You are asked to complete Exercise 3 and to send me your solutions to:

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as a PDF file named HW1-Lastname.pdf on Thu. 7/11 before 12:00.

Exercise 1 (Exponential random variables & kTAM implementation). Recall that an exponential random variable $X$ with parameter $\lambda > 0$ is defined by:

$$\Pr\{X \geq x\} = e^{-\lambda x}.$$

**Question 1.1** Compute $E[X]$.

**Hint.** Recall that if $X$ is a non-negative random variable, then $E[X] = \int_0^\infty \Pr\{X \geq x\} \, dx$.

**Question 1.2** Show that the exponential distribution is memoryless, i.e. if $X$ is exponentially distributed with parameter $\lambda$, then $(\forall t, u \geq 0) \Pr\{X \geq t + u \mid X \geq t\} = \Pr\{X \geq u\}$.

Let $X$ and $Y$ be two independent exponentially distributed random variables with respective parameters $\lambda$ and $\mu$.

**Question 1.3** Show that $\min(X, Y)$ is also exponentially distributed. What is its parameter?

**Question 1.4** What is the probability that $\min(X, Y) = X$?

**Question 1.5** Same questions as the two above for $n$ independent exponentially distributed variables $X_1, \ldots, X_n$ with parameters $\lambda_1, \ldots, \lambda_n$.

**Question 1.6** Assume that a non-negative random variable $X$ is given by its tail distribution $F(x) = \Pr\{X \geq x\}$. Show that $X$ is identically distributed as $F^{-1}(U)$ where $U$ is a uniform random variable in $[0, 1]$.

Describe how to sample an exponential random variable of rate $\lambda$.

**Question 1.7** Propose an algorithm together with a data structure to implement the kTAM model with attachment rate $r_f = k_f[S\text{rand}] = k_f e^{-G_{mc}}$ and detachment rate $r_s,b = k_f e^{-b G_{se}}$ where $b$ is the number of bonds made by the strand with the current aggregate.

Use parameters $k_f = 10^6 / M / sec$, $G_{mc} = 12.9$ and $G_{se} = 6.5$ for the algorithmic phase.

Exercise 2 (Triangle). We consider the family of shapes $S_p, p \geq 1$, of (discretized) right triangles whose horizontal and vertical sizes are respectively $np$ and $n$, for some $n \geq 1$. The figure below shows the shape in $S_4$ for which $n = 4$.
**Question 2.1)** Give a tile set $T_p$ whose final productions are exactly the shapes in $S_p$ at temperature $T^\circ = 2$. How many tiles does it use? Give a generic assembly and indicate the glues, the seed tile and the assembly order on it. No proof asked.

**Exercise 3 (Tileset for simulating cellular automata).** A cellular automaton consists of a finite set of states $Q$, a function $f : Q^3 \rightarrow Q$, called the rule, and an initial configuration $c^0 \in Q^n$. The configuration at time $t + 1$ is obtained from the configuration at time $t$ as follows:

$$c_i^{t+1} = f(c_i^t, c_{i+1}^t, c_{i+2}^t)$$

for $0 \leq i < |c^t| - 2$. The calculation stops at the first time $T$ such that $|c^T| < 3$ and the result of the computation is $c^T_0$. A classic visualization of the computation of a cellular automaton consists of a pyramid where the bottom line is the initial configuration and time goes upwards. Here is an example:

![Example of cellular automaton computation](image)

**Question 3.1)** Propose a finite tile set whose self-assembly simulates the computation of any $Q$-state cellular automata from any initial configuration and whose size is independent of the initial configuration length. Give a generic example of the execution of your assembly for generic computation steps. Give the number of variants of each tile type as a function of $|Q|$. Provide the procedure which selects the tiles used to simulate a given $Q$-state cellular automaton.

**Hint:** Do you need upscaling? Consider reshaping the pyramid to simplify your design.

**Exercise 4 (Probabilistic simulation Turing Machine at $T^\circ = 1$ in 2D).** Recall that in 3D, for any single-tape binary-alphabet Turing machine $M$, there is a tile set which simulates $M$ using a clever trick to encode 0s and 1s. These are encoded with bridges and read using two probes where only one go through the bridge.
Question 4.1) By adjusting the concentrations (and thus the rate at which the different tiles attached), describe a tile set together with concentrations for each tile type, that simulates a given single-tape binary-alphabet Turing machine $M$ with an arbitrary small error $\varepsilon$ for each symbol read in 2D at temperature $T^\circ = 1$. 