. Experimental vs theoretical critical self-buckling height H_c for towers of rectangular cross-sections, for various dimensions and gelatin concentrations. The solid black line has a slope of 1 and emphasizes the excellent agreement between the experimental data and the theoretical predictions.

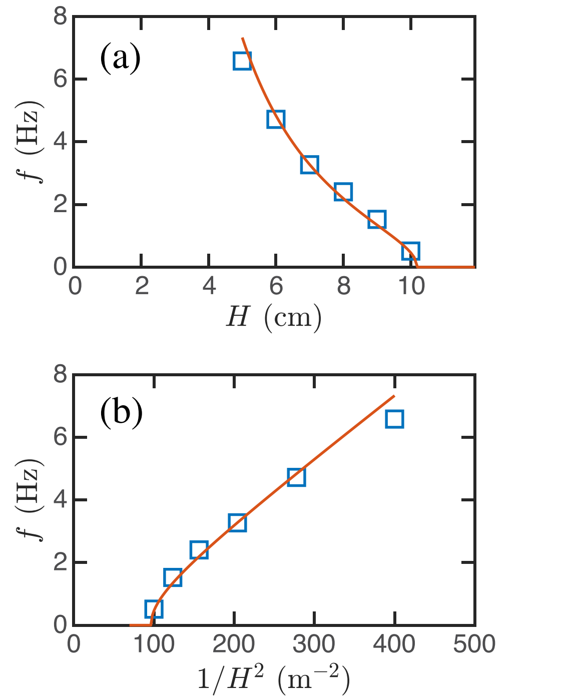

Fig. 5. Frequency f of the free oscillations of a tower (4×2, $C = 16\%$): (a) as a function of its height H and (b) as a function of $1/H^2$. The solid curves are the theoretical predictions from Eq. (7). The frequency vanishes to zero as the height tends toward the critical self-buckling height. The good agreement between experiment and theory validates the hypotheses made and shows that self-buckling is indeed the main limitation to the height of gelatin towers.

experience irreparable fracturing whereas individual bricks can simply separate and be reused.

The bricks were produced using a homemade silicone mold of classic Lego® bricks. We used bricks of width $w = 16$ mm, depth $d = 32$ mm, and height 10 mm, showing 2 rows of 4 studs, henceforth referred to as 4×2 bricks. Cutting these bricks in half creates 4×1 bricks while interlocking them will lead to wider towers. Individual bricks are carefully stacked until the tower falls. Using this simple protocol, the maximum height $H_{\max}$ is bounded between two integer values (between 10 and 11 bricks in Fig. 1, i.e., $10\,\text{cm} < H_{\max} < 11\,\text{cm}$). Figure 4 shows the results obtained for 4×1, 4×2, and 4×4 towers, and for three concentrations $C = 8\%$, $C = 12\%$, and $C = 16\%$. The corresponding maximum heights range from 4 to 16 cm. The experimental measurements are plotted as a function of the theoretical predictions of Eq. (4), using the values of the Young's modulus of Fig. 3.

These results validate the continuous medium approach used for the tower made from stacked bricks, indicating again that a tower made of individual bricks will buckle at the same height as a solid block of gel of identical cross section.

Note, however, that if the cross section is large enough the compressive strength may become the limiting factor. Indeed, the maximum height before the bottom brick of the tower fails is given by the hydrostatic pressure $\rho g H = \sigma_c$. Nevertheless, for a concentration of $C = 8\%$ the corresponding height is as large as 90 cm, whereas it reaches 350 cm for $C = 20\%$. Let us also mention that for such tall towers, the irregularities in the shape of the individual bricks can cause the tower to simply tip over since its center of gravity might not remain above its base.

C. Predicting self-buckling from stable structure self-oscillations

The oscillatory motion of stable 4×2 towers at a concentration of $C = 16\%$ (as in Fig. 1) was studied using a video camera (Ximea, xiQ MQ013MG-ON). The tower is given a gentle push (less than a centimeter in distance) and its free oscillations are recorded at 100 fps. The period is computed from the average duration of the first 5 oscillations yielding an estimated uncertainty of 0.1 Hz. Figure 5 presents the results obtained for towers of height H ranging from 5 to 10 cm. The frequency is plotted as a function of H in panel (a) and as a function of $1/H^2$ in panel (b) [since the frequency of a beam whose weight is neglected scales as $1/H^2$, as indicated in Eq. (5)].

On both panels, the gray line (red online) corresponds to the simplified model presented in Sec. II D, which is seen to match the experimental data quite well. Again, this indicates that the limiting factor in the height of a tower originates from a competition between its elasticity (a stabilizing effect) and its own weight (a destabilizing effect).

Let us mention that the measurement of the frequency of the free oscillations can also constitute a good tool to accurately determine the critical self-buckling height.[21] Stacking up individual bricks until the tower buckles only leads to a precision of one brick height (1 cm in our case) whereas the frequency vs H might simply be interpolated as a straight line (when vanishing to zero) whose intersection with the x-axis gives a value of H_c with a typical accuracy of 1 mm.

IV. TOWARDS HIGHER TOWERS: THE INFLUENCE OF DESIGN AND SHAPES

The shapes of most actual towers and sky-scrapers are not simply rectangular. From the pyramids of Egypt in ancient times to the more modern Eiffel tower and Burj Khalifa, architects have always conceived of creative designs. The choice of complex shapes is not entirely aesthetic (or financial) but is often governed by physical and engineering considerations.[1,22] Two major improvements can be made to increase the critical self-buckling height of an elastic tower:

the structure can be made hollow or tapered. In this section, these two effects are studied through experiments and theoretical predictions. It should be noted, however, that although a hollow structure (for example) can reach a greater self-buckling height, its overall ability to support an additional external weight could be less than the corresponding solid tower. It is also worth mentioning that rather than buckling under their own weight, the biggest challenges facing modern sky-scrapers include buckling and oscillating under strong winds and resistance to earthquakes.[23]

A. Hollow towers

It is seen from Eq. (4) that increasing the second moment of area I (while the mass per unit length is kept constant) pushes back the self-buckling limitation. A simple experiment can be made to demonstrate the increase in stability of hollow towers. Using 4×1 bricks, one can build a simple 4×4 tower and a hollow 5×5 tower that have the same cross-sectional areas, i.e., the same amount of mass per layer (see Fig. 6). The ratio of the second moment of area of the two structures is therefore $(5^4–3^4)4^4=2.125$, showing an outstanding increase in the rigidity of the tower. The critical height of the hollow tower is thus $2.13^{1/3}\simeq1.29$ times that of the solid structure. The difference between the two is clearly visible in Fig. 6, in which the ratio of the two heights corresponds to the predicted value.

The stability of two types of hollow towers built using a variety of concentrations was investigated. We constructed a 5×5 hollow tower (as already mentioned and displayed in Fig. 6) of width 4 cm and also a 9×9 hollow tower (made of 4×1 bricks, with a 7×7 empty hole in the center) of width 7.2 cm (that reached 38 cm in height!). The results are summarized in Table I and are in very good accordance with the theoretical predictions.

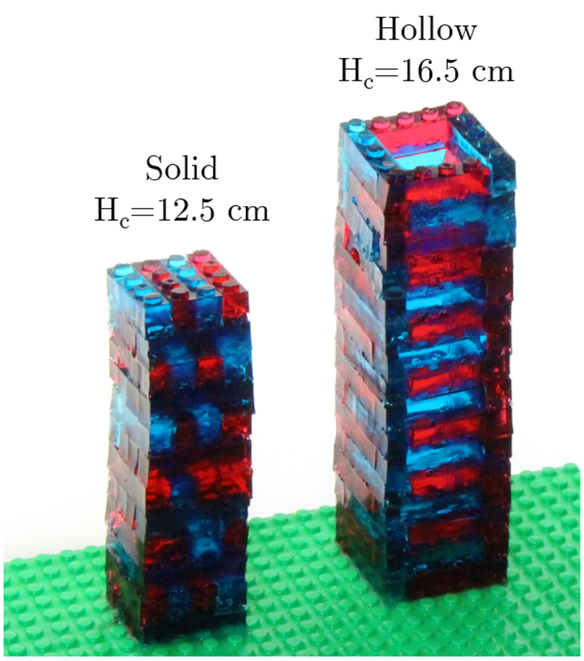

Fig. 6. Picture of solid and hollow towers (built from 4×1 bricks at $C=10\%$) at their maximum height. The solid tower reaches 12.5 ± 0.5 cm while the hollow tower reaches 16.5 ± 0.5 cm; this corresponds to the predicted increase of 29% for hollow towers of identical cross-sectional surface areas A (the same mass per layer).

Table I. Critical self-buckling height (from experiments and theory) of hollow towers for various concentrations and widths. For comparison, the height of solid towers having the same cross-sectional areas (mass per layer) are also given. Values of H_c have an uncertainty of ±0.5 cm.

	Hollow				Solid
C (%)	Overall size	Hole size	$H_c^{\rm exp}$ (cm)	$H_c^{\rm th}$ (cm)	$H_c^{\rm exp}$ (cm)
10	5×5	3×3	16.5	16.1	12.5
30	5×5	3×3	25.5	25.3	20.5
30	9×9	7×7	38.0	36.5	25.5

Another interesting consequence of Eq. (4) is the comparison with a solid tower of identical width. If a linear fraction ϕ (e.g., $\phi=3/5$ for the hollow tower of Fig. 6) of a tower is hollow so that its surface area scales as $1-\phi^2$ and its second moment of area as $1-\phi^4$, then the corresponding maximum height scales as $(1+\phi^2)^{1/3}$, which ranges from 1 to $2^{1/3}\simeq1.260$. It therefore appears that thinner walls allow for taller structures. However, another limitation arises as the walls thin down. For wide structures and thin walls, the walls themselves may start to buckle under the weight of the structure. The determination of the optimal thickness of the wall is a complex problem with no analytical solution.[24–26]

B. Tapered towers

The principle behind the increased rigidity of a tapered tower might seem simple since most of its weight is located at the bottom. Yet the rigidity of such a tower also decreases as the structure tapers, so an exact calculation of the critical self-buckling height H_c is necessary.[27]

Unfortunately, it can be difficult to build smooth tapered towers using individual bricks. Therefore, in this section experiments were conducted using solid wedges and pyramids of gelatin gels, directly molded in homemade molds. The critical height for self-buckling is then measured when holding carefully the base of the tapered structure using both hands, which results is rather large uncertainties (typically 2 cm).

1. Wedges

For a wedge of half-angle α (see Fig. 7), the critical height can be predicted by adapting the results derived in Ref. 16 to wedges

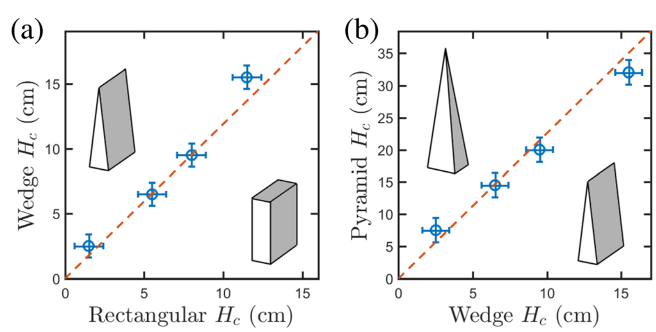

Fig. 7. (a) Experimental maximum height of wedge-shaped tower vs the corresponding predicted critical height of rectangular tower of identical width at their base. The dashed line shows the predicted proportionality (slope of 1.190), which nicely matches the data. As expected, wedges can be 19% taller than rectangular towers before buckling. (b) Experimental maximum height of pyramidal towers vs that of wedge-shaped tower of identical half-angle. The dashed line shows the predicted proportionality (slope of 2.31), in excellent agreement with the experimental data.

$$H_c^{\text{wedge}} = \frac{j_2^2}{6}\frac{E}{\rho g}\tan^2\alpha, \tag{9}$$

where $j_2 \simeq 5.136$ is the first zero of the Bessel function $J_2(x)$. The critical height can be rewritten as a function of the corresponding width $a = 2H_c^{\text{wedge}}\tan\alpha$ at the base, giving

$$H_c^{\text{wedge}} = \left(\frac{j_2^2}{2}\frac{Ea^2}{12\rho g}\right)^{1/3}. \tag{10}$$

In comparison, for a simple rectangular tower of width a [see Eq. (4)] the result is

$$H_c^{\text{rect.}} = \left[\left(\frac{3j_{1/3}}{2}\right)^2 \frac{Ea^2}{12\rho g}\right]^{1/3}. \tag{11}$$

Equations (10) and (11) indicate that a wedge-shaped tapered tower can be approximately $(2j_2^2/9j_{1/3}^2)^{1/3} \simeq 1.190$ times as tall as a rectangular tower of identical width at its base.

Figure 7(a) shows the experimental maximum height before self-buckling for wedges of various half-angles and for various concentrations. The experimental data are plotted as a function of the corresponding theoretical value of $H_c^{\text{rect.}}$ for rectangular towers and the dashed-line has the predicted slope of 1.190. The agreement between the experiments and the prediction is very satisfactory, showing that wedge-shaped towers are indeed $\simeq$19% taller than their rectangular counterparts.

2. *Pyramids*

The shape can be further improved by building a tower that tapers down towards the tip in both directions, giving the shape of a pyramid. For a square cross-section, the critical height of such a pyramid of half-angle α (between opposing faces of the pyramid) is given by[16]

$$H_c^{\text{pyramid}} = \frac{j_3^2}{4}\frac{E}{\rho g}\tan^2\alpha, \tag{12}$$

where $j_3 \simeq 6.380$ is the first zero of the Bessel function $J_3(x)$. Note that in Ref. 16 the results are derived for towers of circular cross sections, but the results still apply to rectangular cross sections, using the corresponding values of I and A.

For the same half-angle, the height of a pyramid can therefore be as much as $3j_3^2/2j_2^2 \simeq 2.31$ times that of a wedge-shaped tower. The maximum height of pyramids and wedges of identical half-angles were experimentally measured for various values of α and various concentrations. Figure 7(b) shows a comparison between the two shapes; the slope of the dashed line is the predicted ratio. Note that while it is somewhat surprising that pyramids of a given half-angle can be more than twice as tall as wedges with the same half-angle, the difference in height is not quite so dramatic when the comparison is made for equal base widths. In this situation, a pyramid can be approximately $2.31^{1/3} \simeq 1.32$ as tall as the corresponding wedge-shaped tower of identical base width.

V. CONCLUSION

We have shown that the main limiting factor of the stability of gelatin-based brick towers is buckling under their own weight. The classical theory of elasticity provides a continuum approach whose predictions (for critical height and oscillation frequency of a stable tower) are in good agreement with experimental measurements. Two improvements that can help push the limits of self-buckling have been demonstrated: structures can be made hollow or tapered. Combining both techniques, we were able to build a brick tower (pyramidal and hollow, 12×12 at its base, with $C = 30\%$) as tall as 56 cm.

As an extension, the mechanical properties of the gels could be investigated further. Indeed, gelatin-based gels are not simply elastic but display a viscoelastic behavior (reminiscent of the properties of silly putty[28]). The storage modulus (which characterizes the elasticity) is typically larger than the loss modulus (which characterizes the viscous behavior), but for low concentrations both quantities are of the same order of magnitude.[29] Although as a first-order approximation the material can be considered purely elastic, the bricks can undergo slow deformations when submitted to a constant pressure (a phenomenon known as creep flow,[30,31] typically occurring within a few minutes). These properties can affect the stability of the brick-towers. Stacks whose height is less than but close to the expected critical height for self-buckling might be initially stable when built, but can slowly become unstable and buckle within minutes.

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[a]Electronic mail: nicolas.taberlet@ens-lyon.fr

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Geissler Tube with Fluorescent Flowers

The aluminum anode of this discharge tube is formed in the shape of leaves and flowers. On the surface of the metal are small crystals of calcium sulphide and zinc sulphide, which glow green and violet when stuck by electrons from the cathode at the top. This example is at Franklin and Marshall College. (Picture and Notes by Thomas B. Greenslade, Jr., Kenyon College)