HW1 Molecular Programming

M2IF SCR1

24.10.2019 - **Due on Thu. 7/11 before 12:00**

You are asked to complete Exercise 3 and to send me your solutions to:

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as a PDF file named **HW1-Lastname.pdf** on **Thu. 7/11 before 12:00**.

■ **Exercise 1 (Exponential random variables & kTAM implementation).** Recall that an *exponential random variable* X with parameter $\lambda > 0$ is defined by: $(\forall x \geqslant 0)$ $\Pr\{X \geqslant x\} = e^{-\lambda x}$.

▶ ***Question 1.1)*** *Compute* $\mathbb{E}[X]$*.*

▷ Hint. *Recall that if X is a non-negative random variable, then $\mathbb{E}[X] = \int_0^\infty \Pr\{X \geqslant x\}dx$.*

▶ ***Question 1.2)*** *Show that the exponential distribution is* memoryless*, i.e. if X is exponentially distributed with parameter λ, then $(\forall t, u \geqslant 0)$ $\Pr\{X \geqslant t+u \,|\, X \geqslant t\} = \Pr\{X \geqslant u\}$.*

Let X and Y be two independent exponentially distributed random variables with respective parameters λ and μ.

▶ ***Question 1.3)*** *Show that* $\min(X, Y)$ *is also exponentially distributed. What is its parameter?*

▶ ***Question 1.4)*** *What is the probability that* $\min(X, Y) = X$*?*

▶ ***Question 1.5)*** *Same questions as the two above for n indenpendent exponentially dsitributed variables $X_1, \ldots, X_n$ with parameters $\lambda_1, \ldots, \lambda_n$.*

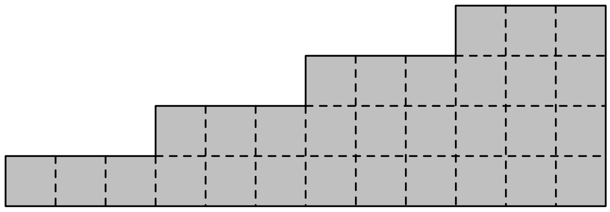
▶ ***Question 1.6)*** *Assume that a non-negative random variable X is given by its tail distribution $F(x) = \Pr\{X \geqslant x\}$. Show that X is identically distributed as $F^{-1}(U)$ where U is a uniform random variable in $[0, 1]$.*

Describe how to sample an exponential random variable of rate λ.

▶ ***Question 1.7)*** *Propose an algorithm together with a data structure to implement the kTAM model with attachement rate $r_f = k_f[Strand] = k_f\, e^{-G_{mc}}$ and detachment rate $r_{s,b} = k_f\, e^{-b \cdot G_{se}}$ where b is the number of bonds made by the strand with the current agregate.*

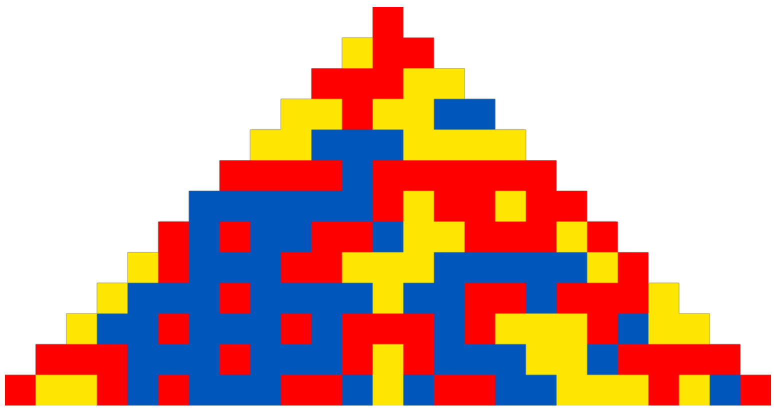
Use parameters $k_f = 10^6/M/sec$, $G_{mc} = 12.9$ and $G_{se} = 6.5$ for the algorithmic phase.

■ **Exercise 2 (Triangle).** We consider the family of shapes $\mathcal{S}_p$, $p \geqslant 1$, of (discretized) right triangles whose horizontal and vertical sizes are respectively np and n, for some $n \geqslant 1$. The figure below shows the shape in $\mathcal{S}_3$ for which $n = 4$.



▶ ***Question 2.1)*** *Give a tileset T_p whose final productions are exactly the shapes in S_p at temperature $T^\circ = 2$. How many tiles does it use? Give a generic assembly and indicate the glues, the seed tile and the assembly order on it.* No proof asked.

■ **Exercise 3 (Tileset for simulating cellular automata).** A cellular automaton consists of a finite set of *states* Q, a function $f : Q^3 \to Q$, called the *rule*, and an initial configuration $c^0 \in Q^*$. The configuration at time $t+1$ is obtained from the configuration at time t as follows: $c_i^{t+1} = f(c_i^t, c_{i+1}^t, c_{i+2}^t)$ for $0 \leqslant i < |c^t| - 2$. The calculation stops at the first time T such that $|c^T| < 3$ and the result of the computation is c_0^T. A classic visualization of the computation of a cellular automaton consists of a pyramid where the bottom line is the initial configuration and time goes upwards. Here is an example:

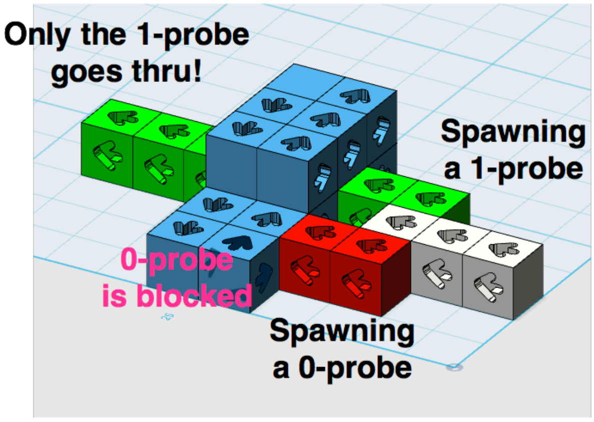


$$\text{for the rule } f(x,y,z) = \begin{cases} \textcolor{yellow}{\blacksquare} & \text{if } \{x,y,z\} = \{\textcolor{red}{\blacksquare}\} \text{ or } \{\textcolor{yellow}{\blacksquare}, \textcolor{blue}{\blacksquare}\} \\ \textcolor{blue}{\blacksquare} & \text{if } \{x,y,z\} = \{\textcolor{yellow}{\blacksquare}\} \text{ or } \{\textcolor{blue}{\blacksquare}, \textcolor{red}{\blacksquare}\} \\ \textcolor{red}{\blacksquare} & \text{if } \{x,y,z\} = \{\textcolor{blue}{\blacksquare}\} \text{ or } \{\textcolor{yellow}{\blacksquare}, \textcolor{red}{\blacksquare}\} \text{ or } \{\textcolor{yellow}{\blacksquare}, \textcolor{blue}{\blacksquare}, \textcolor{red}{\blacksquare}\} \end{cases}$$

▶ ***Question 3.1)*** *Propose a finite tileset whose self-assembly simulates the computation of any Q-state cellular automata from any initial configuration and whose size is independent of the initial configuration length. Give a generic example of the execution of your assembly for generic computation steps. Give the number of variants of each tile type as a function of $|Q|$. Provide the procedure which selects the tiles used to simulate a given Q-state cellular automaton.*

▷ Hint. *Do you need upscaling? Consider reshaping the pyramid to simplify your design.*

■ **Exercise 4 (Probabilistic simulation Turing Machine at $T^\circ = 1$ in 2D).** Recall that in 3D, for any single-tape binary-alphabet Turing machine M, there is a tile set which simulates M using a clever trick to encode 0s and 1s. These are encoded with bridges and read using two probes where only one go through the bridge:



► ***Question* 4.1)** *By adjusting the concentrations (and thus the rate at which the different tiles attached), describe a tile set together with concentrations for each tile type, that simulates a given single-tape binary-alphabet Turing machine M with an arbitrary small error ε for each symbol read in 2D at temperature $T^\circ = 1$.*