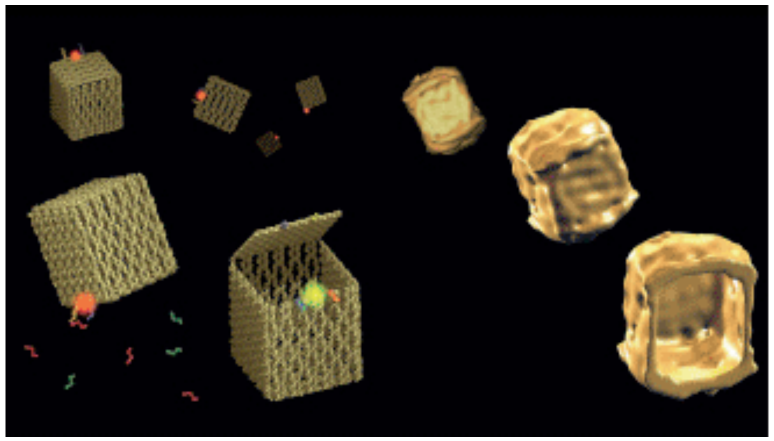
Oritatami:
A computational model for cotranscriptional folding

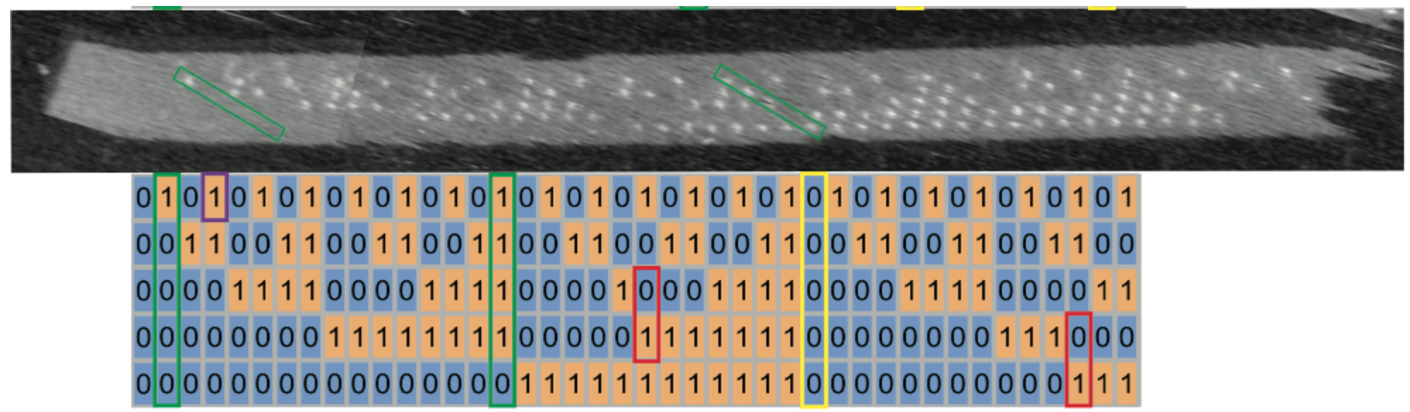
Nicolas **Schabanel**
CNRS - LIP, ENS Lyon & IXXI - France

Context: Biomolecular Computing & Engineering

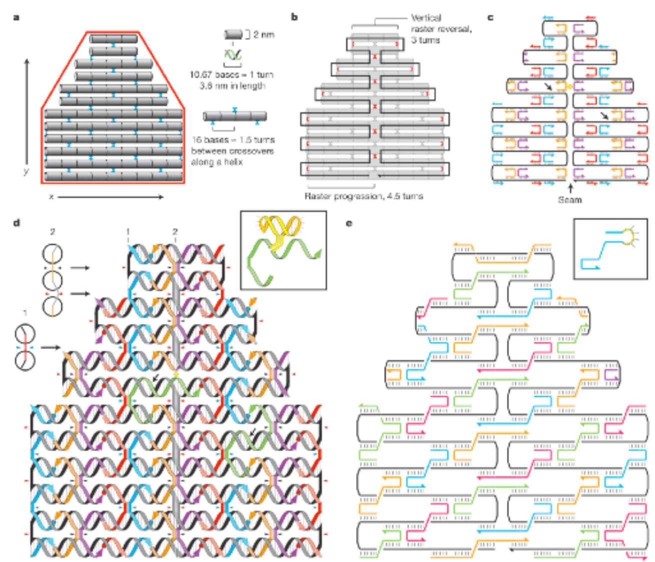
~100 nm

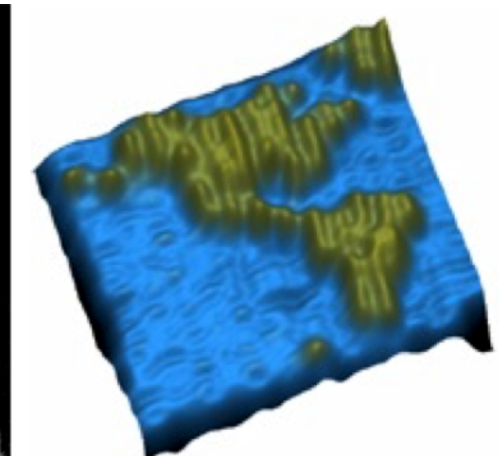


Andersen et al, 2009

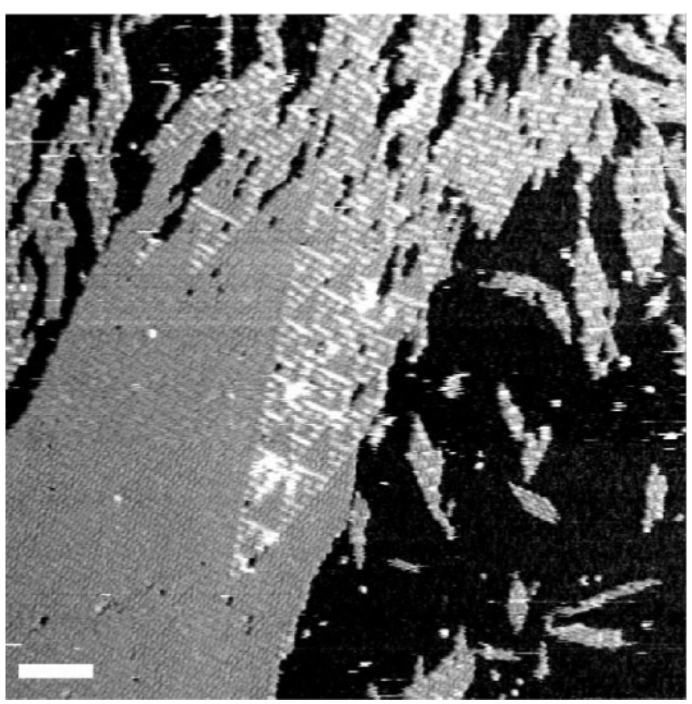


Constantine Evans, PhD Thesis, Caltech 2014

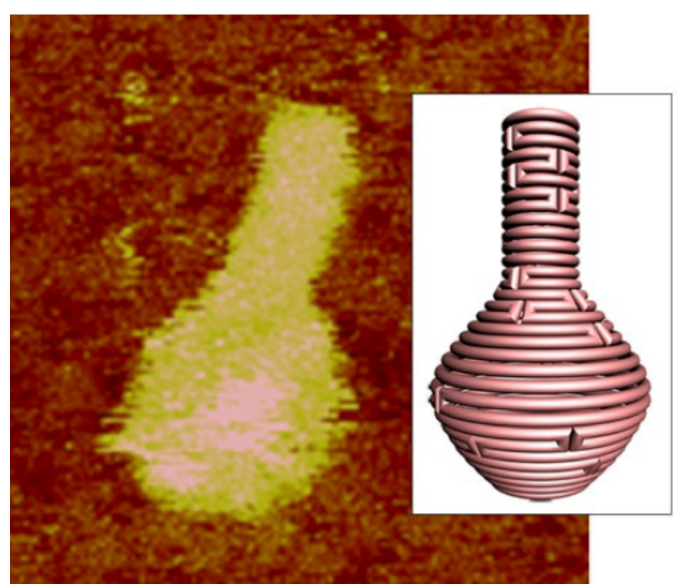




Rothemund, Nature 2006



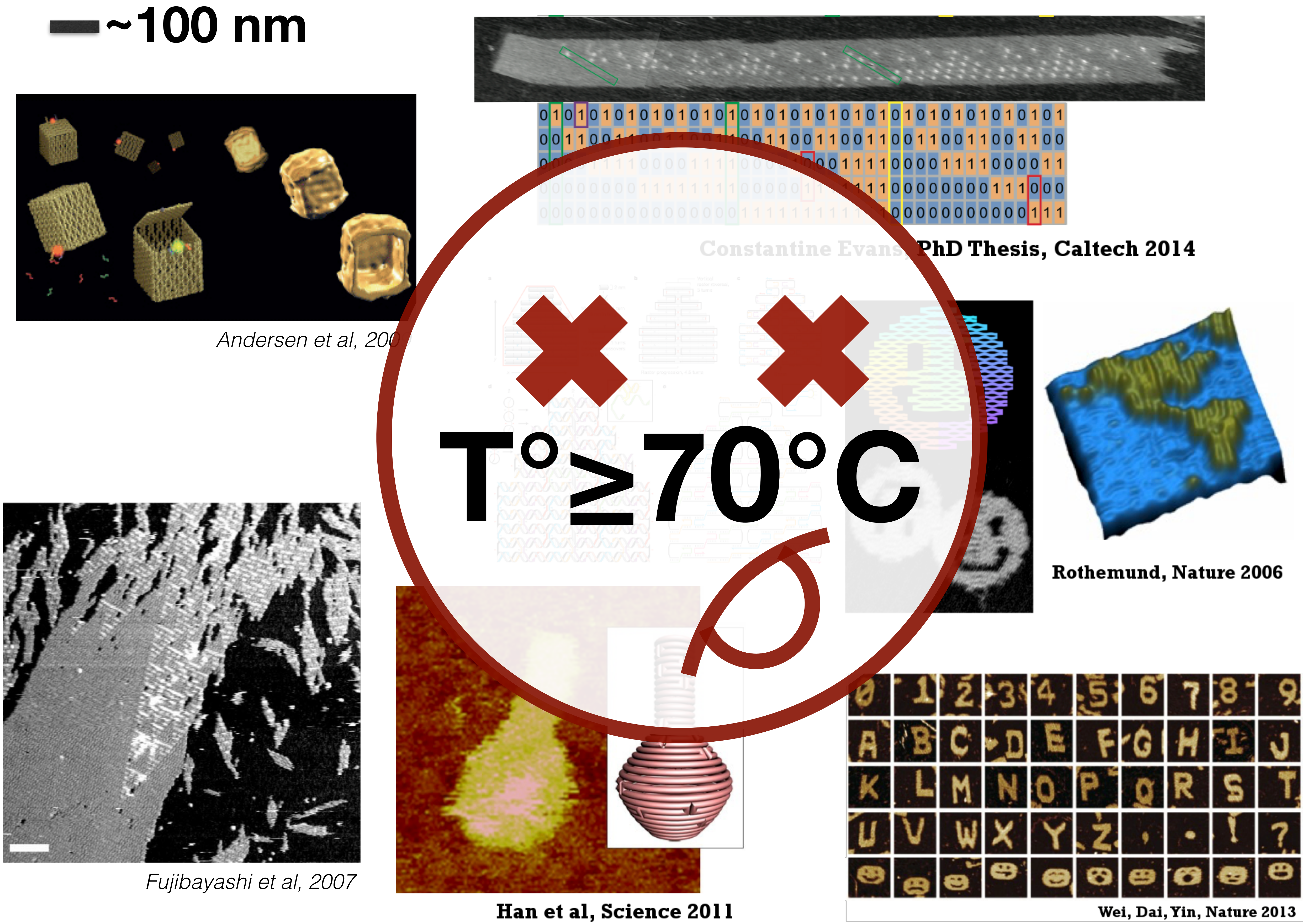
Fujibayashi et al, 2007



Han et al, Science 2011

Wei, Dai, Yin, Nature 2013

Context: Biomolecular Computing & Engineering



Andersen et al, 200

Constantine Evans, PhD Thesis, Caltech 2014

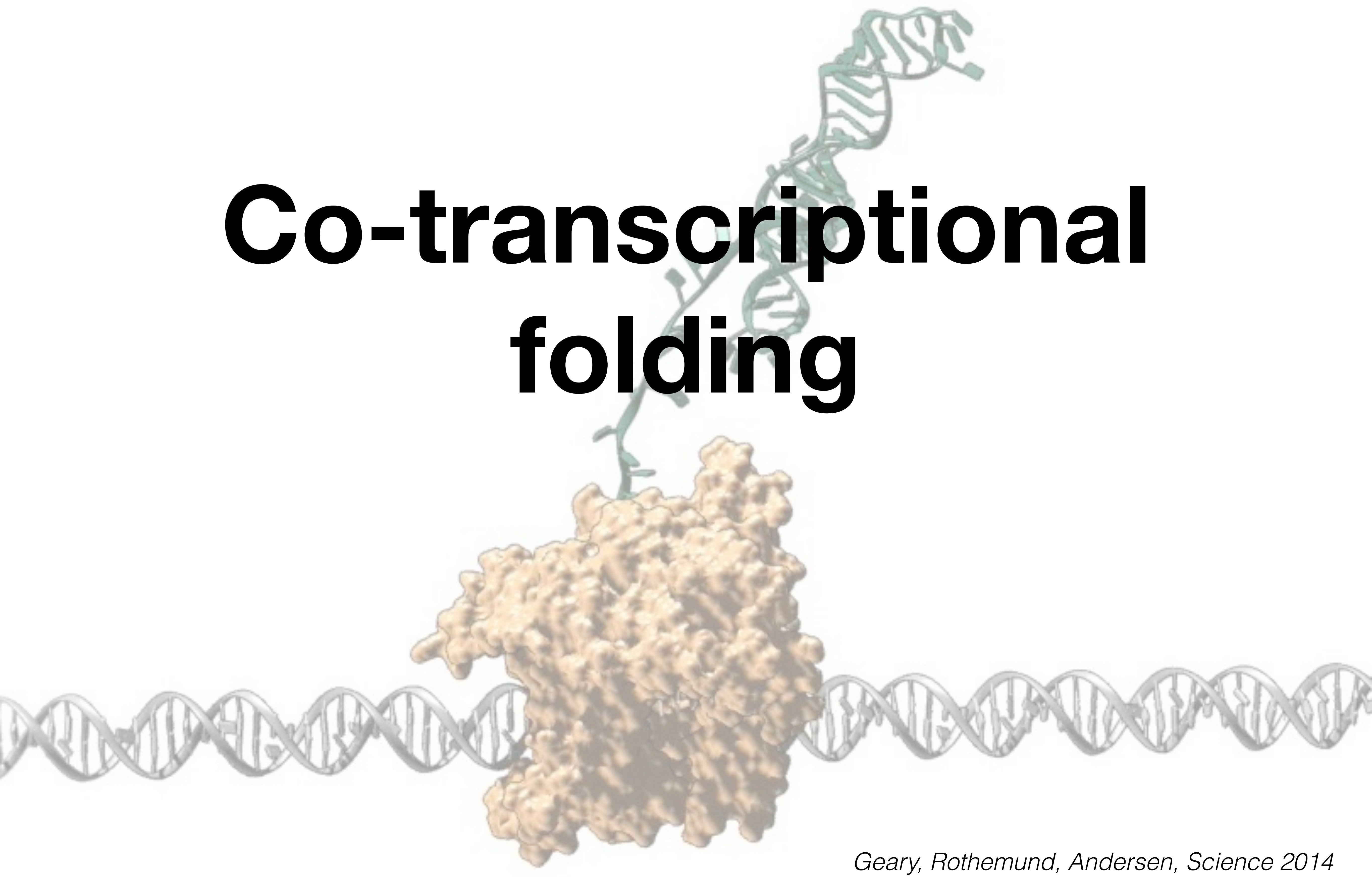
Rothemund, Nature 2006

Fujibayashi et al, 2007

Han et al, Science 2011

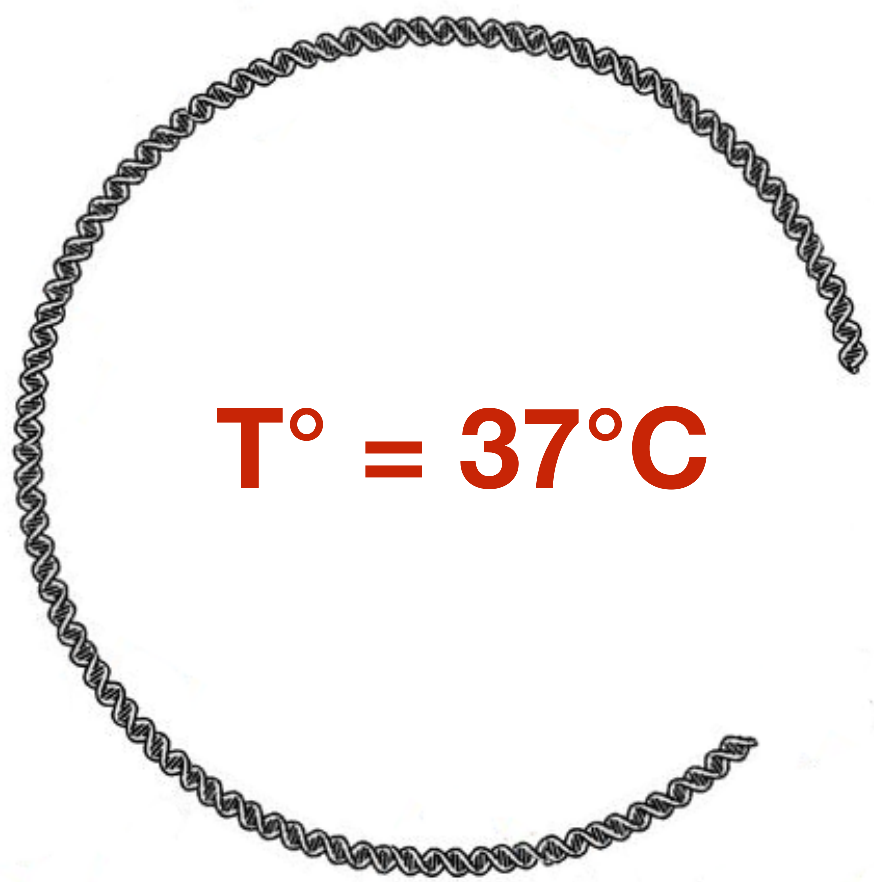
Wei, Dai, Yin, Nature 2013

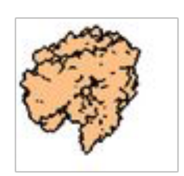
Co-transcriptional folding

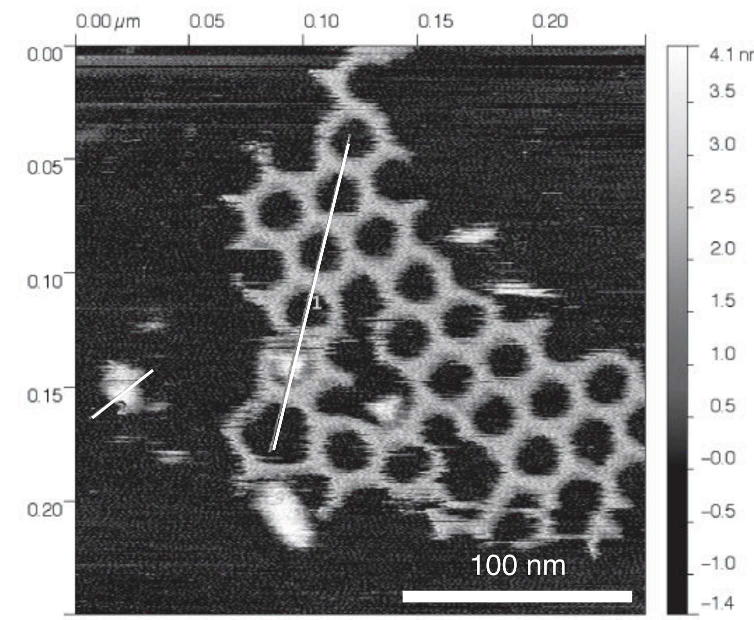


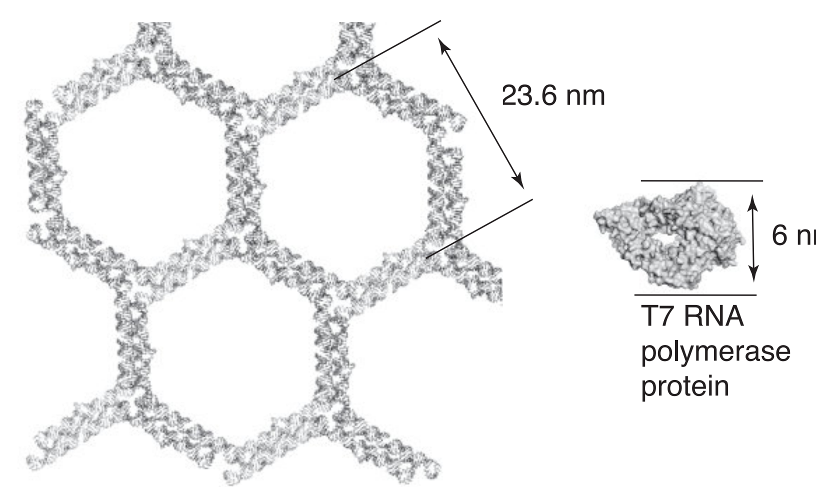
Geary, Rothemund, Andersen, Science 2014

RNA co-transcriptional folding



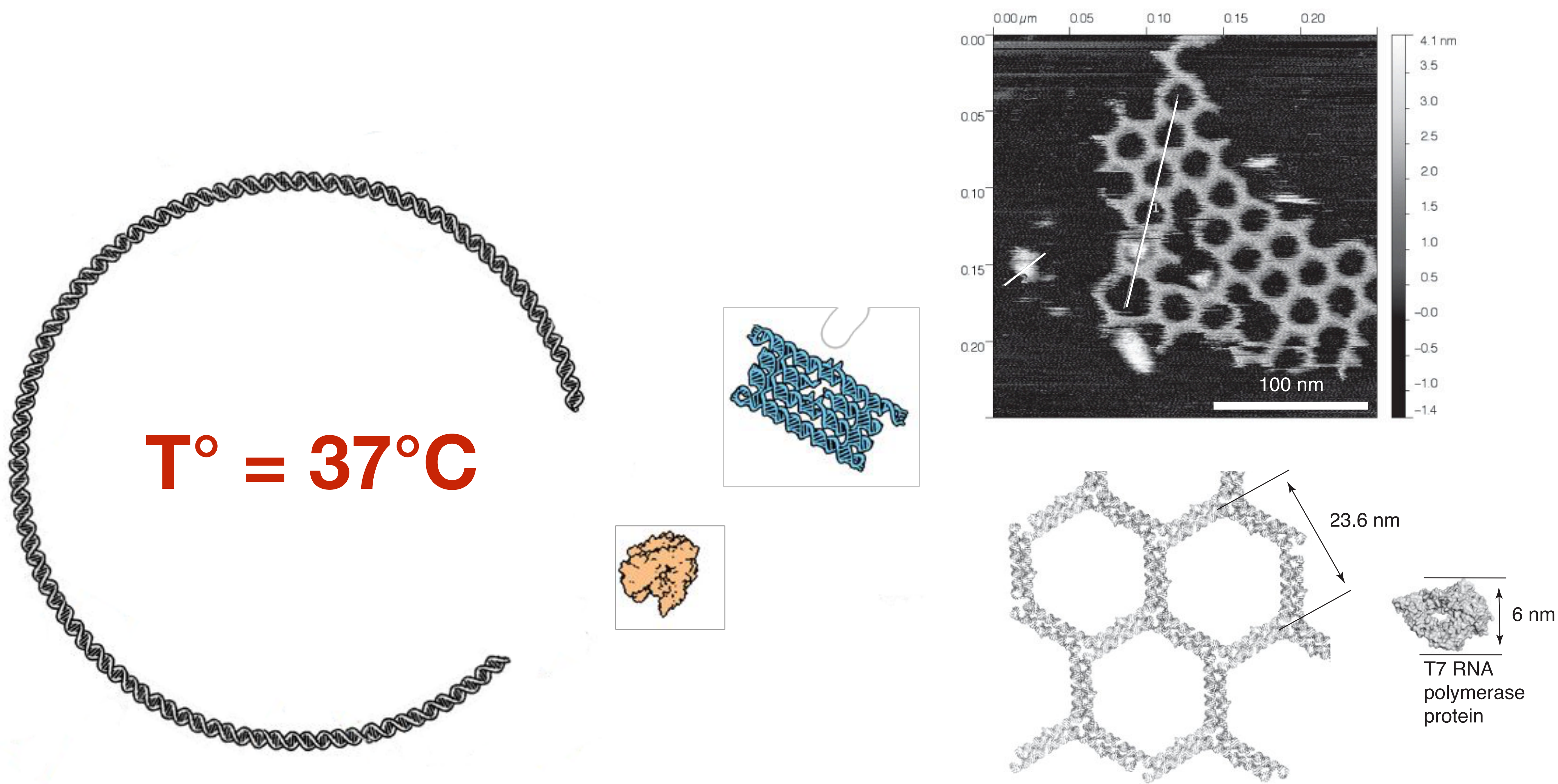






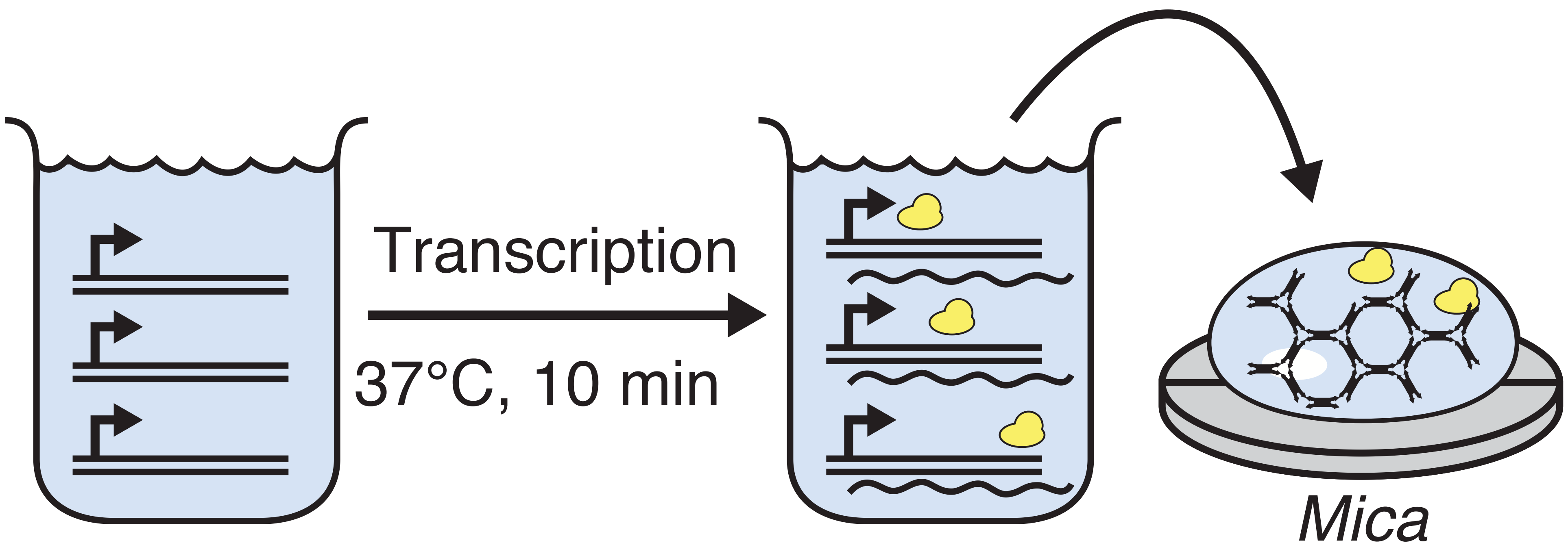
Geary, Rothemund, Andersen, Science 2014

RNA co-transcriptional folding

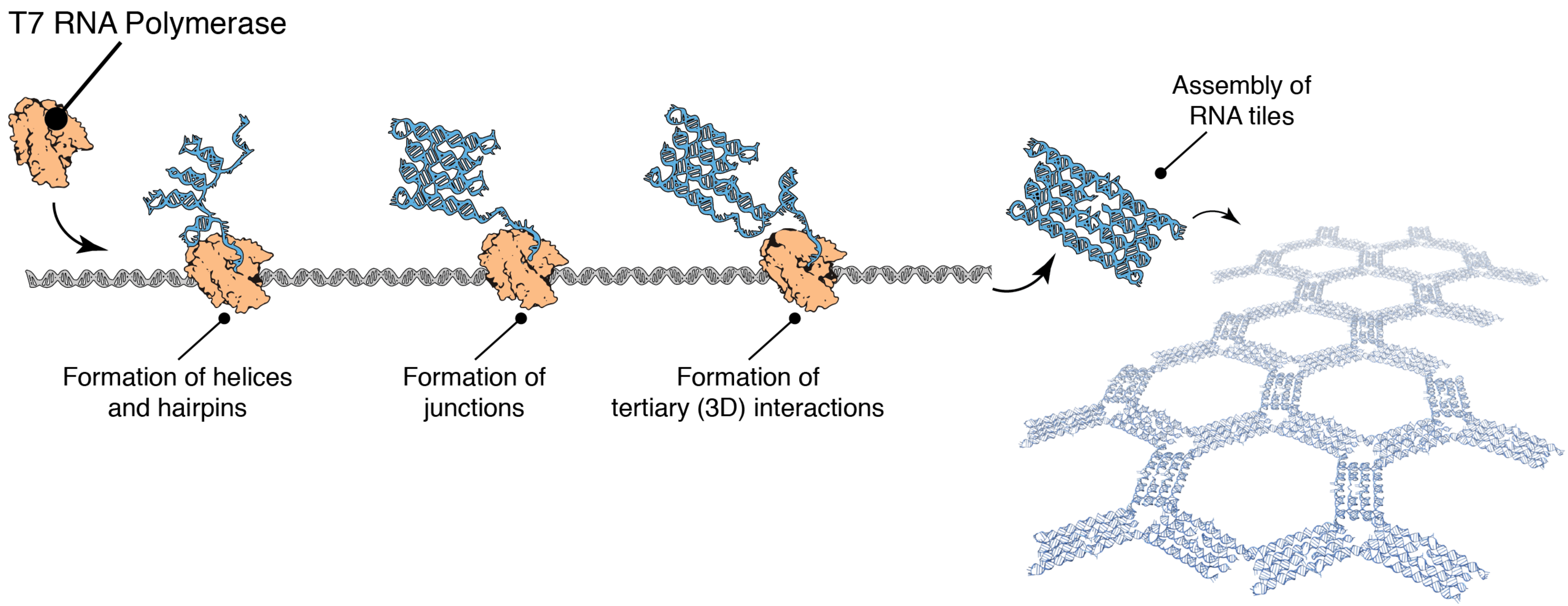


Geary, Rothemund, Andersen, Science 2014

Protocol



RNA Origami in Real Time



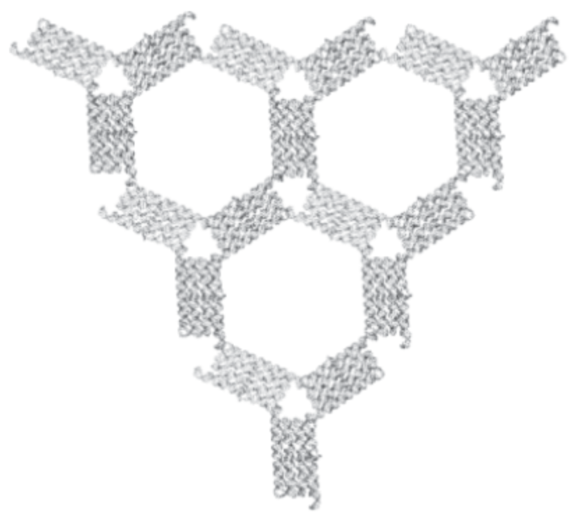
T7 RNA polymerase produces RNA directionally from 5’ to 3’, **at a rate much slower than the RNA folds up (few microseconds).**

The polymerase reads the DNA gene, and becomes an RNA origami production factory, **synthesizing a new RNA origami roughly every 1 second.**

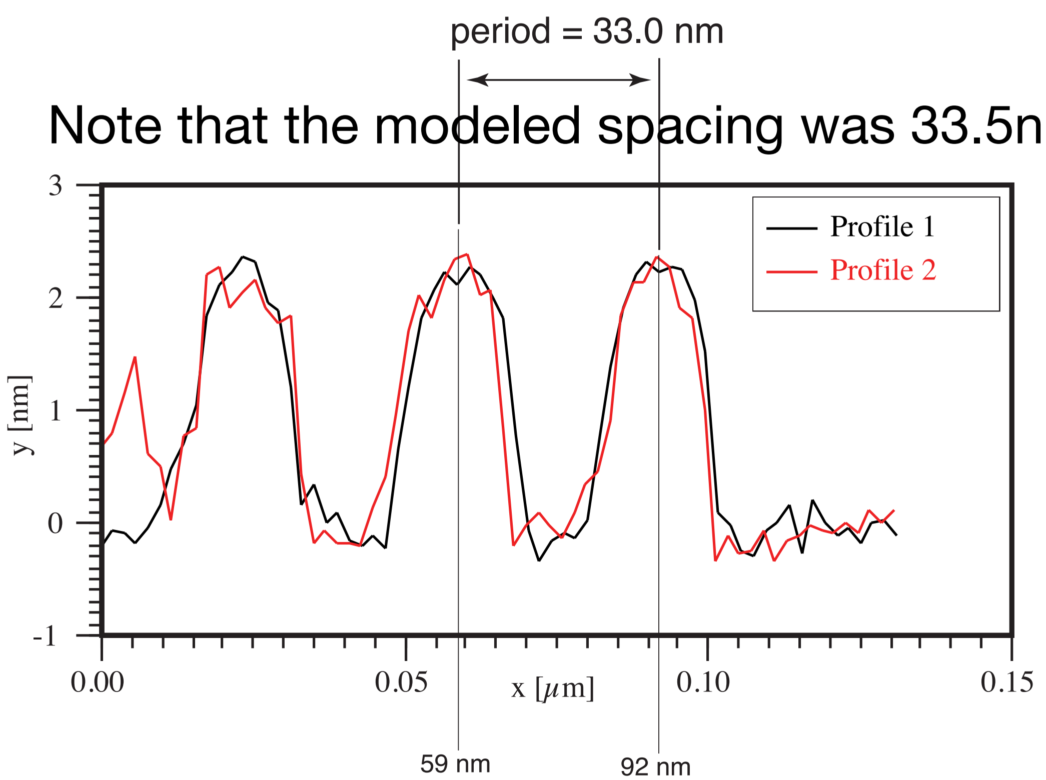
Slide by Cody Geary

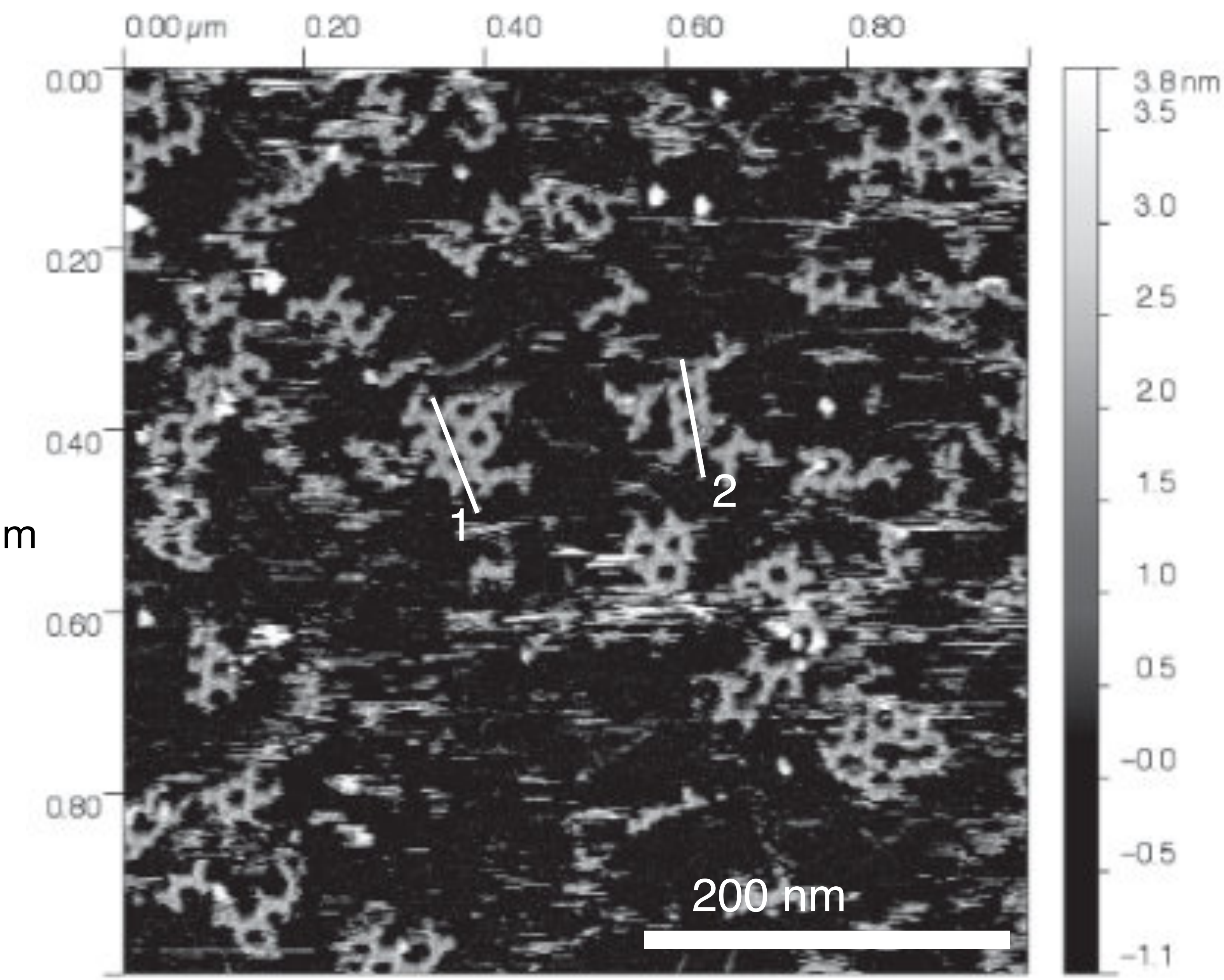
Westhof and Leontis (Science, 2014)

AFM imaging of 4H-AE co-transcriptional assembly



Note that the modeled spacing was 33.5nm





Geary et al (2014) Science

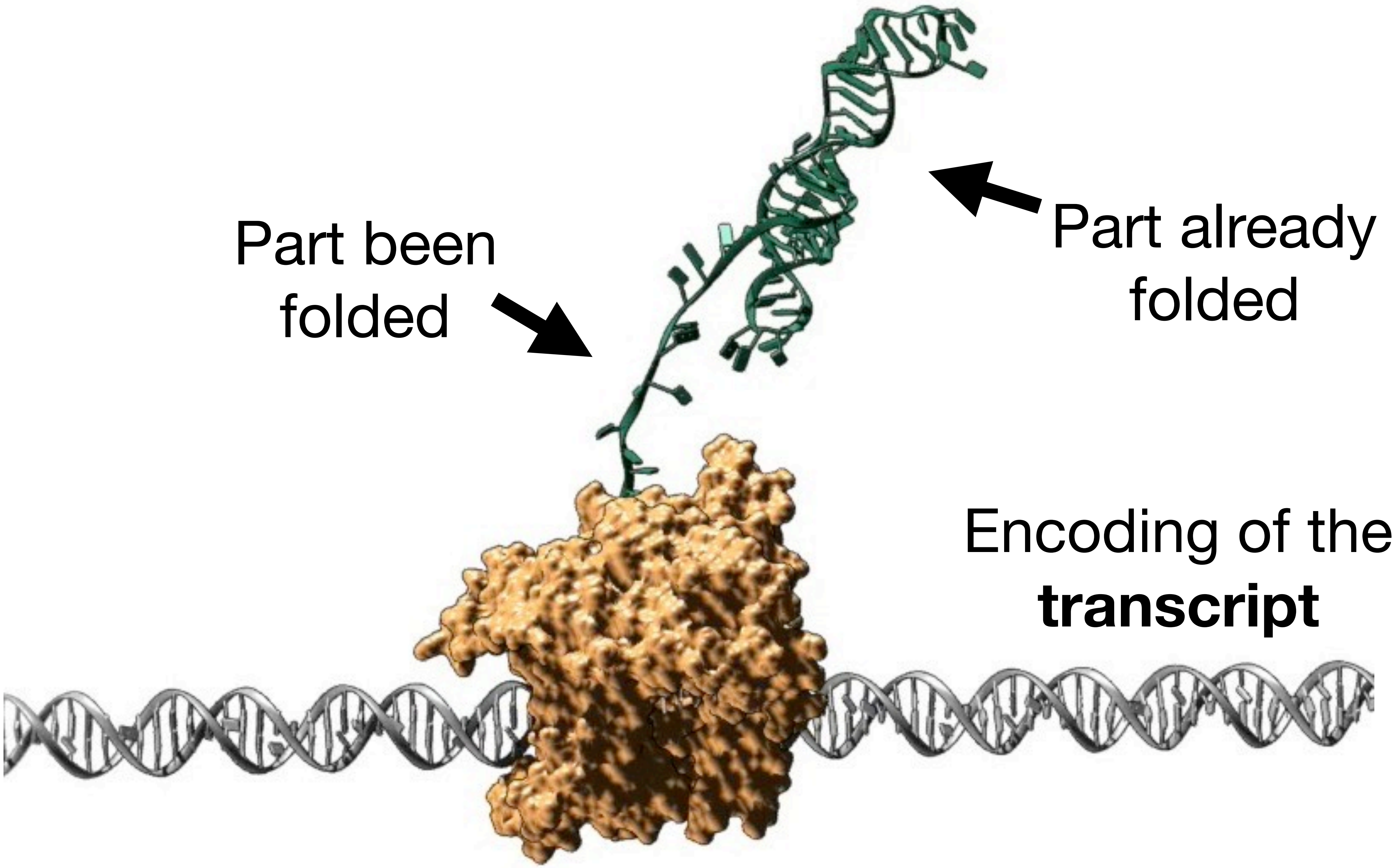
Oritatami:

A computational model for co-transcriptional folding

Geary, Meunier, Schabanel, Seki MFCS 2016

RNA Folding

(Real time: ~1 second)



Video: Geary

Oritatami:
A model for co-transcriptional folding

The program:

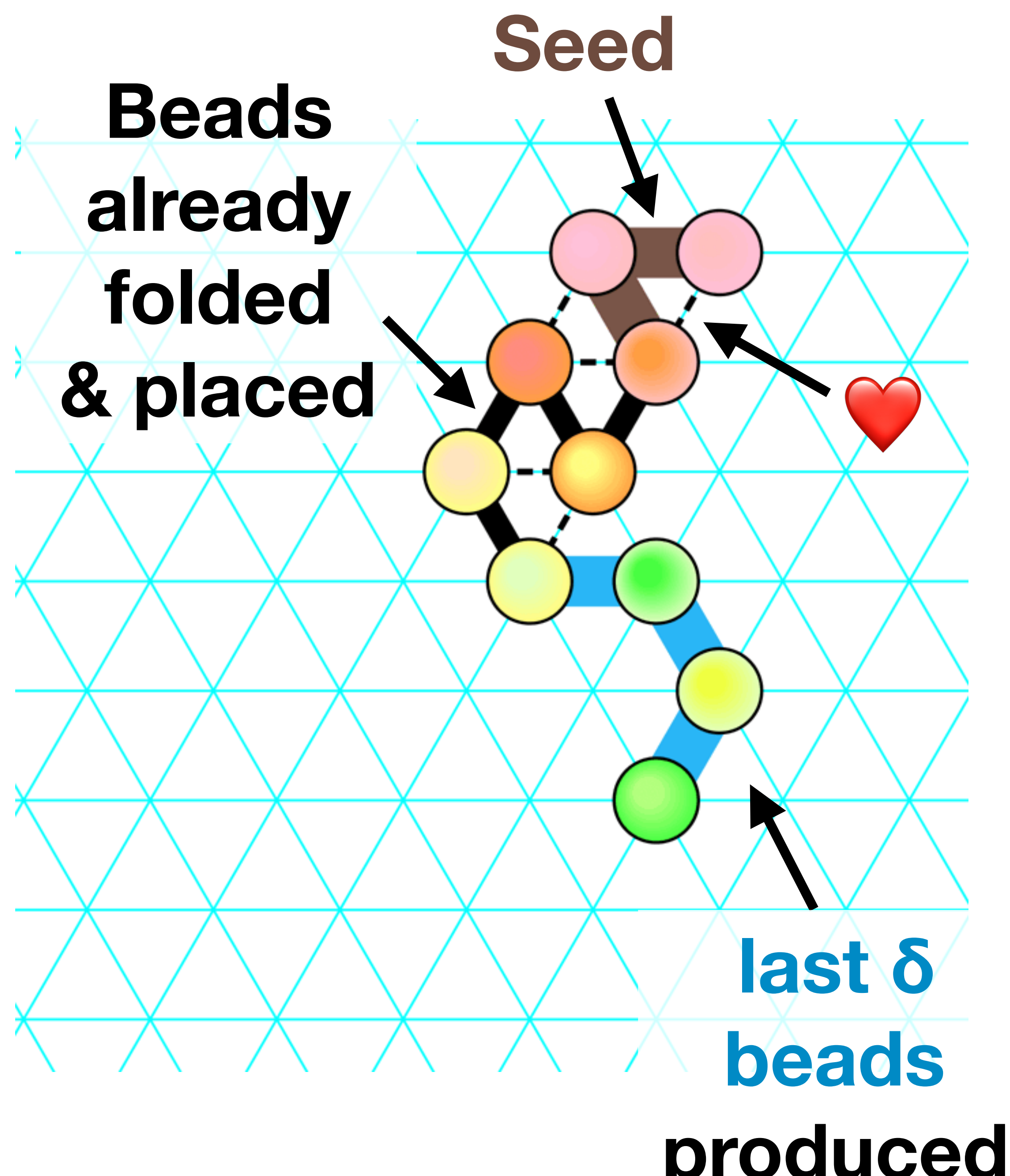
- a **sequence** of **bead types** (the **transcript**)

The instructions:

- the **rule** **a**❤️**b** if bead types **a** and **b** attract each other

The input configuration:

- Some beads placed beforehand (the **seed**)



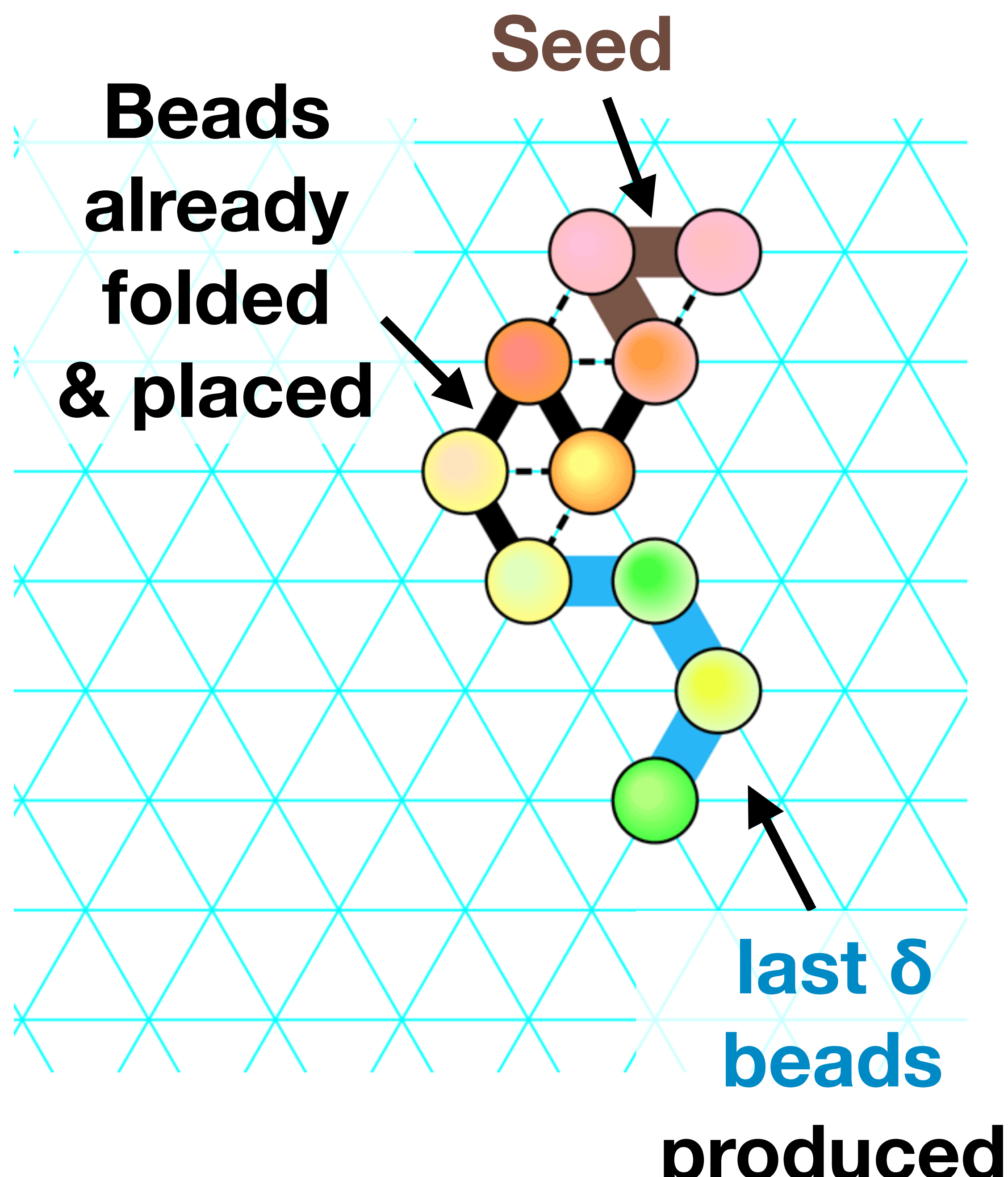
Geary, Meunier, Schabanel, Seki MFCS 2016

Oritatami:
A model for co-transcriptional folding

The dynamics

- Starting from the seed, the sequence is *produced one bead at a time*
- **Only the δ last produced beads** are free to move and explore the accessible positions to settle in the ones **maximizing the number of bonds**
- All other beads remain in their last locations

here, delay **δ = 3**



Geary, Meunier, Schabanel, Seki MFCS 2016

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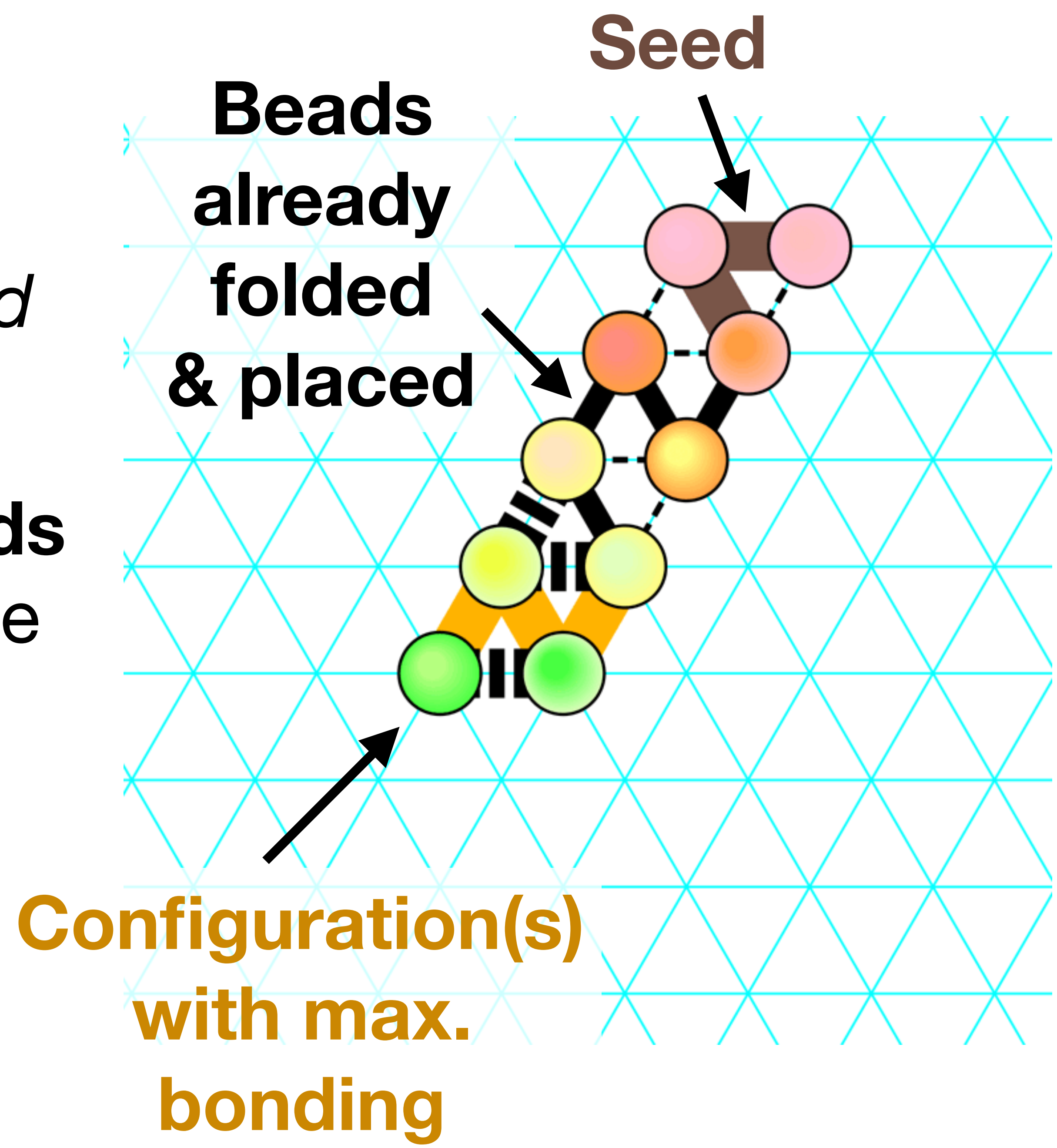

Geary, Meunier, Schabanel, Seki MFCS 2016

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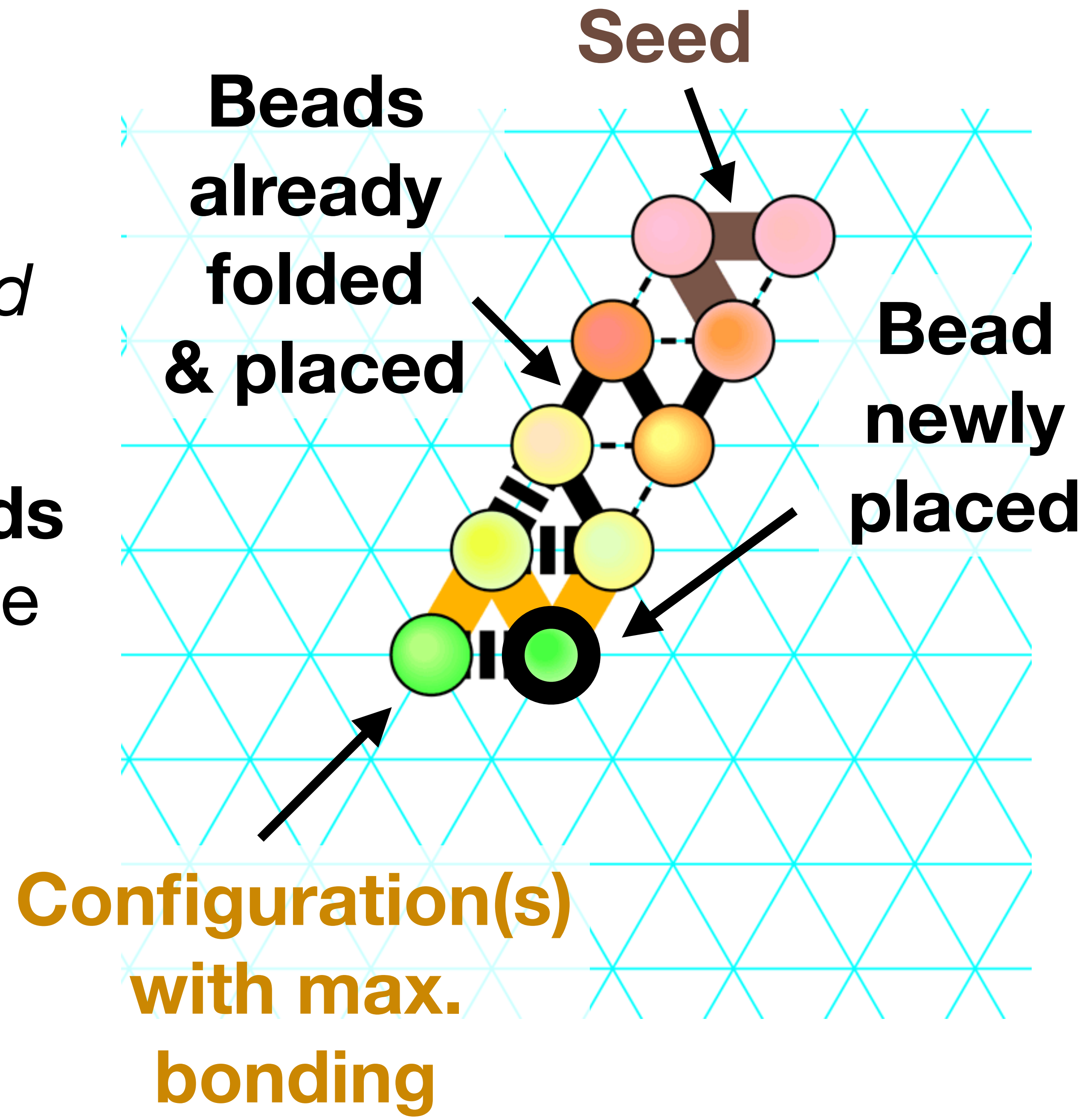

Geary, Meunier, Schabanel, Seki MFCS 2016

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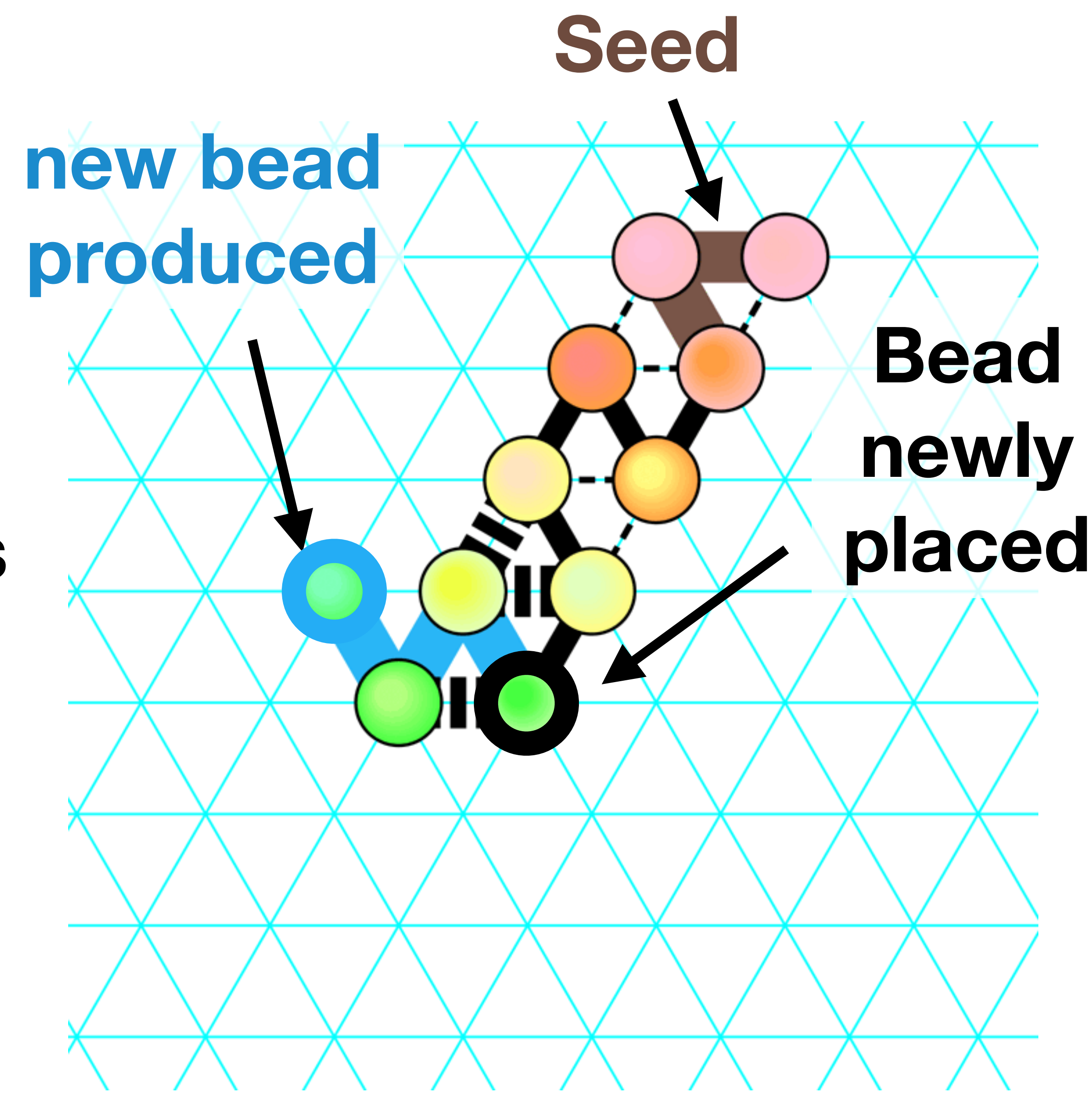

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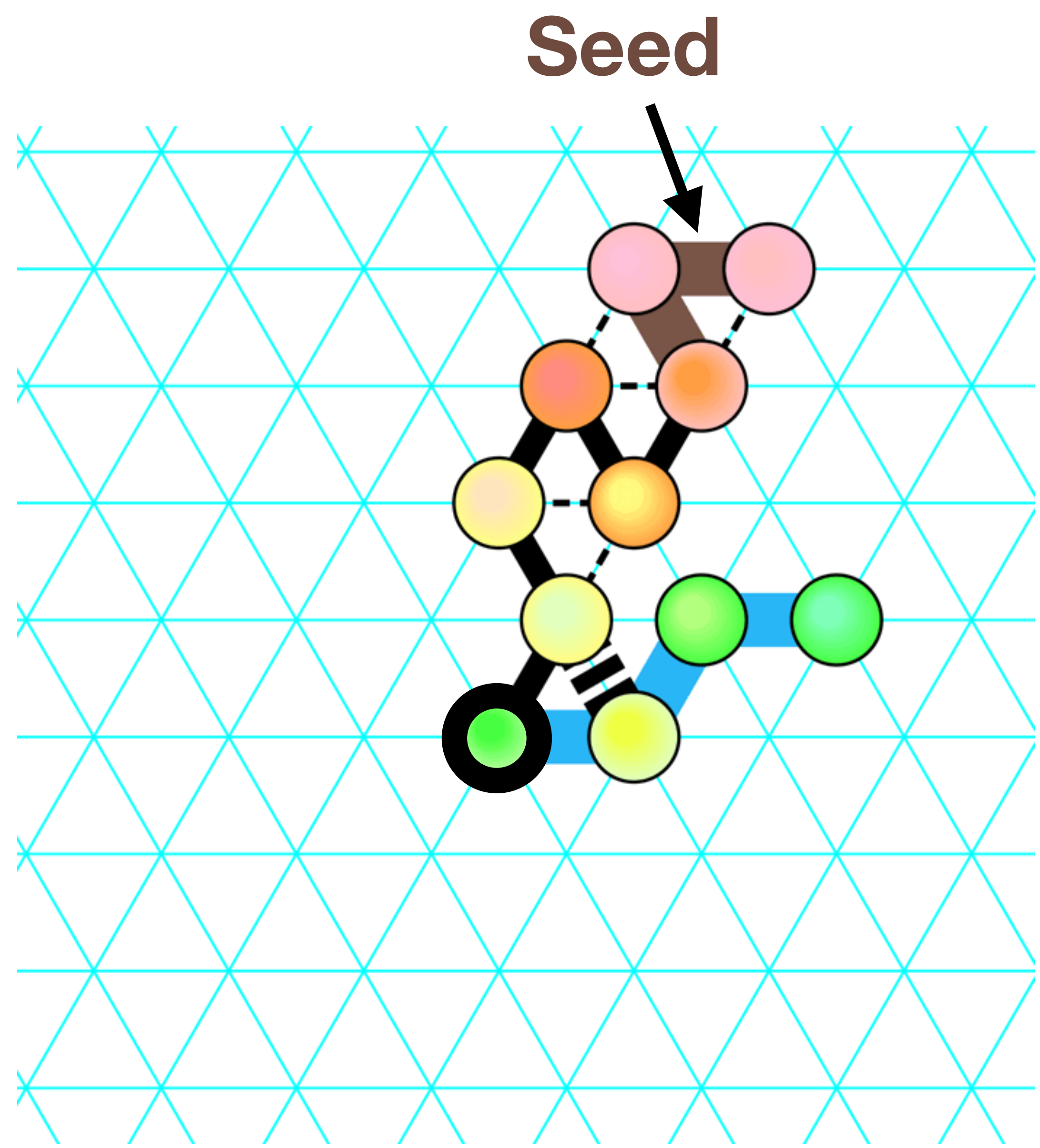

Geary, Meunier, Schabanel, Seki MFCS 2016

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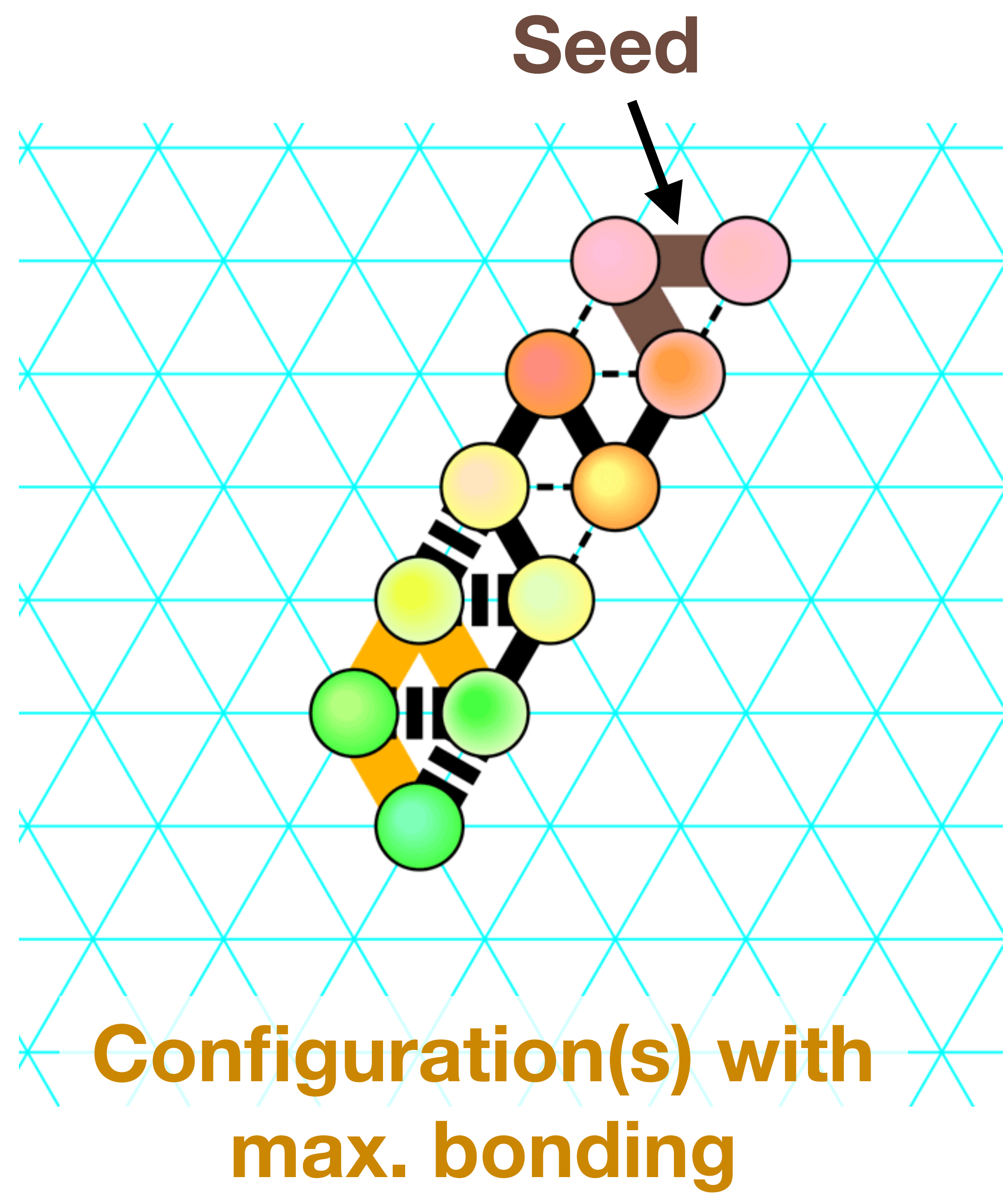

Geary, Meunier, Schabanel, Seki MFCS 2016

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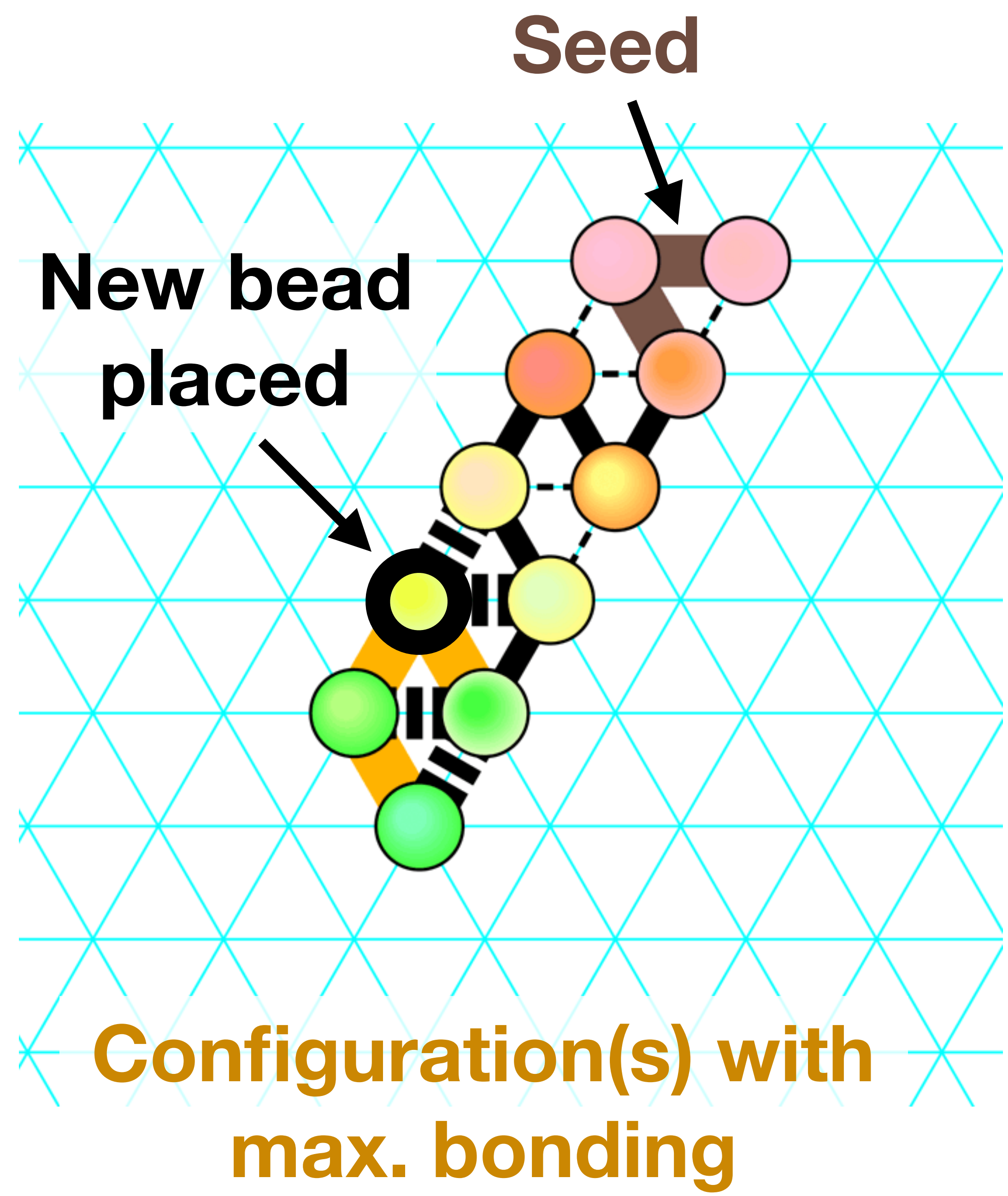

Geary, Meunier, Schabanel, Seki MFCS 2016

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There might be <u>several</u> configurations with max. bonding

Geary, Meunier, Schabanel, Seki MFCS 2016

Oritatami:
A model for co-transcriptional folding

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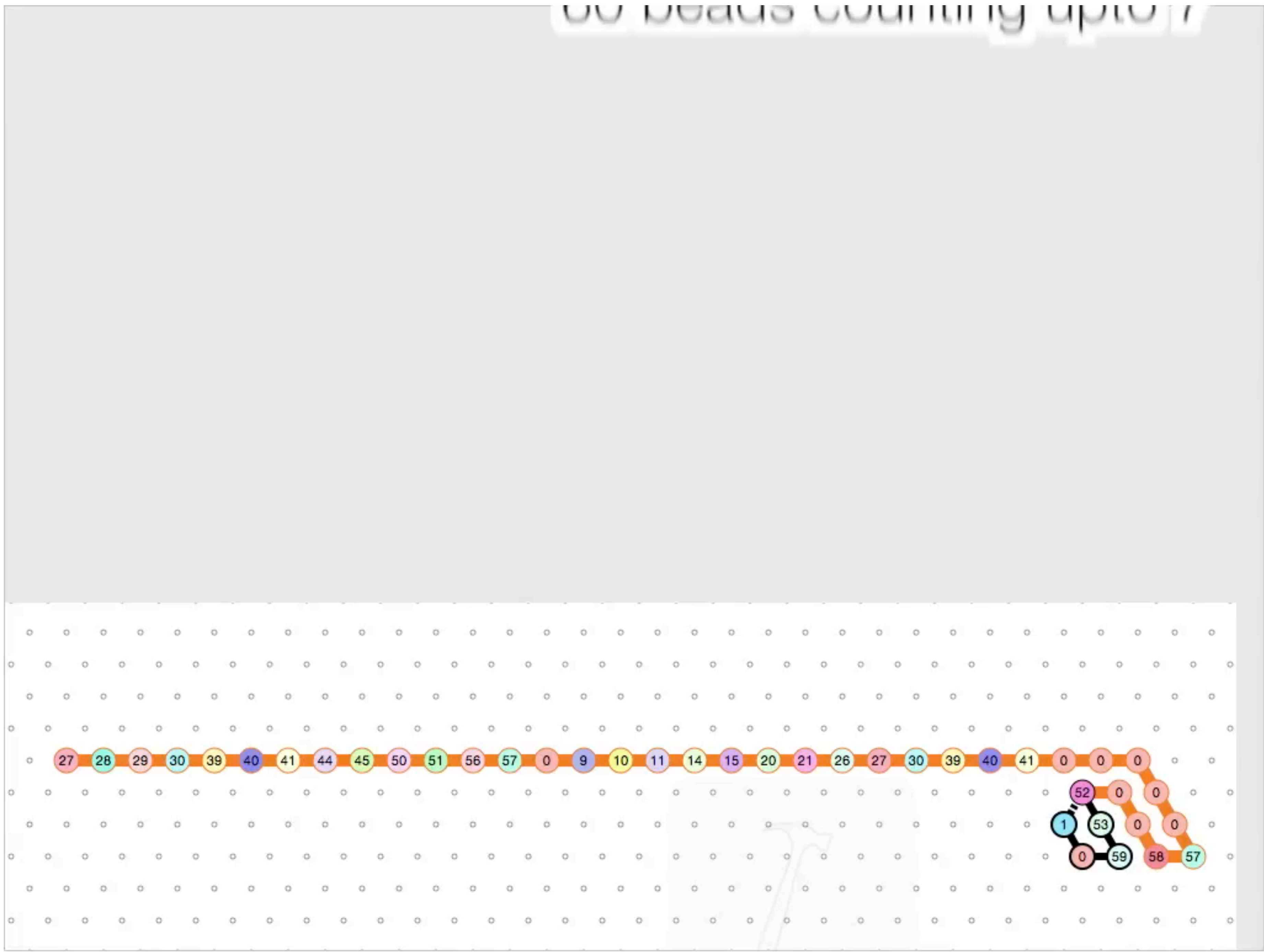
- All other beads remain in their last locations

The bead has same position in all maximal extension ⇒ *deterministic*

There might be <u>several</u> configurations with max. bonding

Geary, Meunier, Schabanel, Seki MFCS 2016

Oritatami
A first example



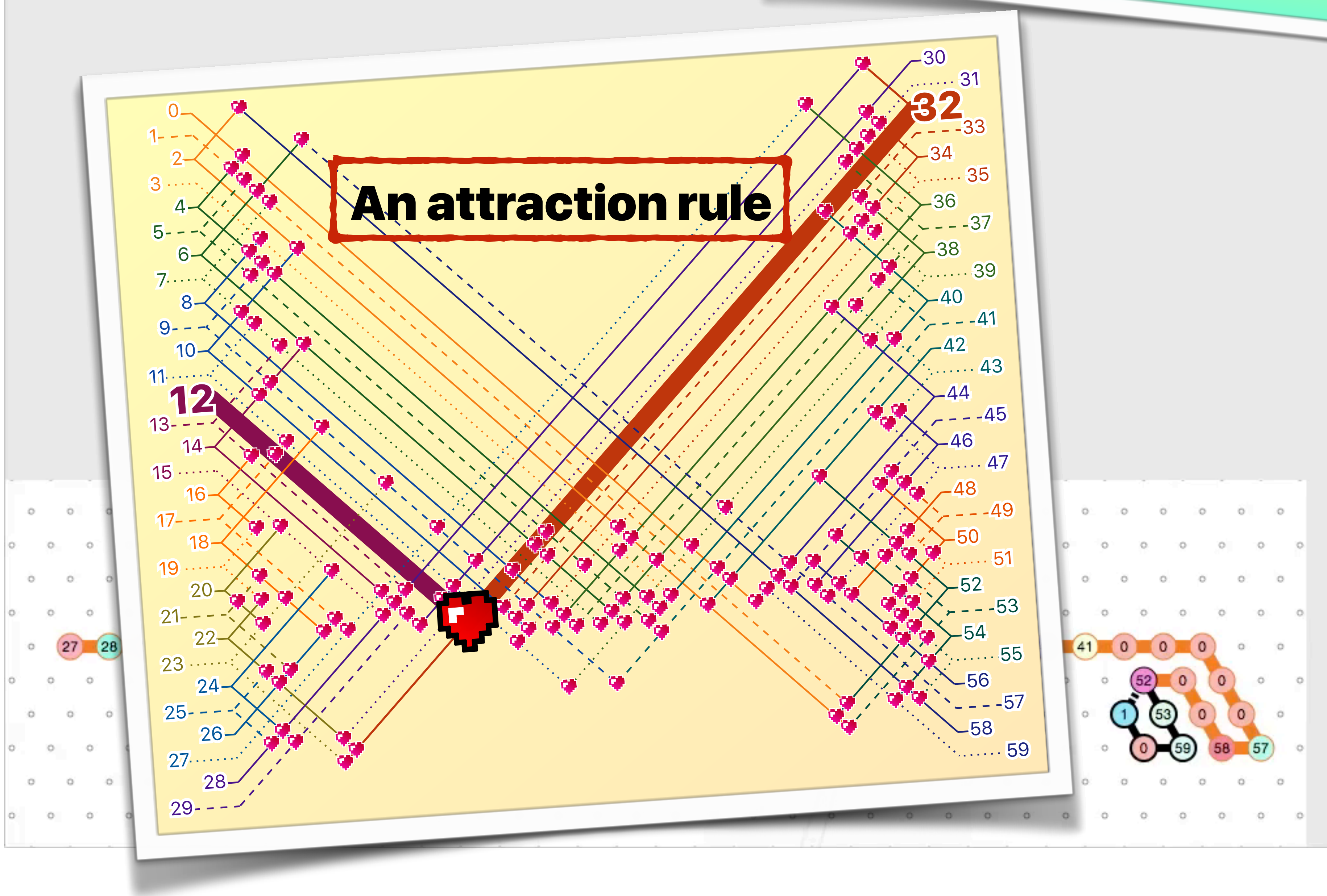
A binary counter made of a **single** 60-beads periodic molecule folding upon itself

Geary Meunier Seki Schabanel 2015

Oritatami

A first example

The molecule: `0,…,59,0,…,59,0,…`



of a **single** 60-
beads
periodic
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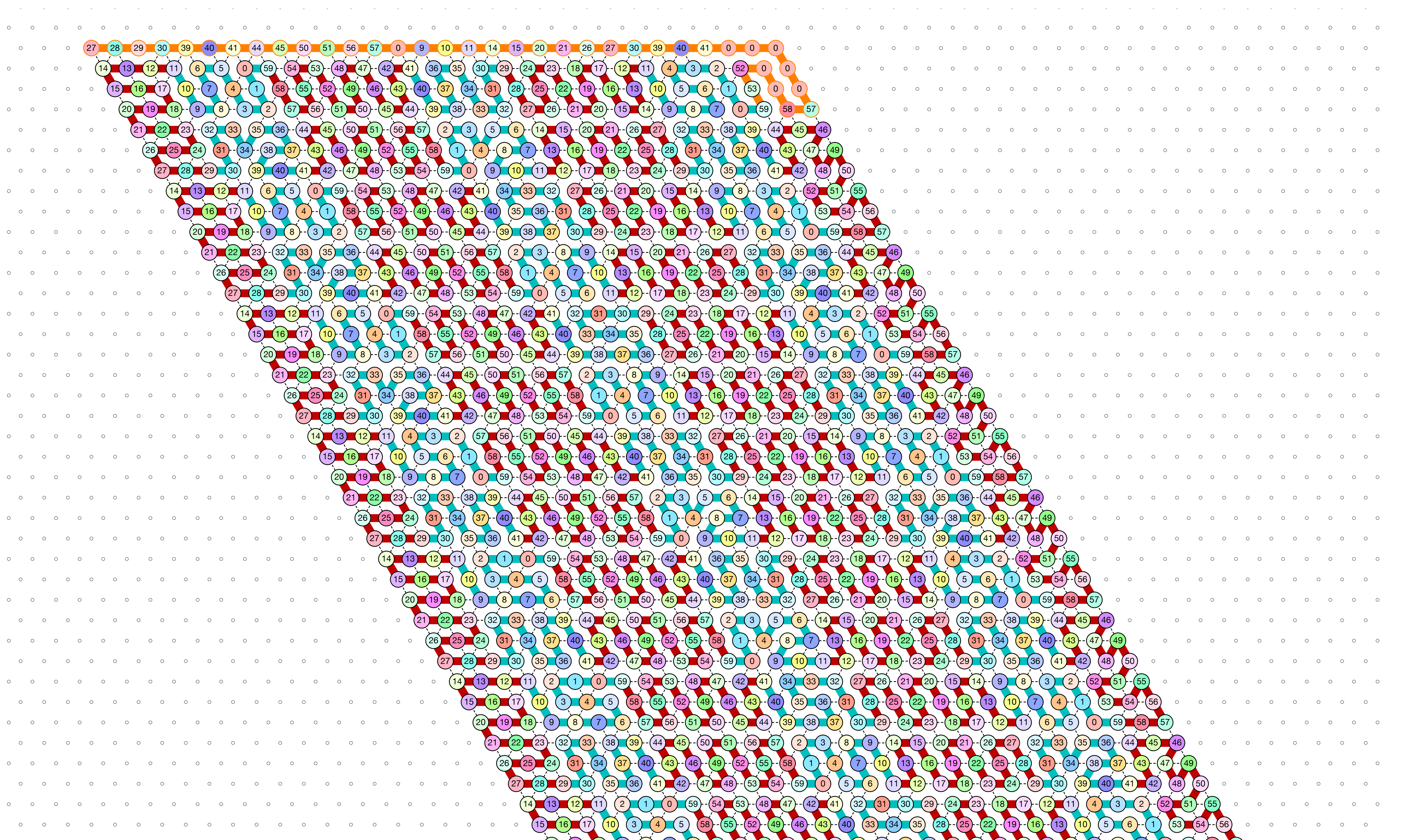
folding upon
itself

Geary Meunier Seki Schabanel 2015

Oritatami. A binary counter

Information is encoded in the geometry



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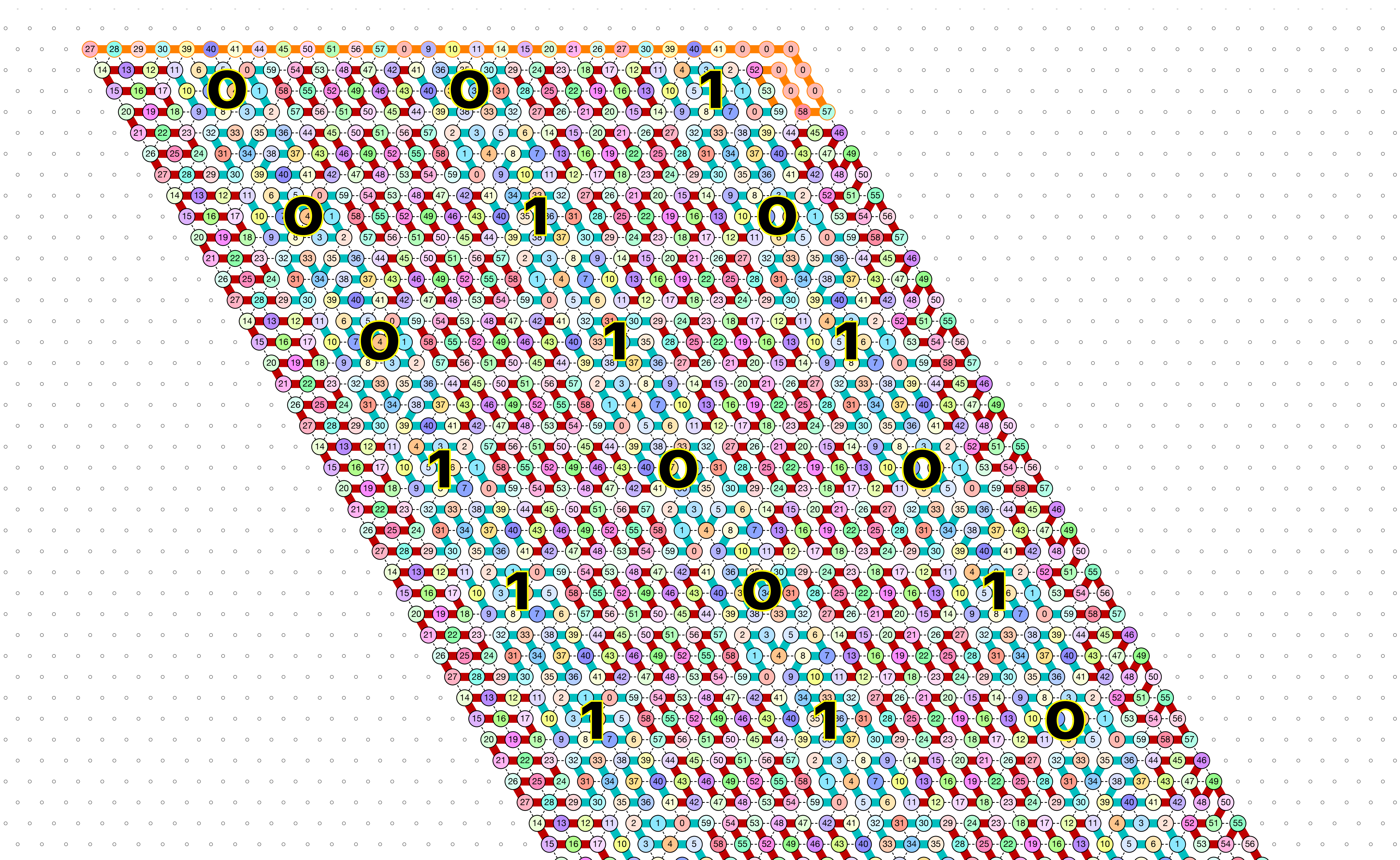

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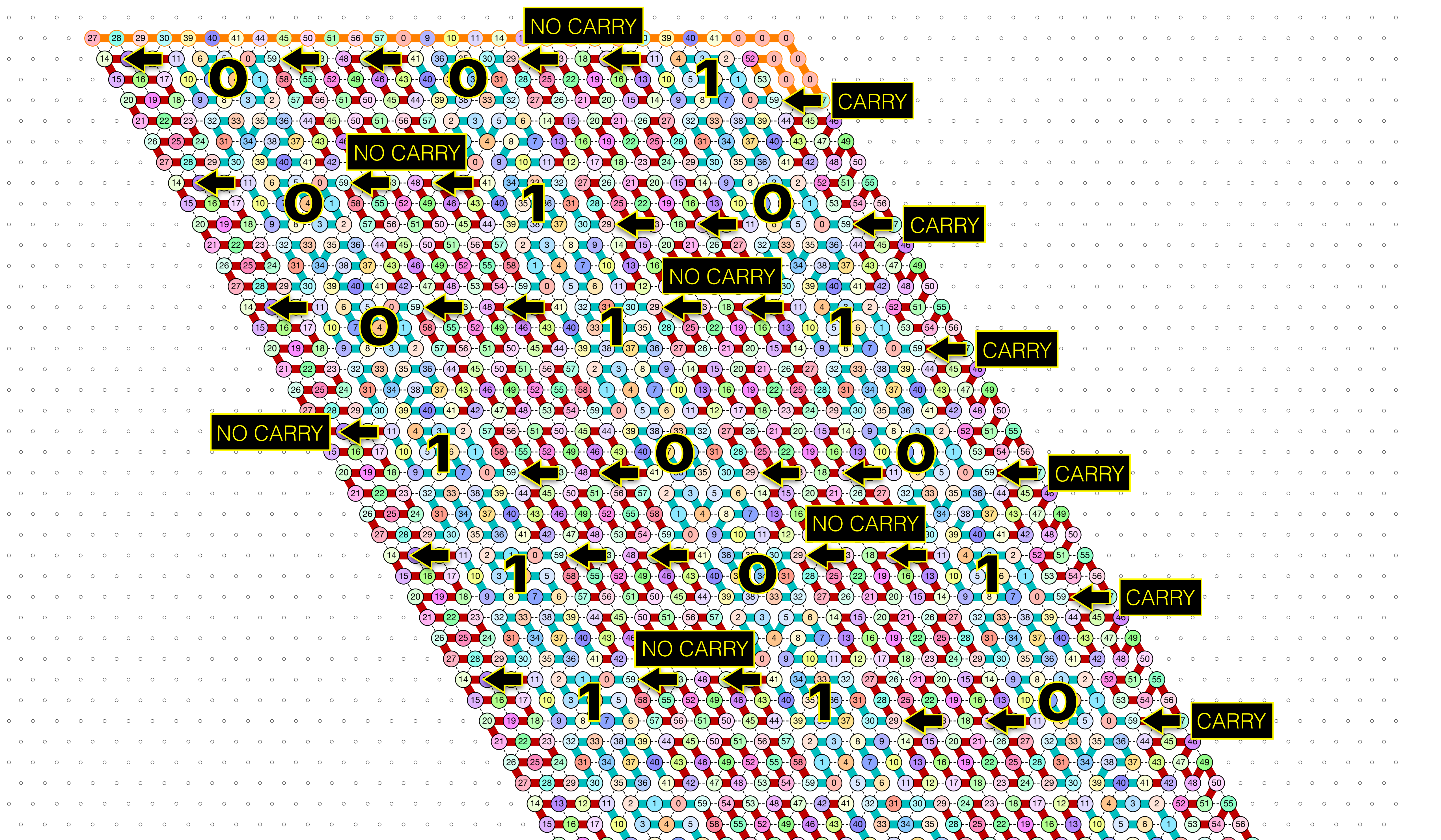
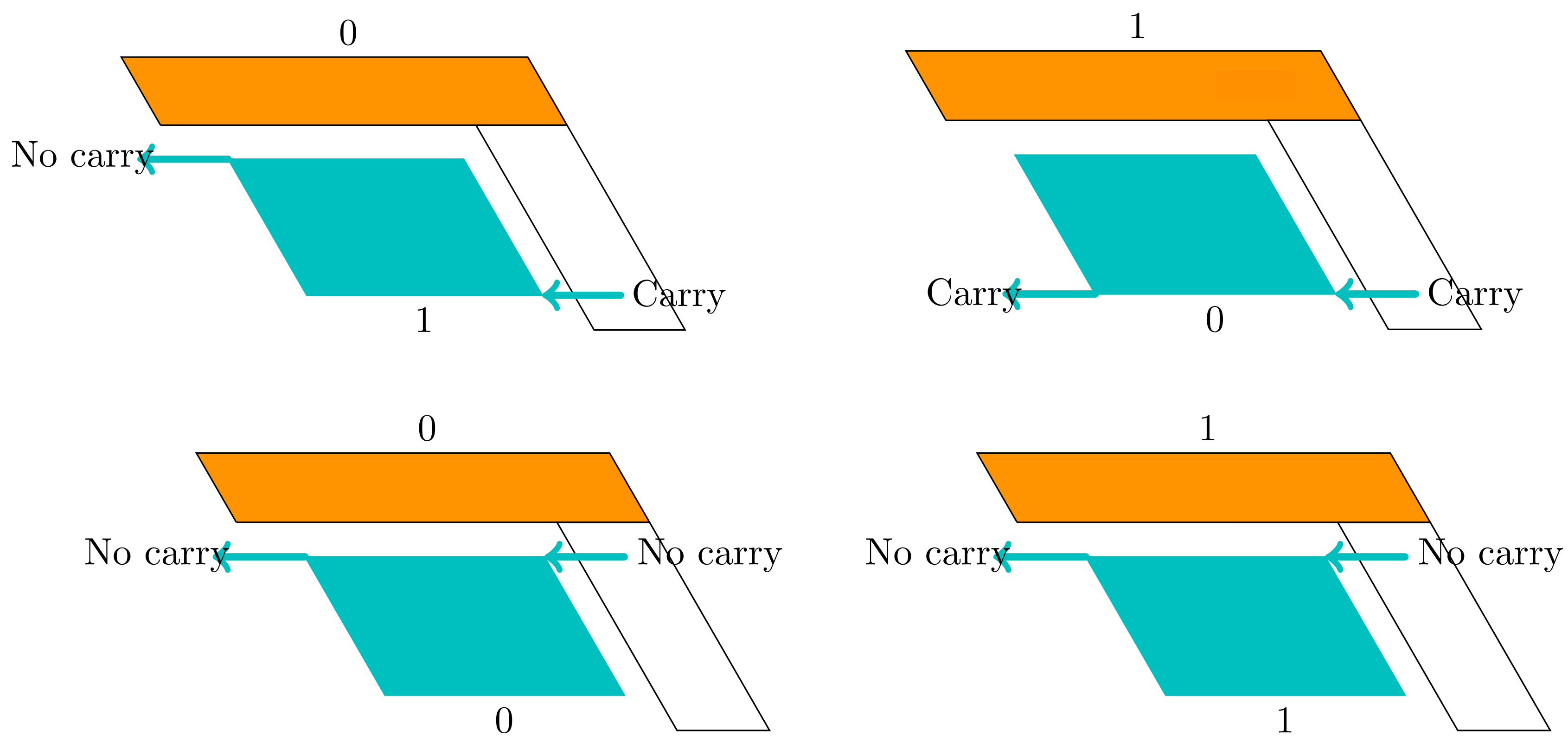


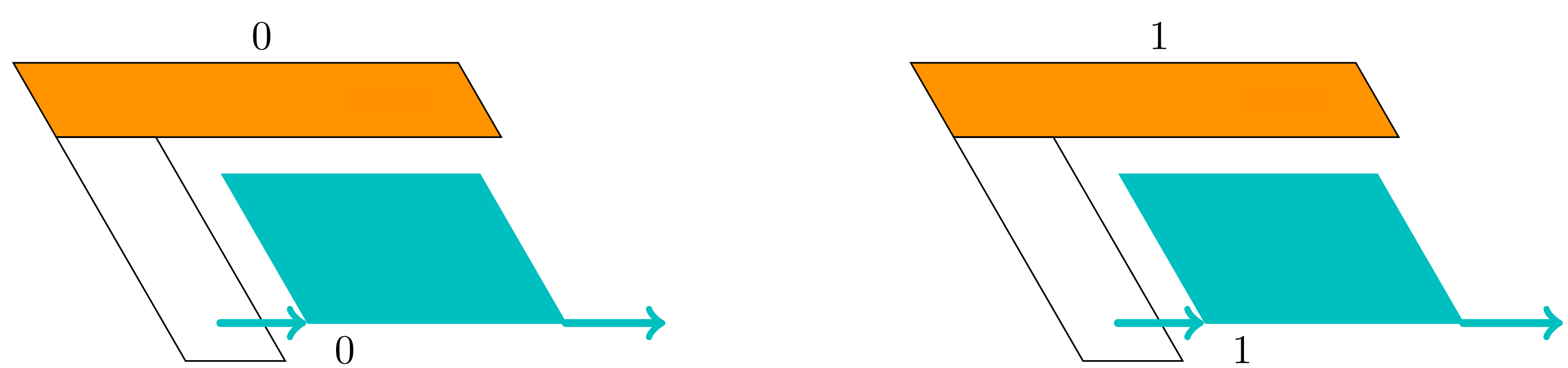
Oritatami. A binary counter

Information is encoded in the geometry

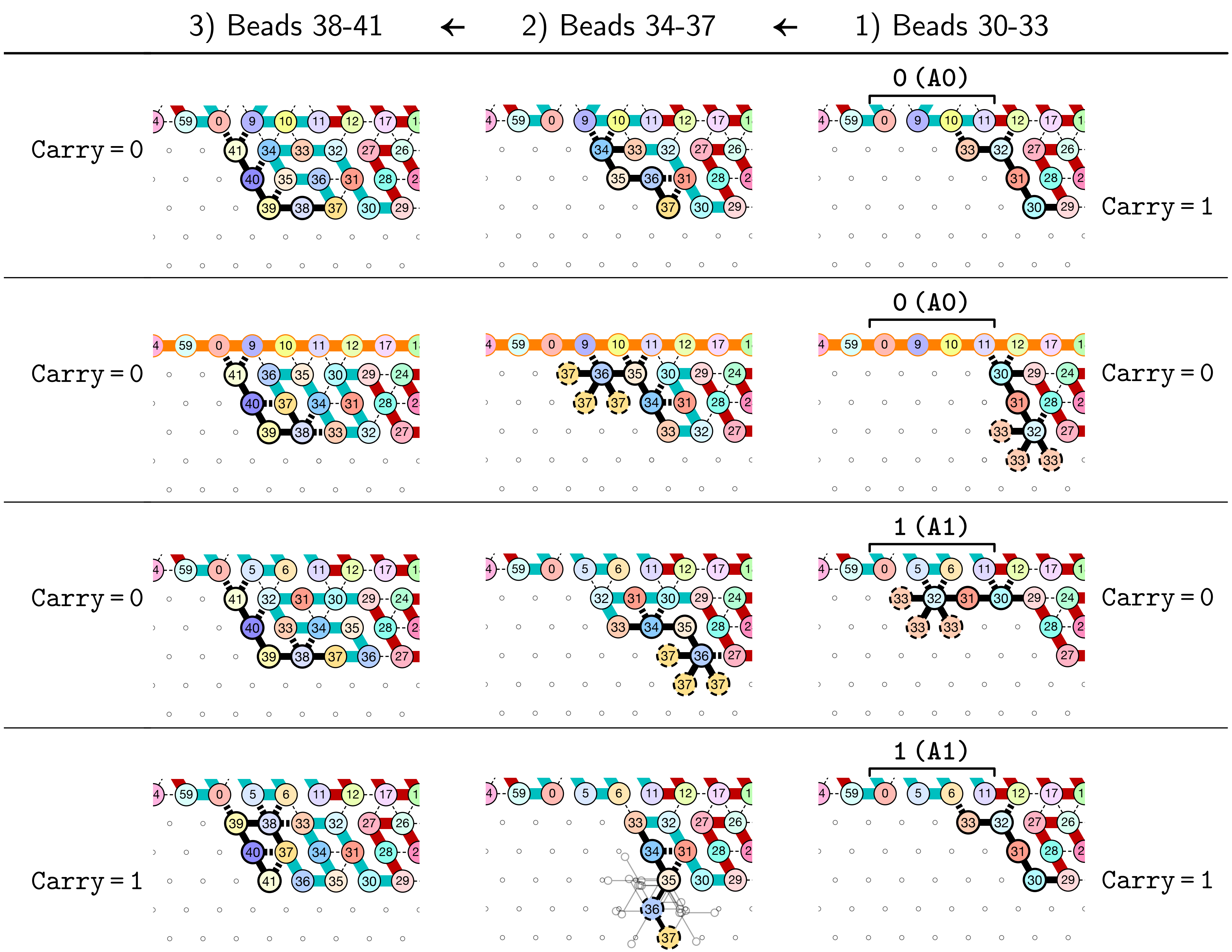
Carry propagation



Line feed



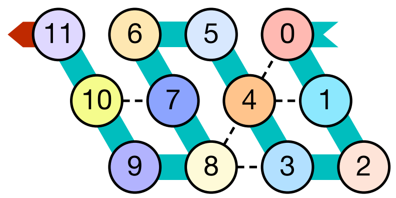
How does computation work?



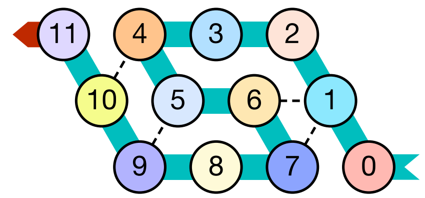
Proving the binary counter:
First, define the *bricks*

- Module A, First Half-Adder (beads 0–11):

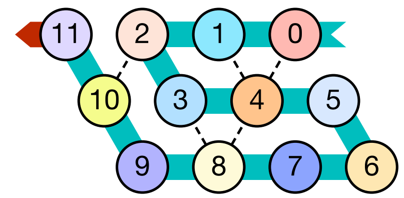
Zig Direction ⟵



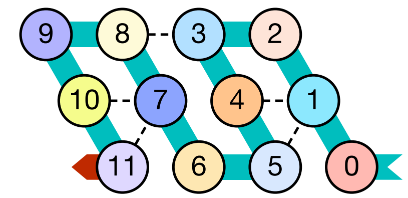
Brick A00



Brick A01

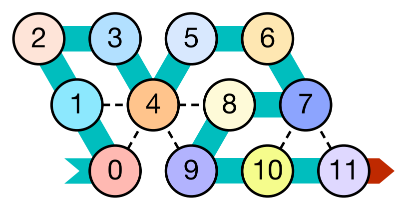


Brick A10

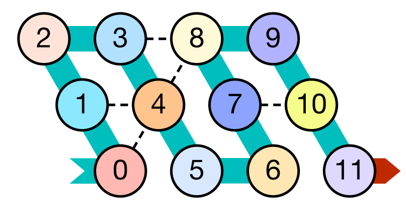


Brick A11

Zag Direction ⟶



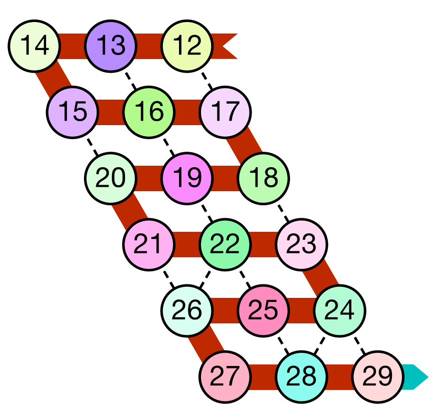
Brick A0



Brick A1

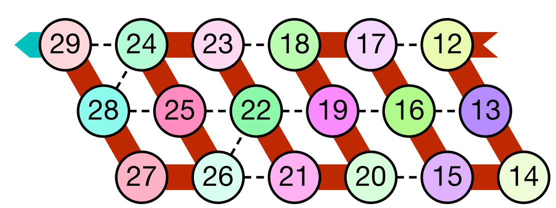
- Module B, Left-Turn module (beads 12–29)

Left Turn ↪ (Zig-to-Zag turn)

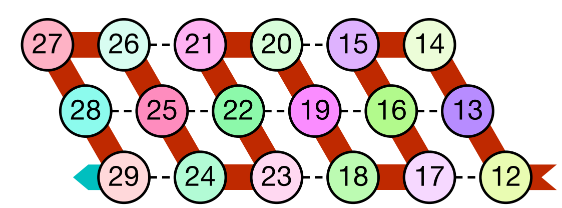


Brick BT

Zig Direction ⟵

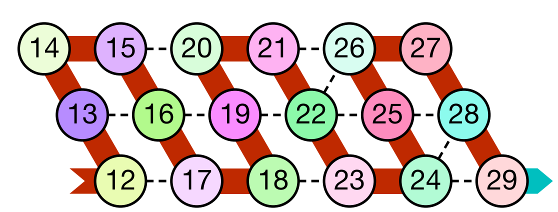


Brick B0



Brick B1

Zag Direction ⟶

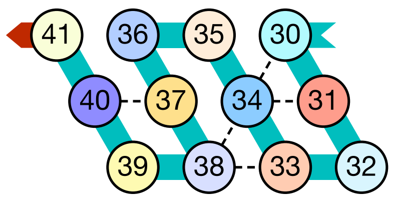


Brick B2

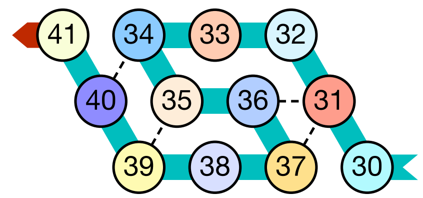
Proving the binary counter:
First, define the *bricks*

- Module C, Second Half-Adder (beads 30–41)

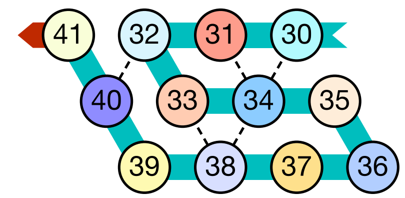
Zig Direction ⟵



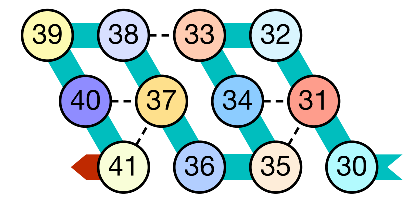
Brick C00



Brick C01

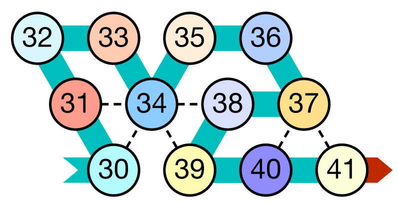


Brick C10

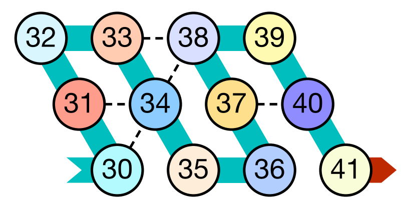


Brick C11

Zag Direction ⟶



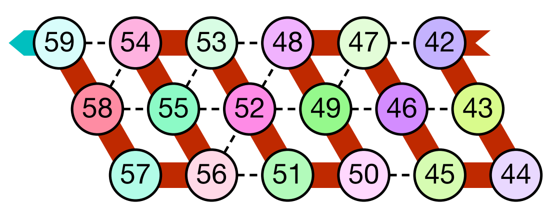
Brick C0



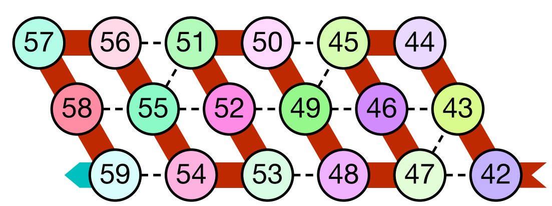
Brick C1

- Module D, Right-Turn module (beads 42–59)

Zig Direction ⟵

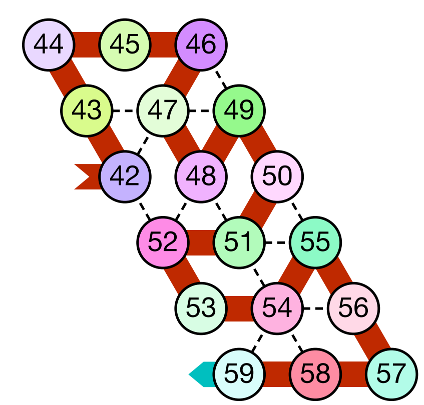


Brick D0



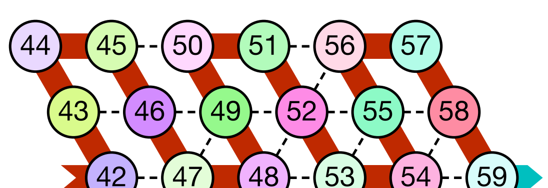
Brick D1

Right Turn ⟻ (Zag-to-Zig turn)



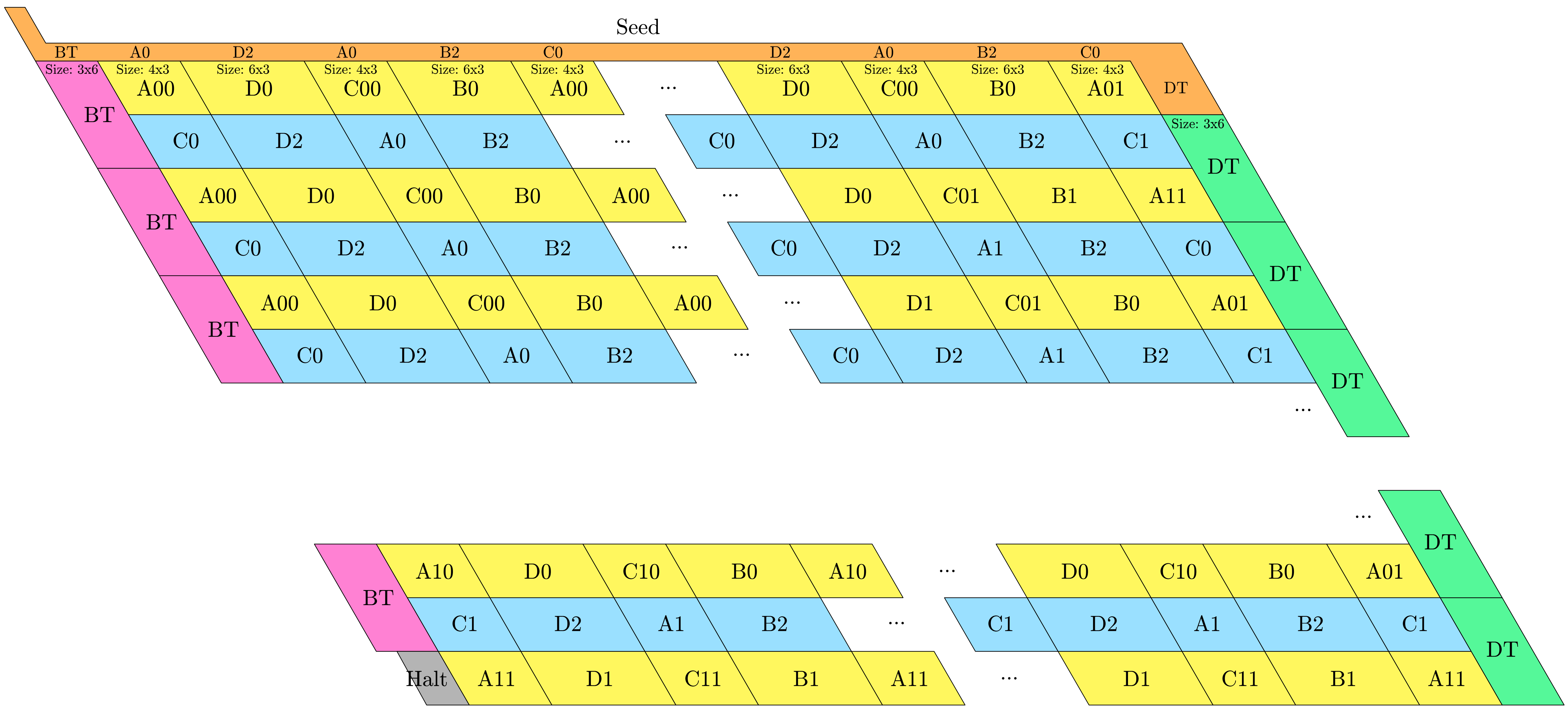
Brick DT

Zag Direction ⟶



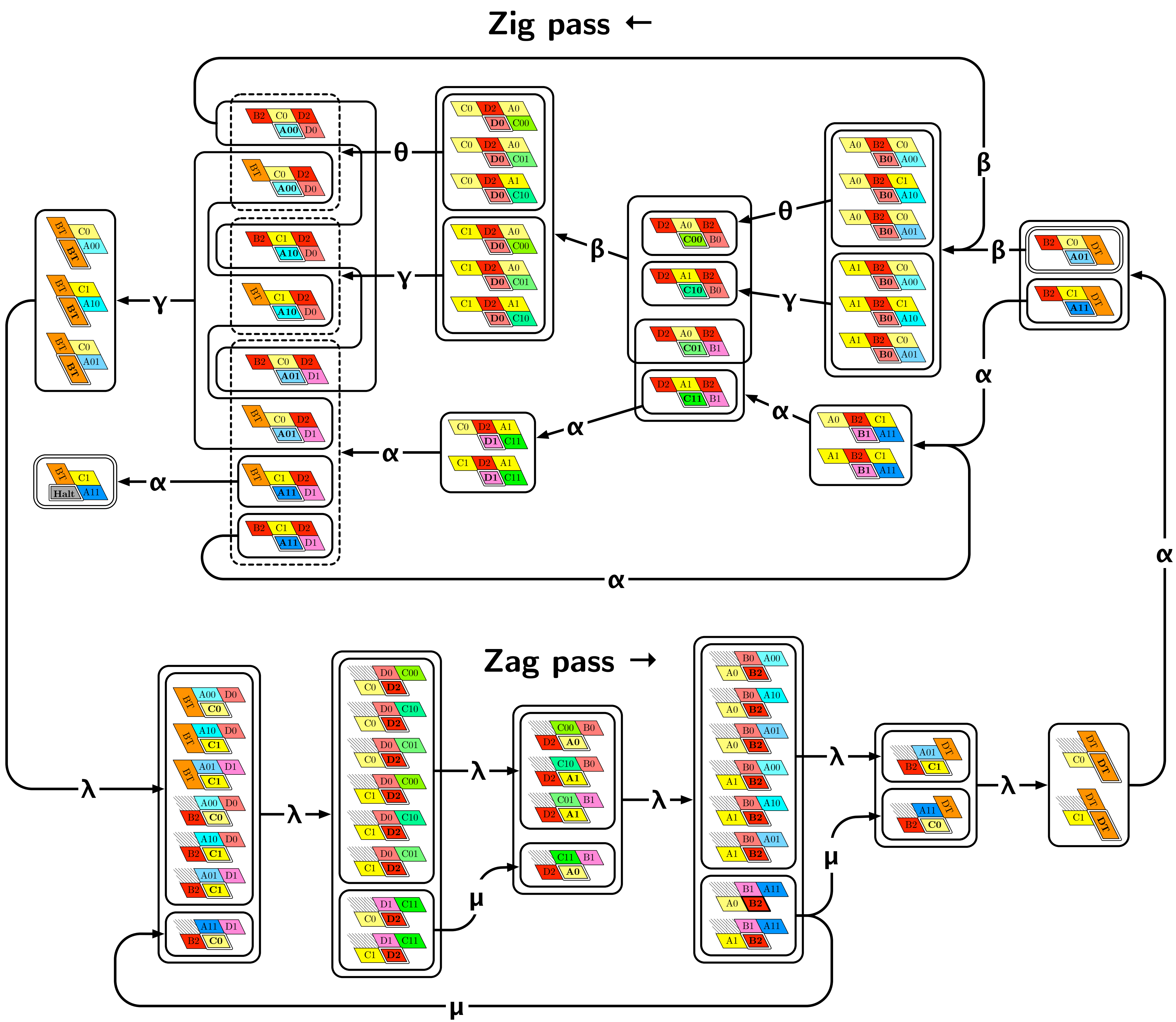
Brick D2

2nd, describe the final folding

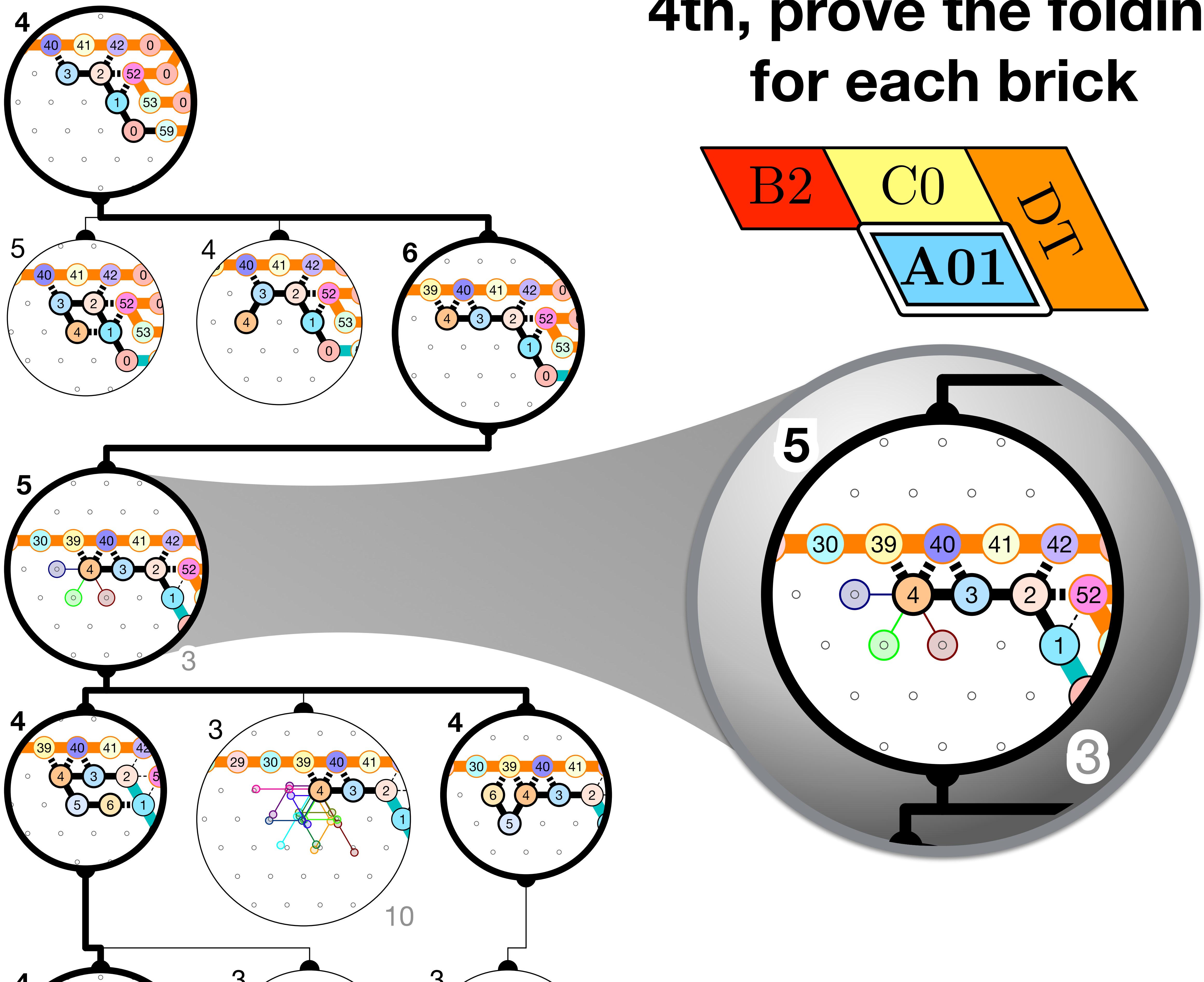


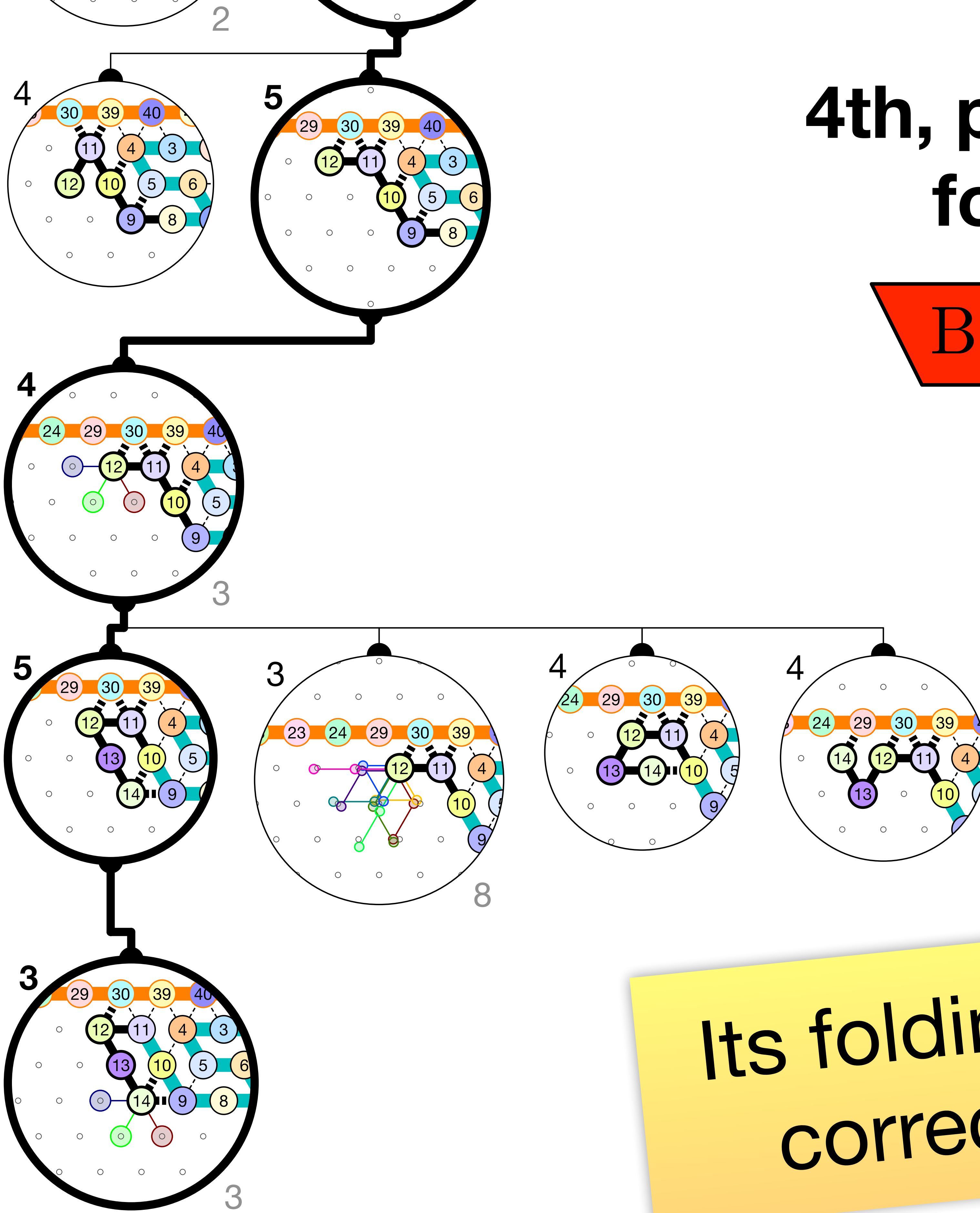
We prove that the molecule folds like this by induction

3rd, enumerate all the environments for each brick

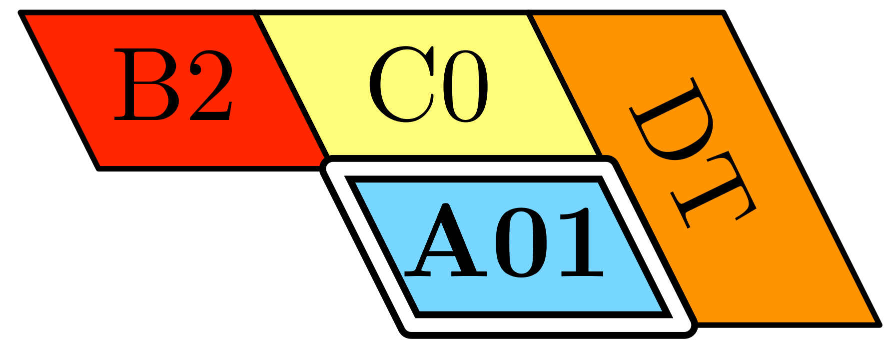


4th, prove the folding for each brick



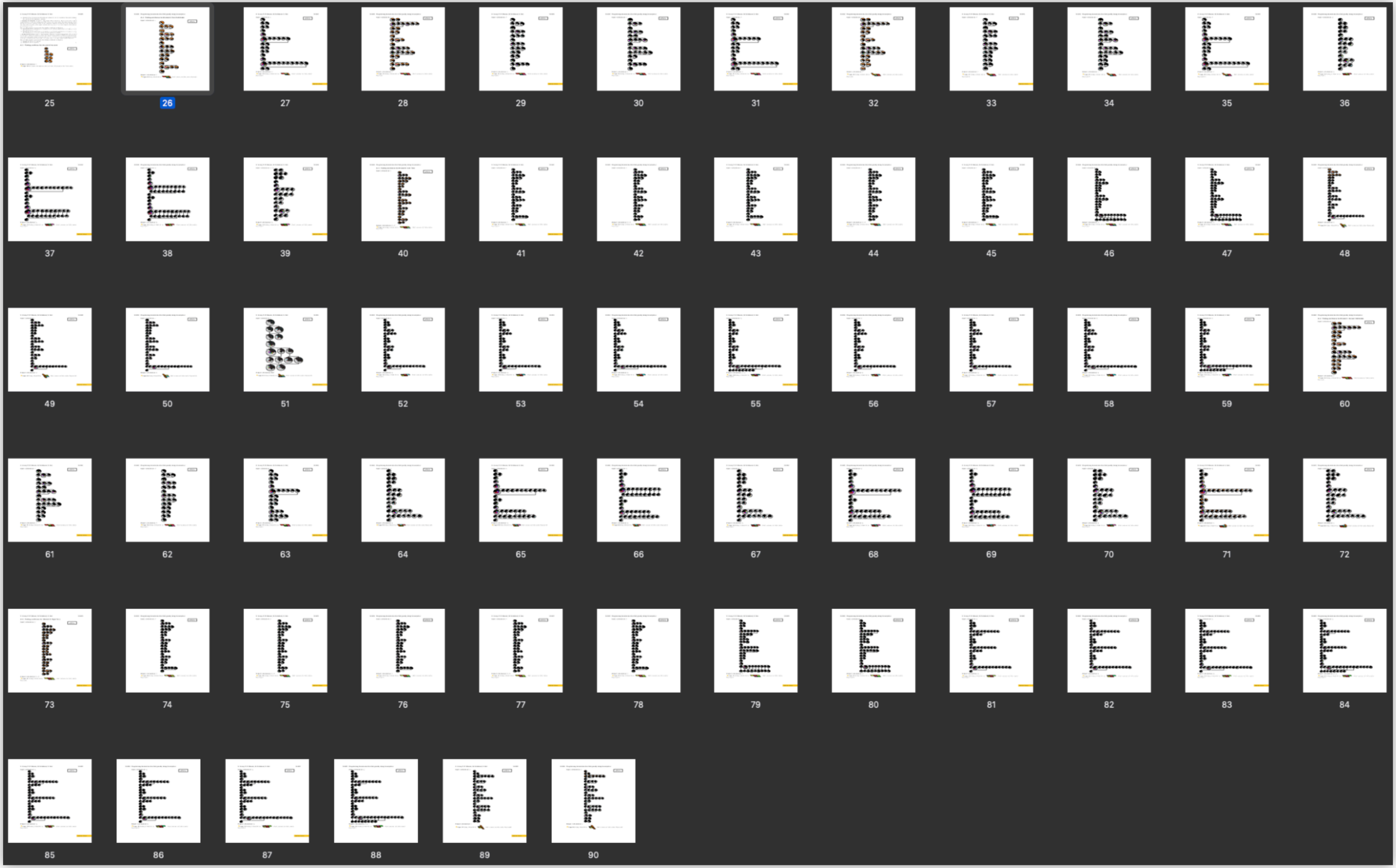


4th, prove the folding for each brick

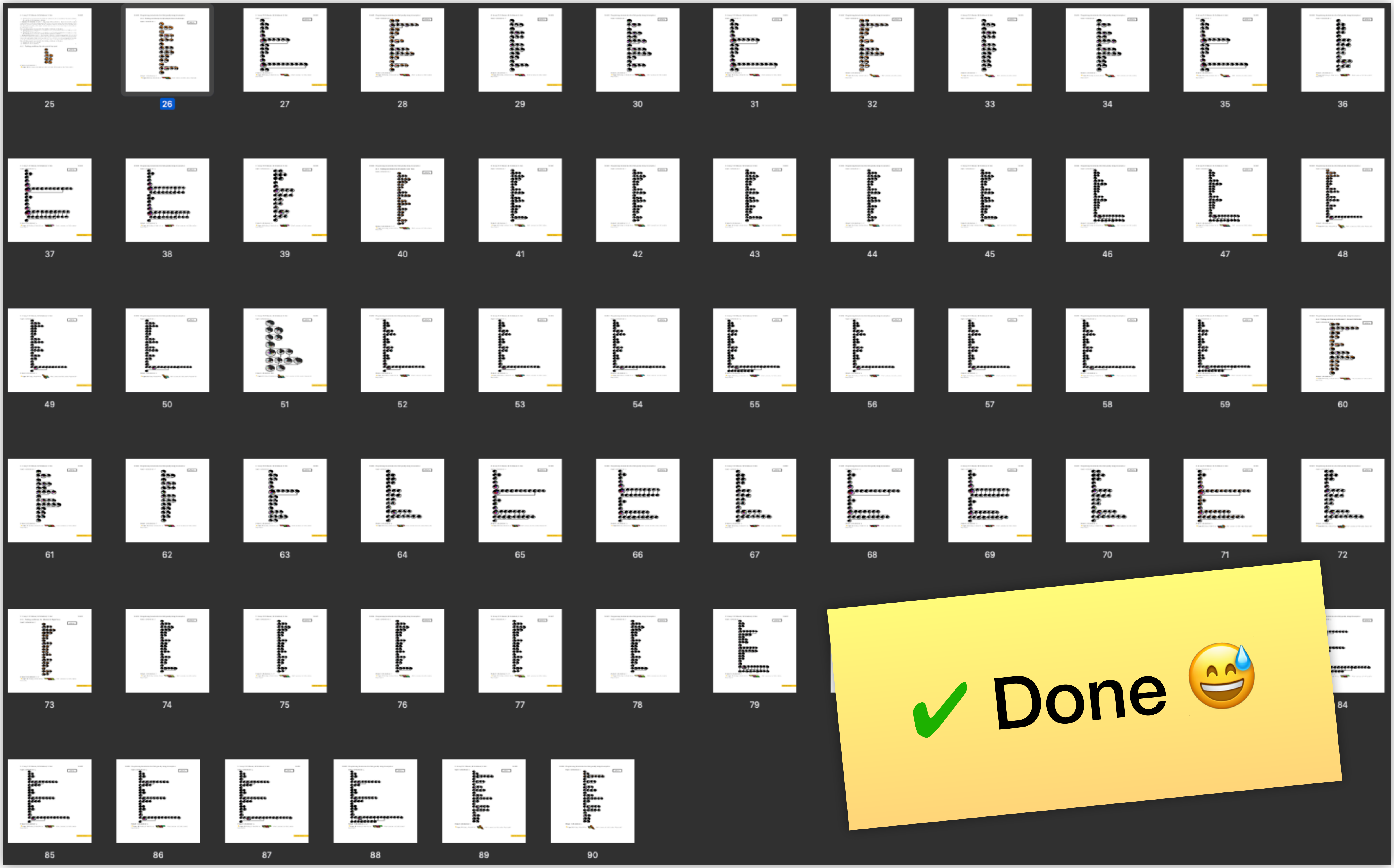


Its folding is correct !

Repeat for each brick in each environment



Repeat for each brick in each environment



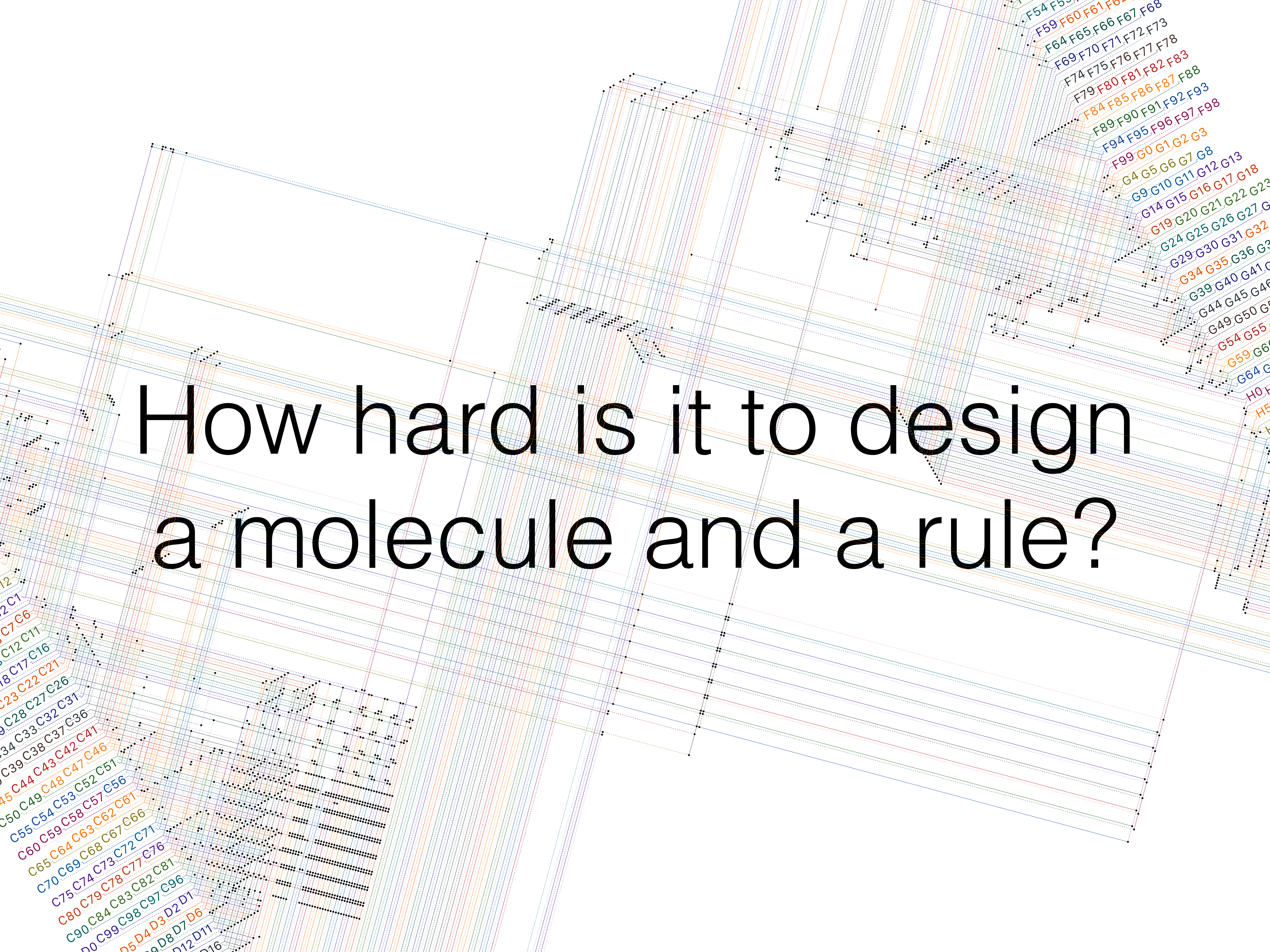
Binary counter: conclusion

- **Theorem.** There is a 60-periodic molecule that simulates a binary counter using 60 bead types and delay 3.

Back to general oritatami

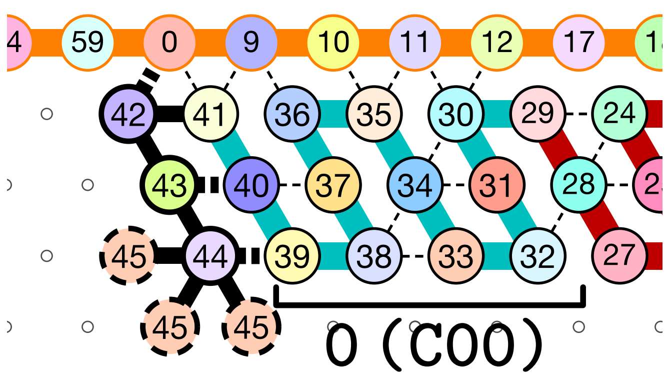
- How hard is it to design a rule?
  - NP-hard… but FPT, thus feasible!
- What can it compute?
  - Simulates any Turing Machine… *efficiently!*

How hard is it to design a molecule and a rule?

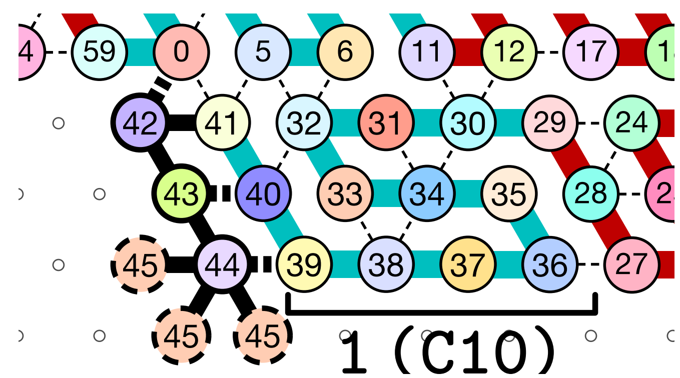


The first challenge: Designing the desired shapes

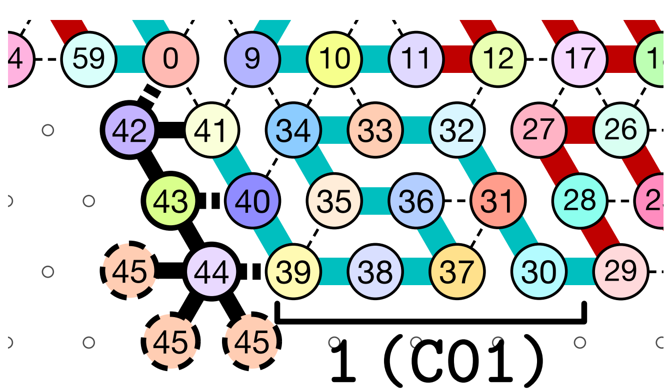
- Design shapes for which a **common** rule ❤️ exists



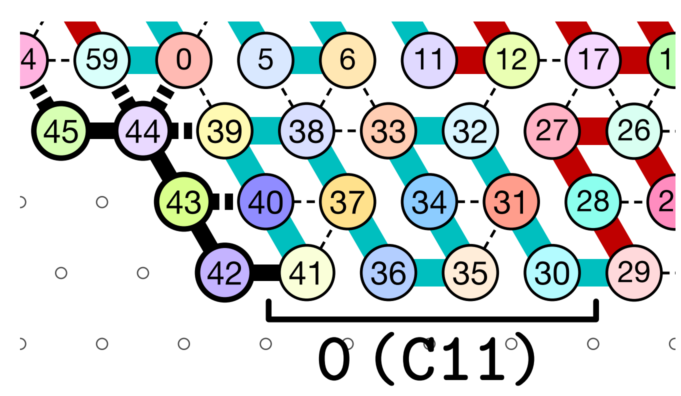
0+0 = 0 + no C



1+0 = 1 + no C



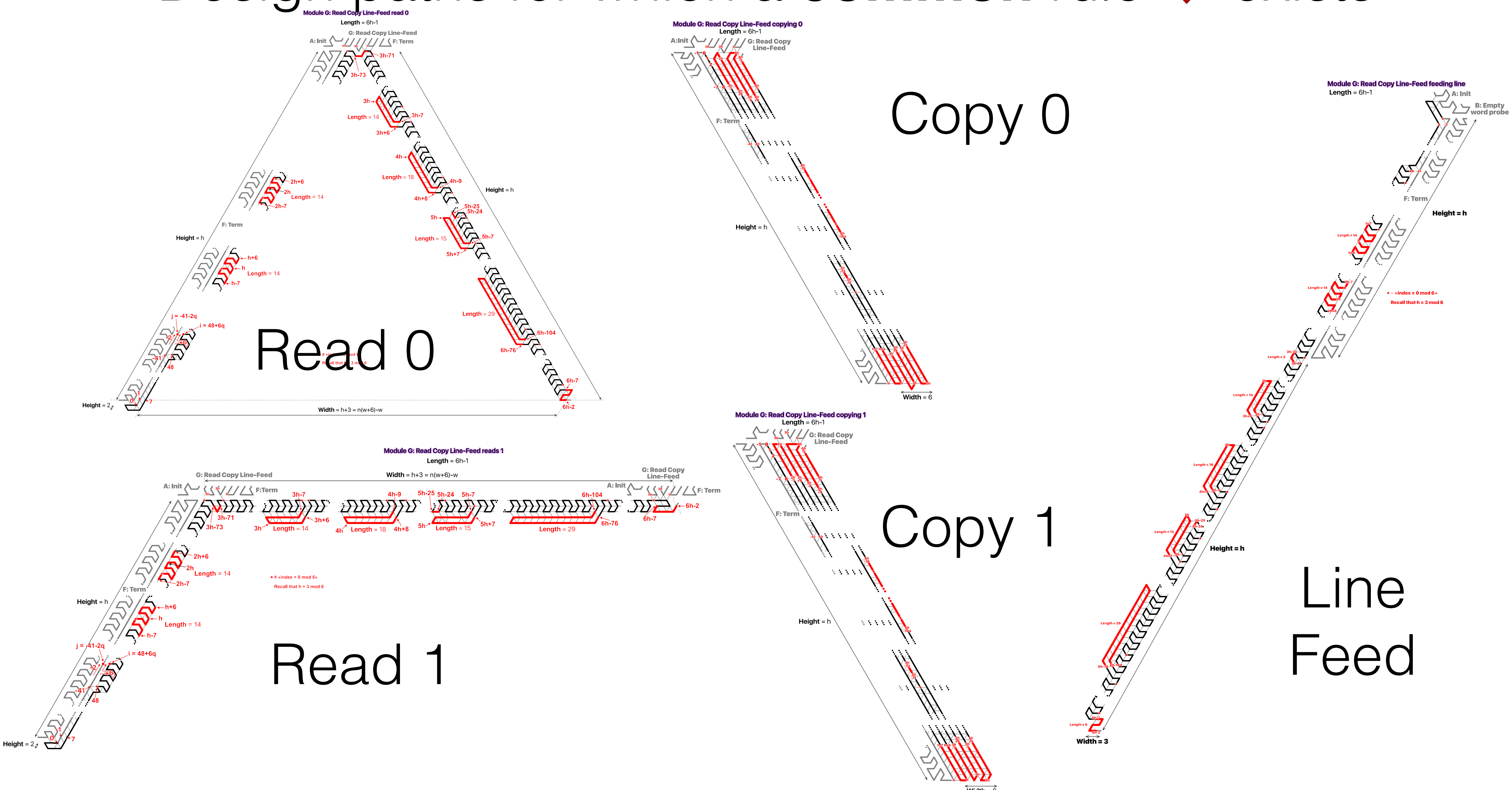
0+1 = 1 + no C



1+1 = 0 + C

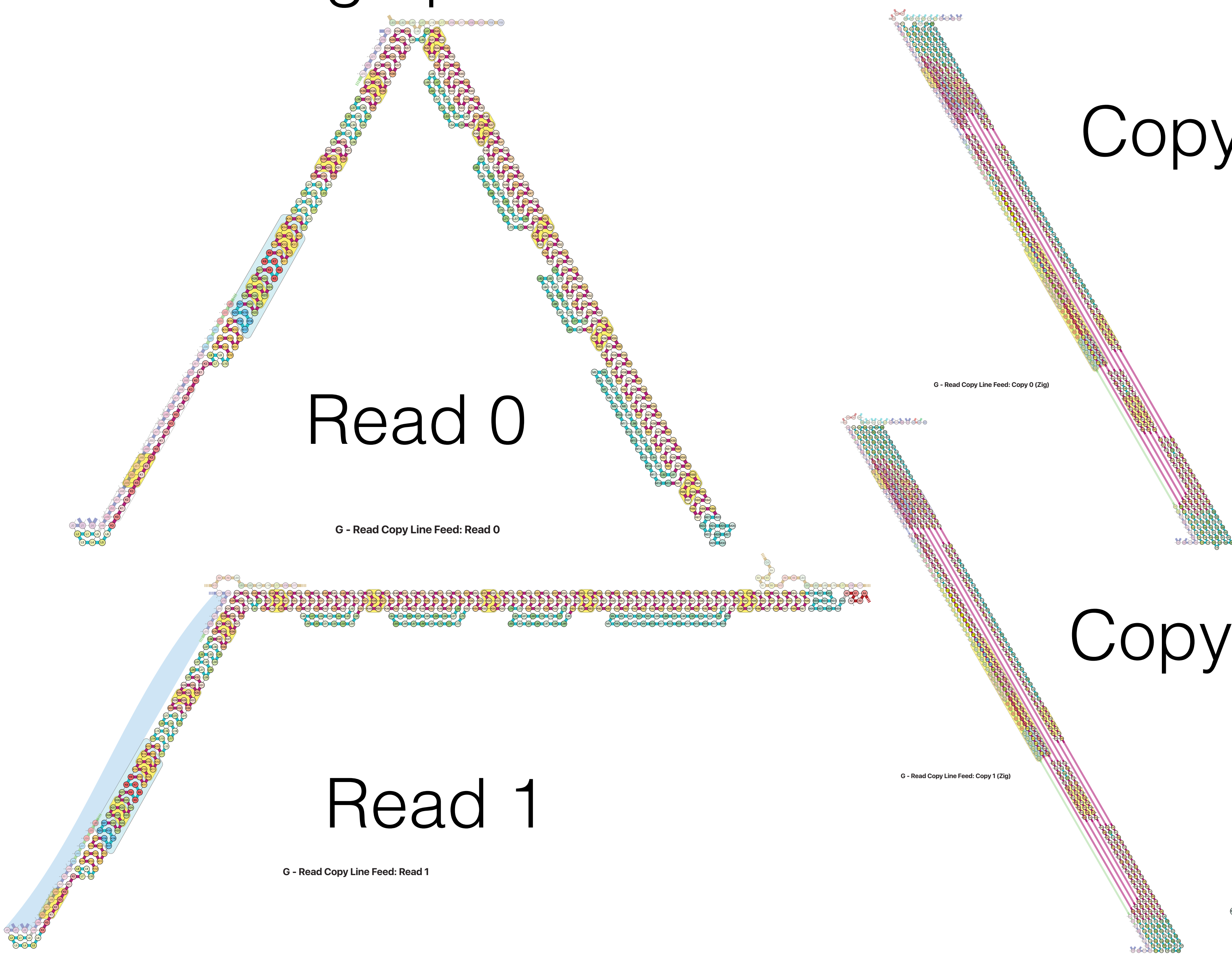
The first challenge: Designing the desired paths

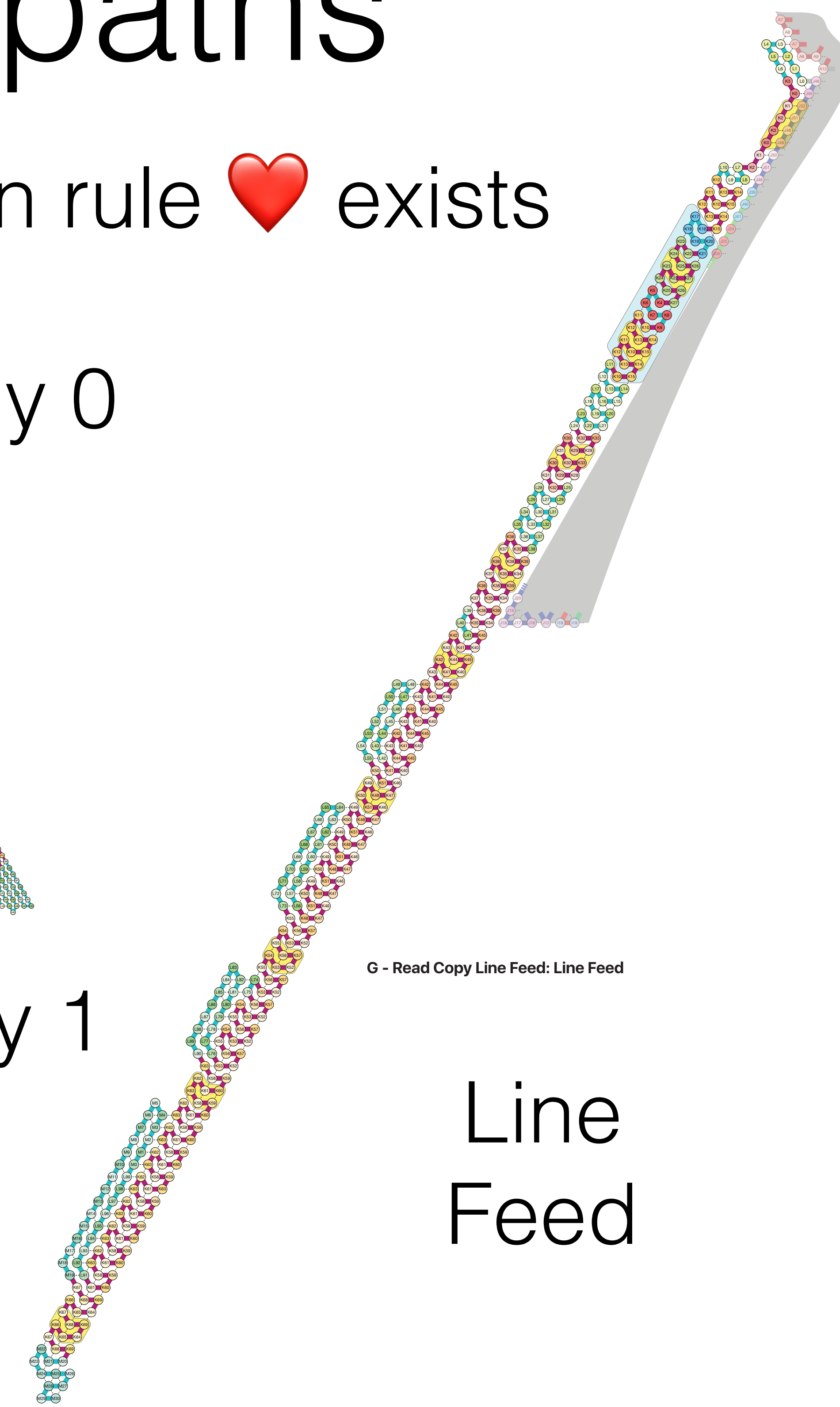
- Design paths for which a **common** rule ❤️ exists



The first challenge: Designing the paths

- Design paths for which a common rule ❤️ exists



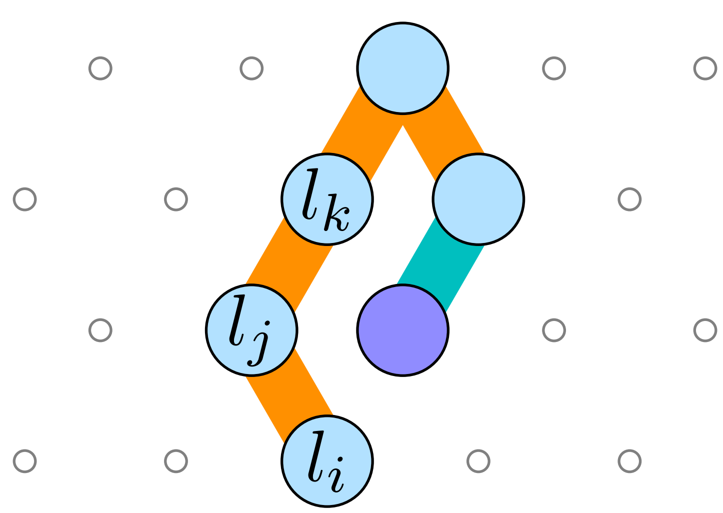


Oritatami design is NP-hard

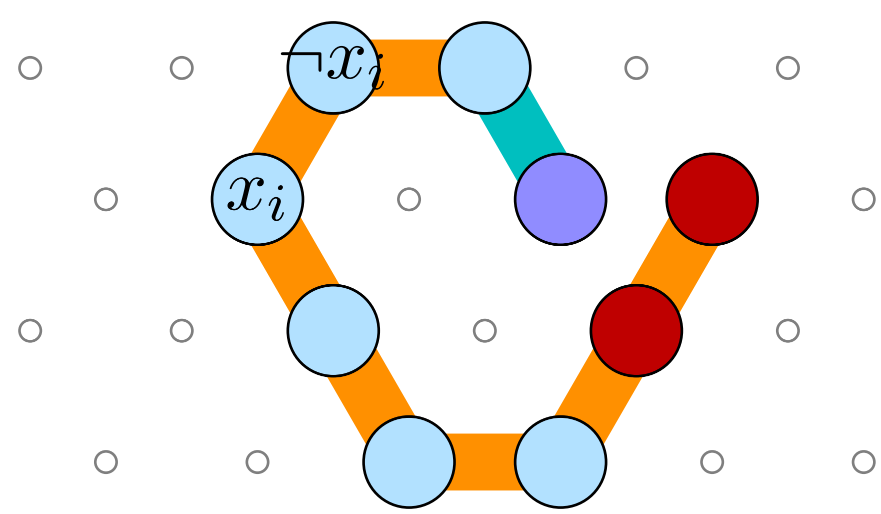
INPUT:	a delay time δ, a list of $n > 0$ seeds $\sigma_1, \sigma_2, \ldots, \sigma_n$, and a list of n conformations $c_1, c_2, \ldots, c_n$ of the same length l
OUTPUT:	an attraction rule $\heartsuit$ such that for all $i \in \{1, 2, \ldots, n\}$, Oritatami system $\mathcal{O}_i = (s, \sigma_i, \heartsuit, \delta)$ deterministically folds into conformation c_i, where s is the sequence of length l such that for all $i \in \{1, 2, \ldots, l\}$, $s_i = i$.

The reduction *(length=1, δ arbitrary)*

Ensures it binds to at least one litteral in $l_i \vee l_j \vee l_k$



Ensures it binds to at most one of x_i and $\neg x_i$



The second challenge: Designing the rule ❤️

Theorem. There is a **FPT algorithm** with respect to ***L*** that designs **in linear time in *L*** (but exponential in ***k*** and ***δ***) a **rule** ❤️ that folds the **sequence *1,…,L* of length *L*** into ***k*** **prescribed conformations** when folded in ***k*** **prescribed environments.**

Proof. • **Locality:** each bead only sees a bounded number (exponential in δ) of other beads when folded.

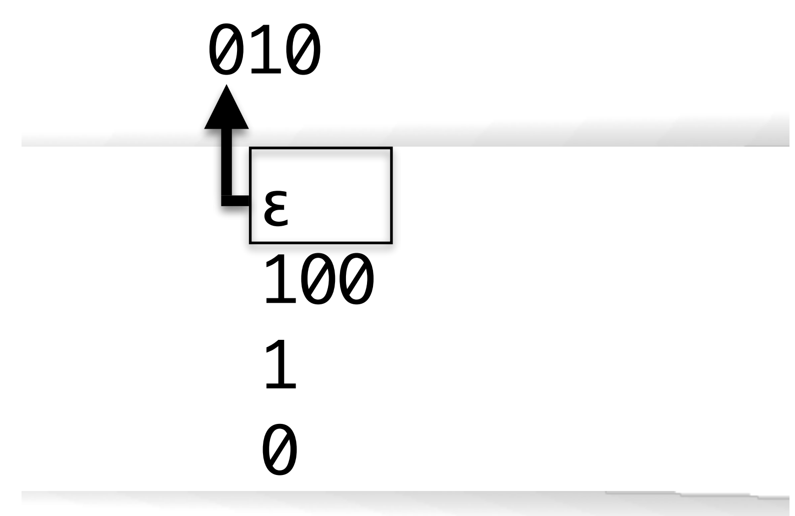
- Then, compute all valid local rules for each of these neighborhoods
- And use dynamic programming to decide whether there is a global rule compatible with at least one of the local rule for each environment.

Oritatami is Turing complete

Skipping Cyclic Tag Systems

- A finite **cyclic sequence** of finite binary **code words** with a pointer p to one of them
- An initial binary **tape word** (the input)
- **Dynamics:**
  - *If the tape word is empty (ε):* halt
  - *If the 1st letter of the tape word is 0:* delete the 0 and increment the pointer p
  - *If the 1st letter of the tape word is 1:* delete the 1, append to the tape word the code word at position p+1 and increase p by +2

Example.



Skipping Cyclic Tag Systems

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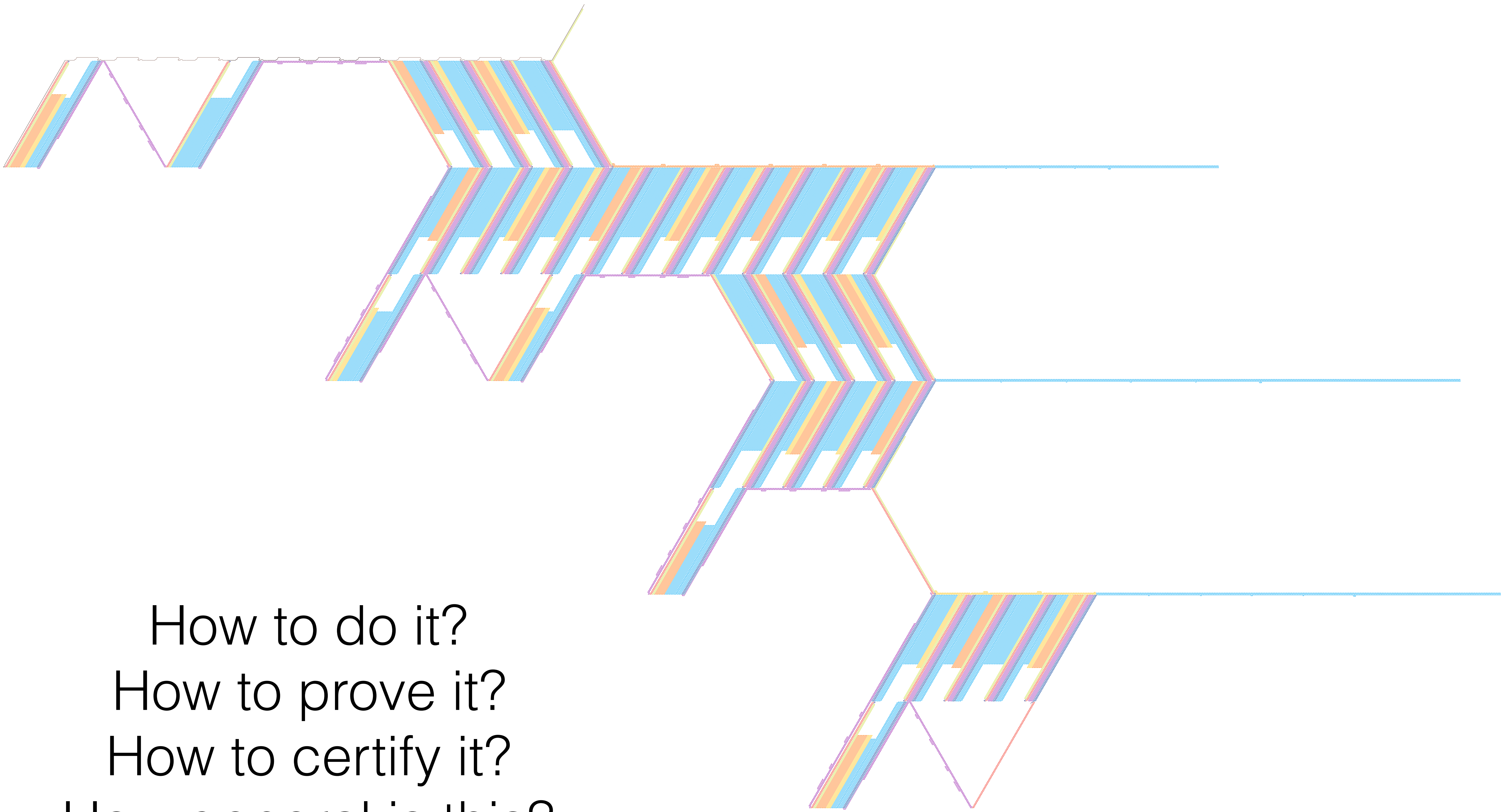
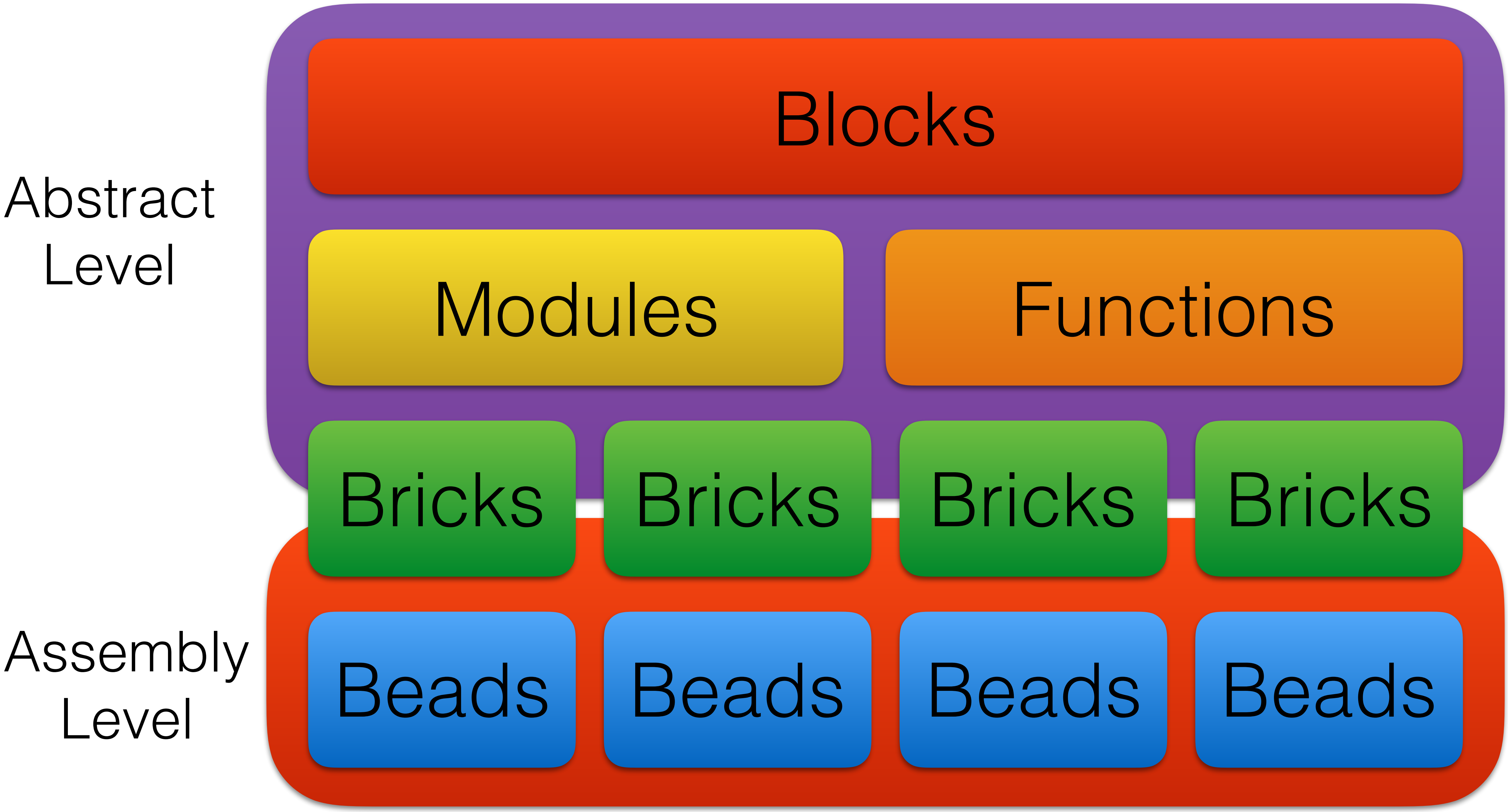
- **Theorem** [Neary, Woods, 2006] Cyclic tag systems simulate any Turing machine with only a **quadratic** slow down

The simulation

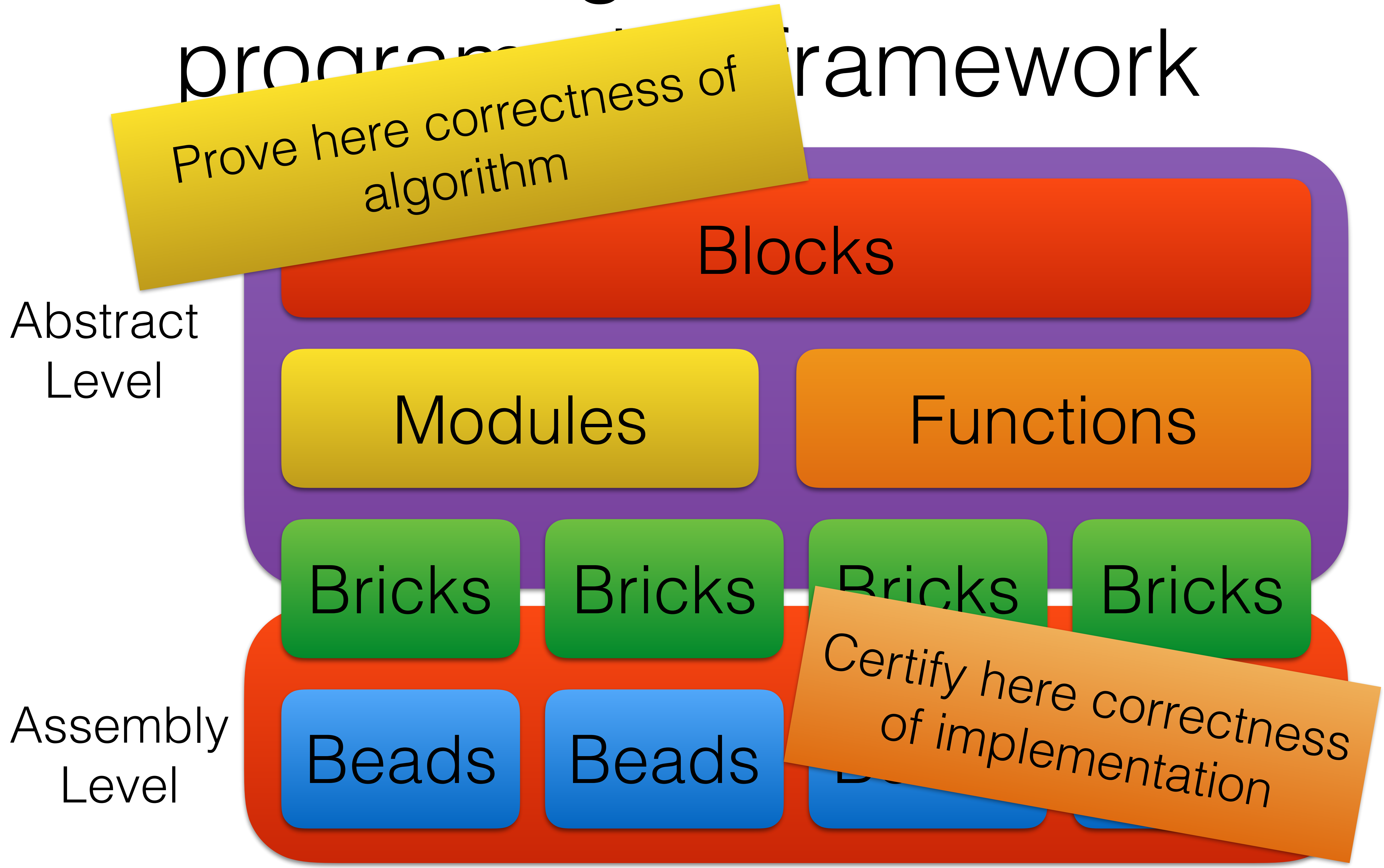


How to do it?
How to prove it?
How to certify it?
How general is this?

A general programming framework



A general
progra … framework



General programming tools

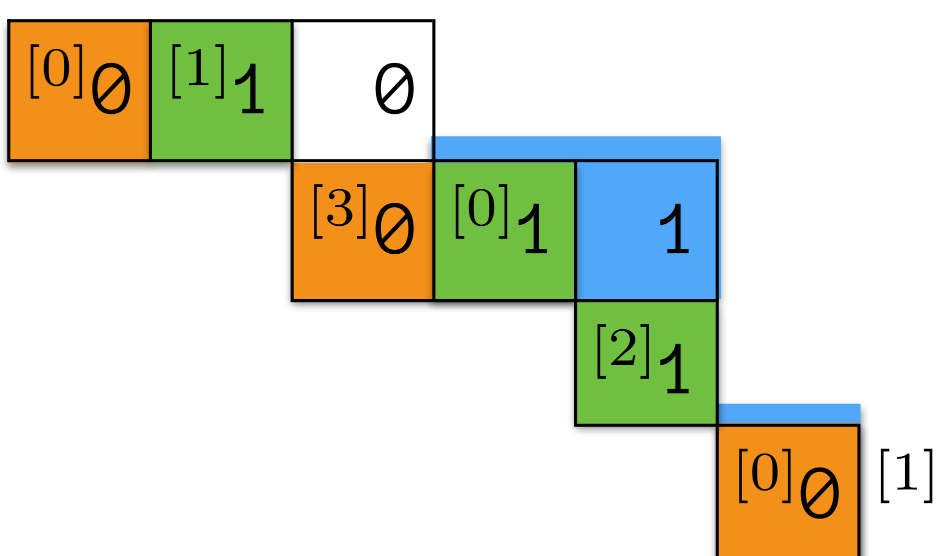
- **State**
  - Area entry point
  - Position in Molecule
- **Logic**
  - Sliding shapes
  - Bouncing gliders
  - Geometry
  - Expanding shapes
- **Goto**
  - Offsets
  - Socks
  - Exponential coloring
  - Hiding

Back to our simulation

Trimmed space-time diagram

Consider the following productions: $p = \langle \overset{[0]}{\texttt{110}}, \overset{[1]}{\epsilon}, \overset{[2]}{\texttt{11}}, \overset{[3]}{\texttt{0}} \rangle$

$$^{[0]}\texttt{010} \rightarrow {}^{[1]}\texttt{10} \xrightarrow[\text{[2]:}\texttt{11}]{\text{Append}} {}^{[3]}\texttt{011} \rightarrow {}^{[0]}\texttt{11} \xrightarrow[\text{[1]:}\epsilon]{\text{Append}} {}^{[2]}\texttt{1} \xrightarrow[\text{[3]:}\texttt{0}]{\text{Append}} {}^{[0]}\texttt{0} \rightarrow {}^{[1]}\ \texttt{Halt}$$



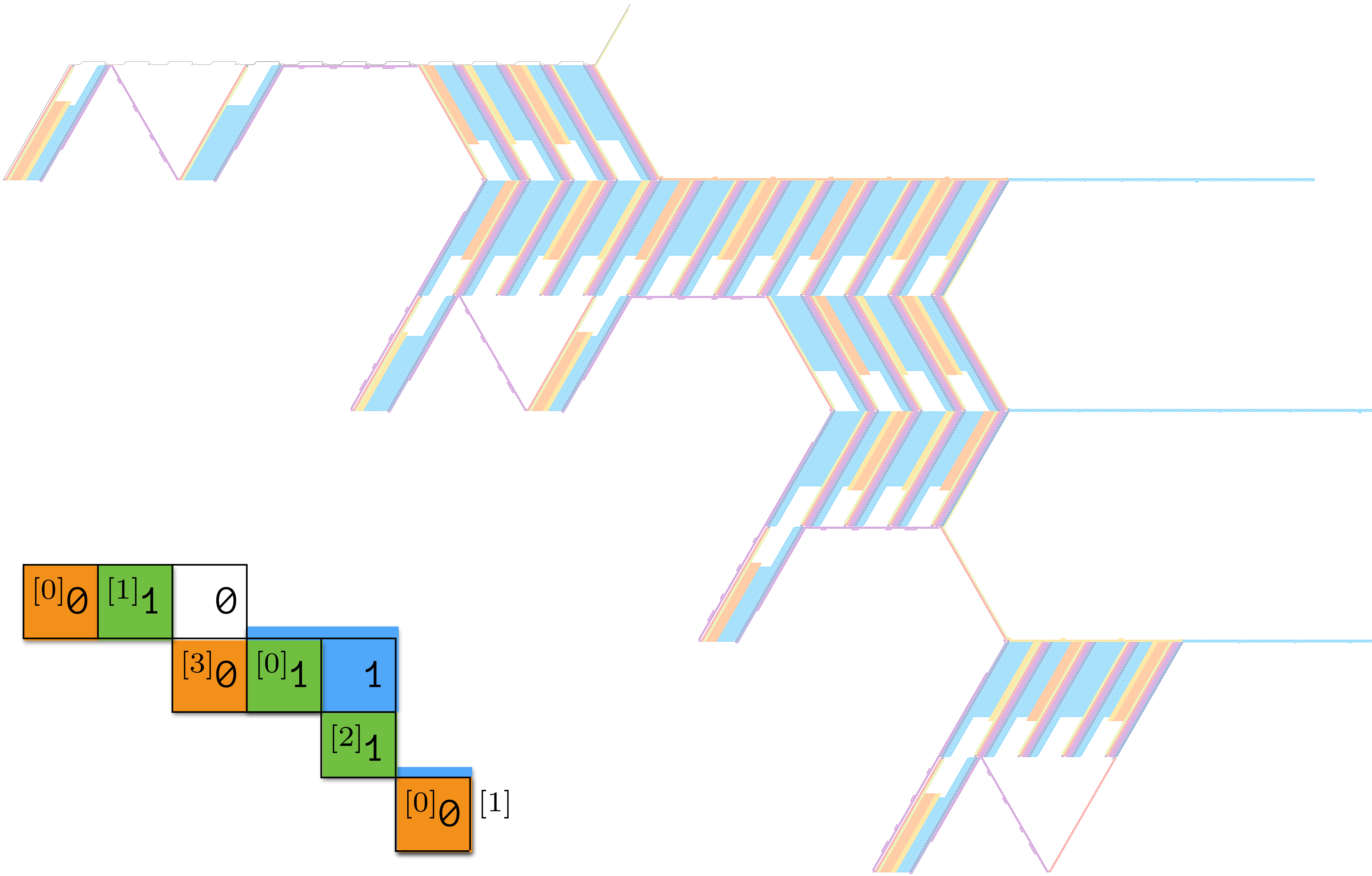
$\longrightarrow$ Append $[2]:\texttt{11}$

$\longrightarrow$ Append $[1]:\epsilon$

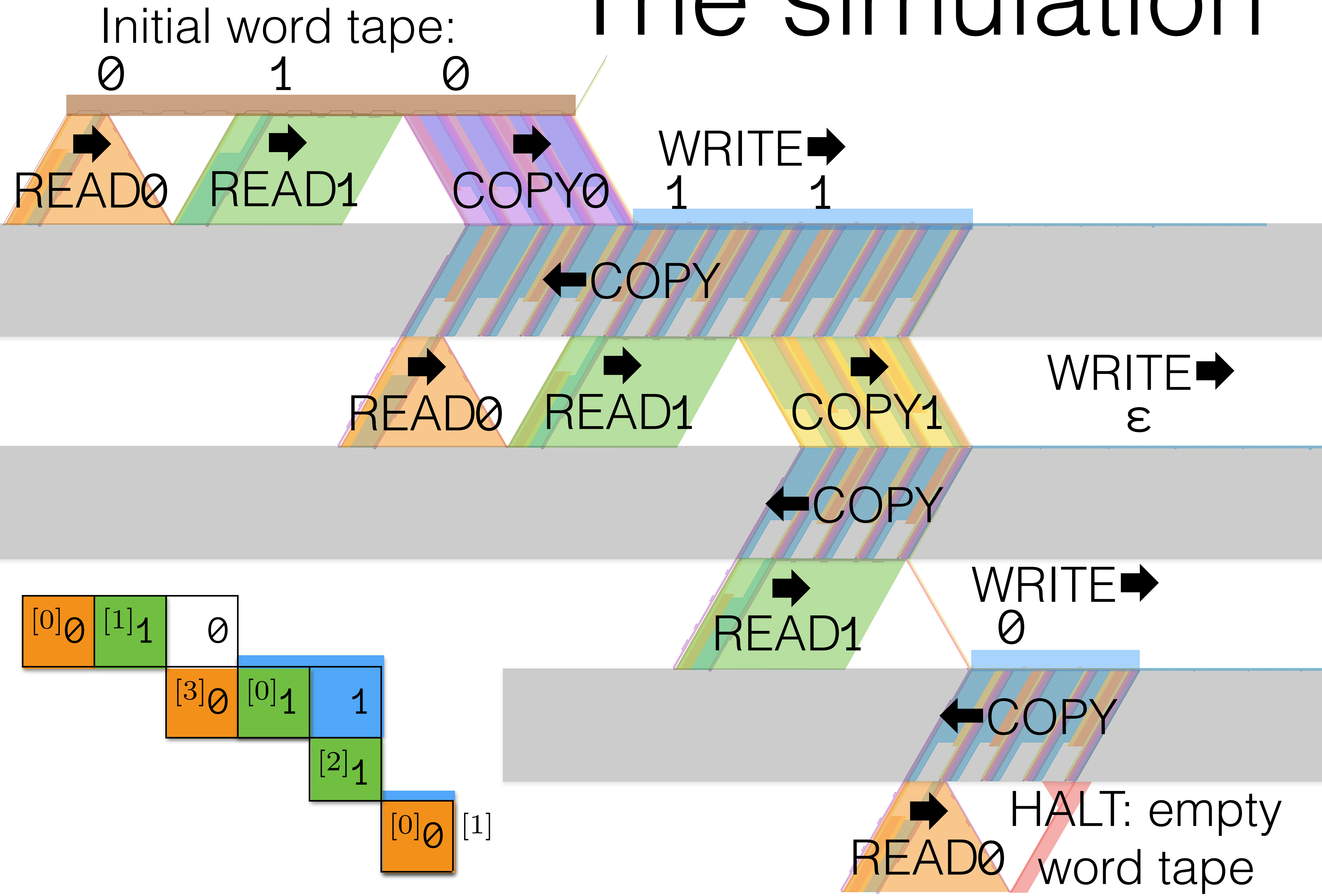
$\longrightarrow$ Append $[3]:\texttt{0}$

$\longrightarrow$ Halt $[1]:\epsilon$

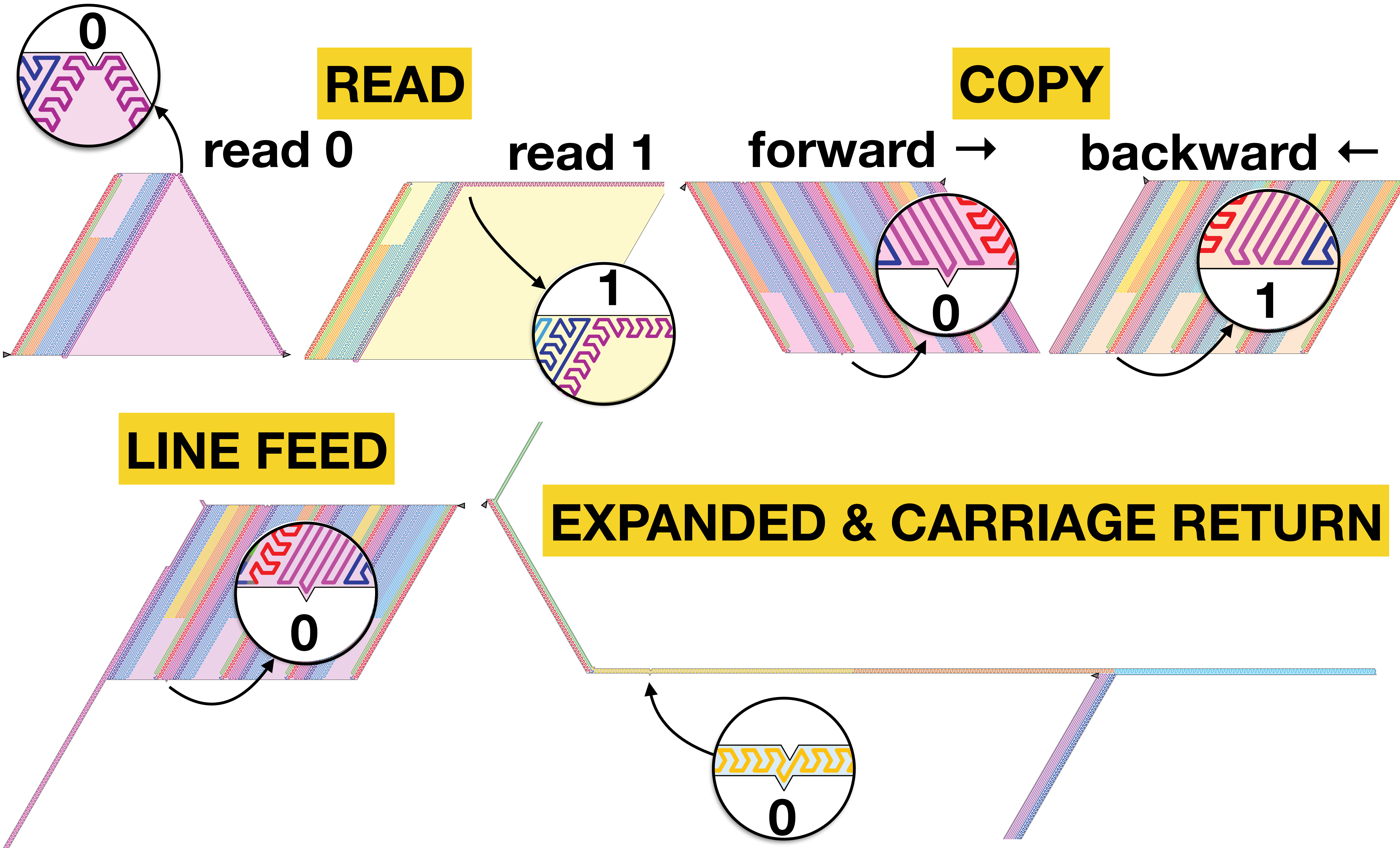
The simulation



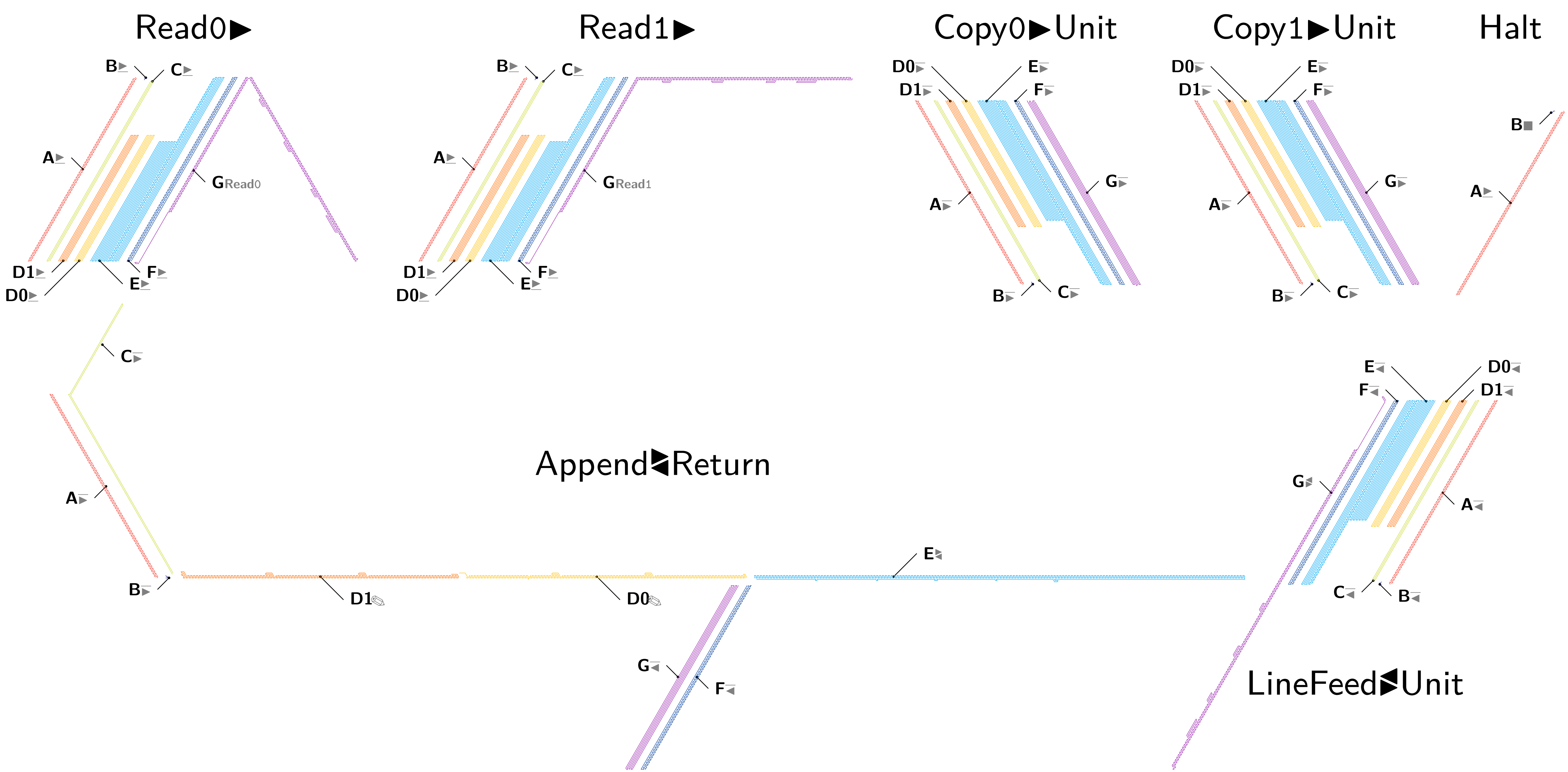
The simulation



The blocks


READ
read 0
0
read 1
1
COPY
forward →
0
backward ←
1
LINE FEED
0
EXPANDED & CARRIAGE RETURN
0

The bricks inside each block

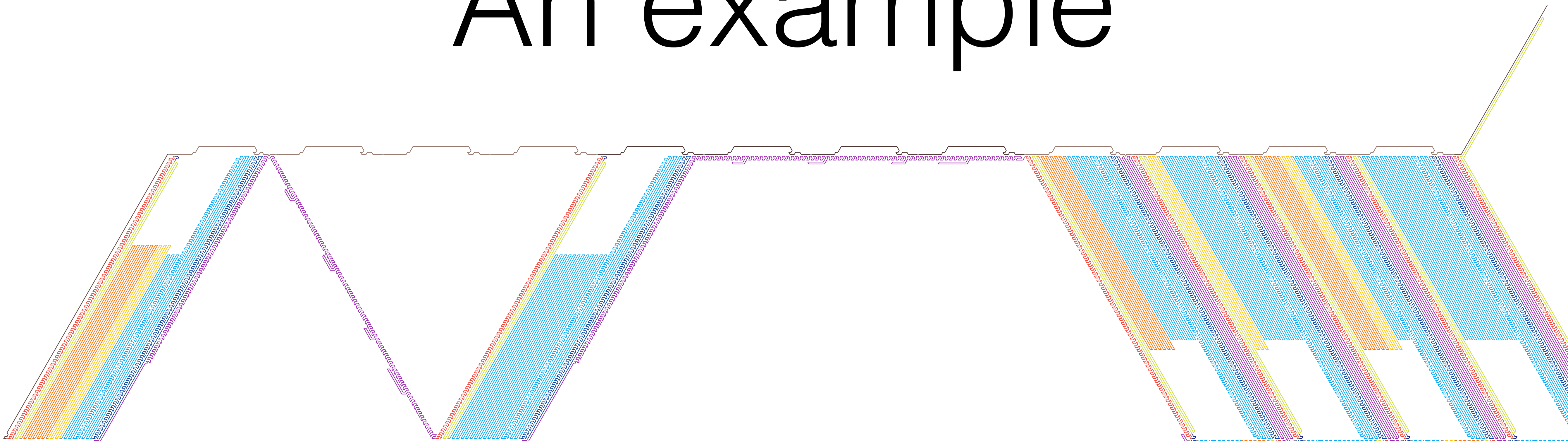


How do we implement several functions

(i.e. folding into different bricks)

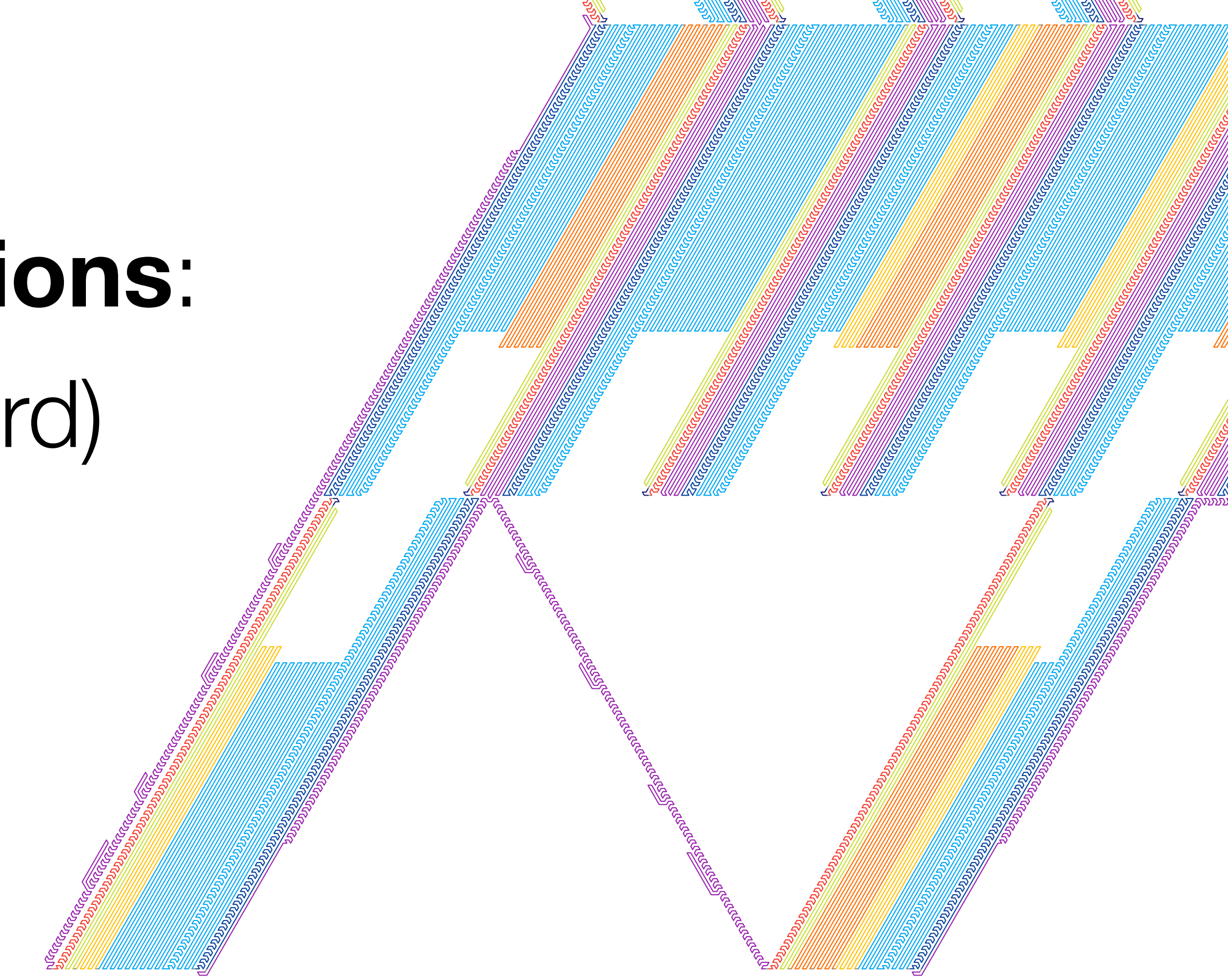
in a given module?

An example

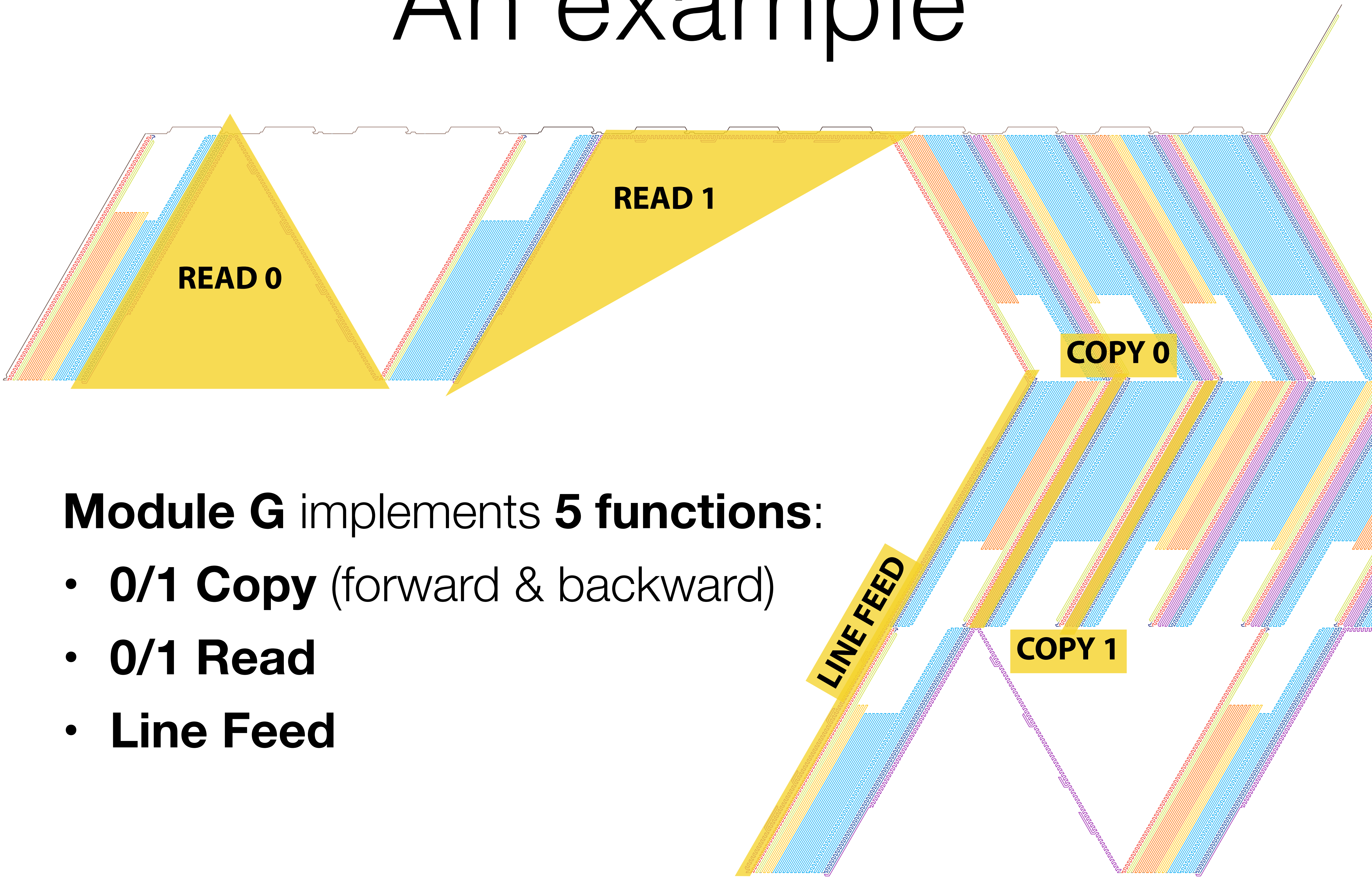


Module G implements **5 functions**:

- **0/1 Copy** (forward & backward)
- **0/1 Read**
- **Line Feed**



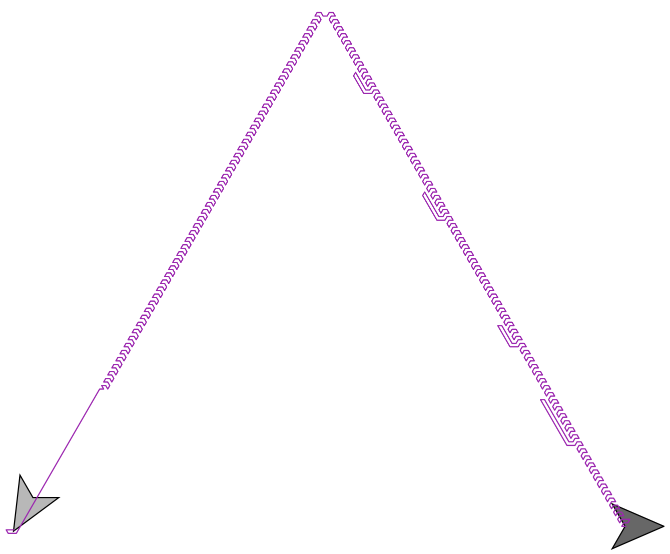
An example



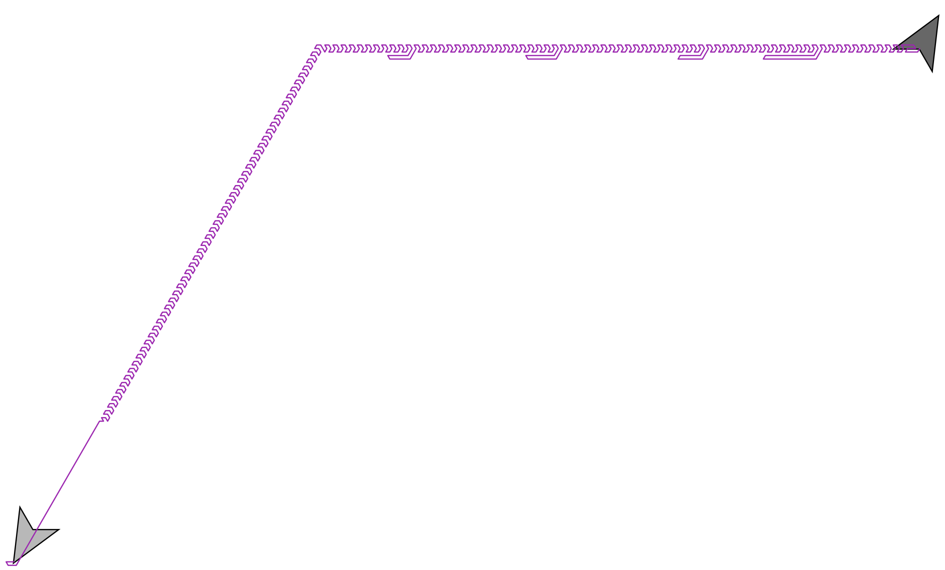
Module G implements **5 functions**:

- **0/1 Copy** (forward & backward)
- **0/1 Read**
- **Line Feed**

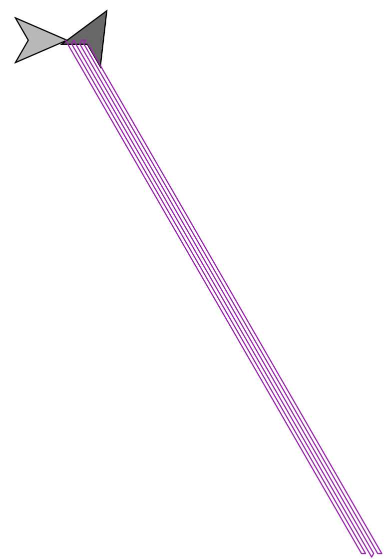
The various bricks for module G



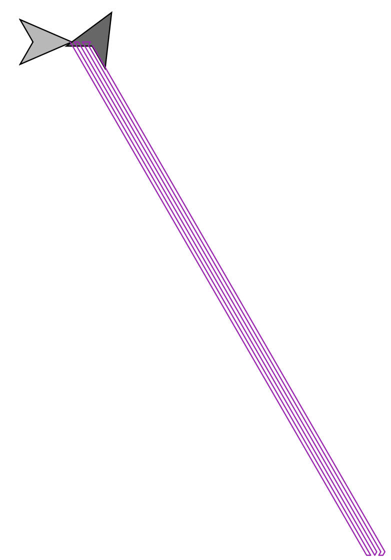
(a) The brick G ▶ Read0.



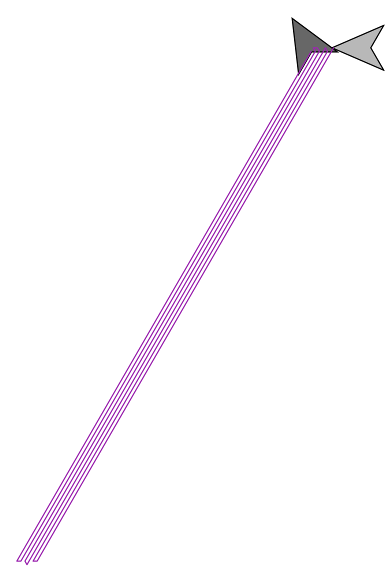
(b) The brick G ▶ Read1.



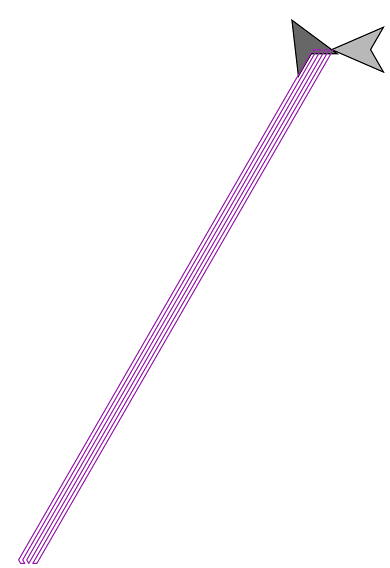
(a) The brick G ▶̄ Copy0.



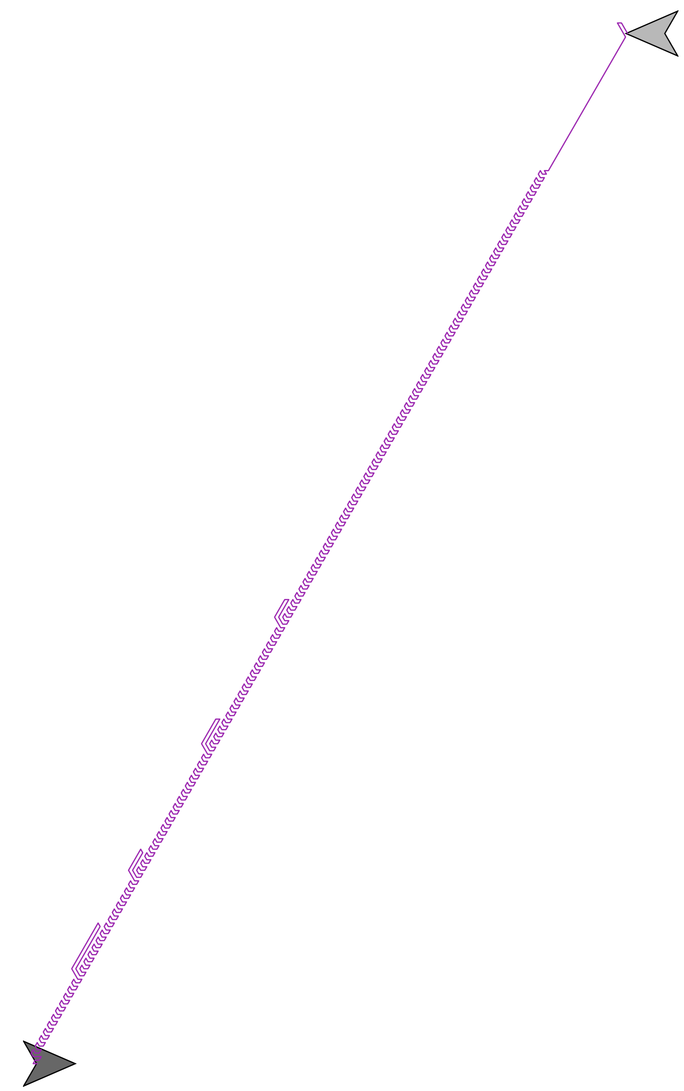
(b) The brick G ▶̄ Copy1.



(c) The brick G ◀ Copy0.

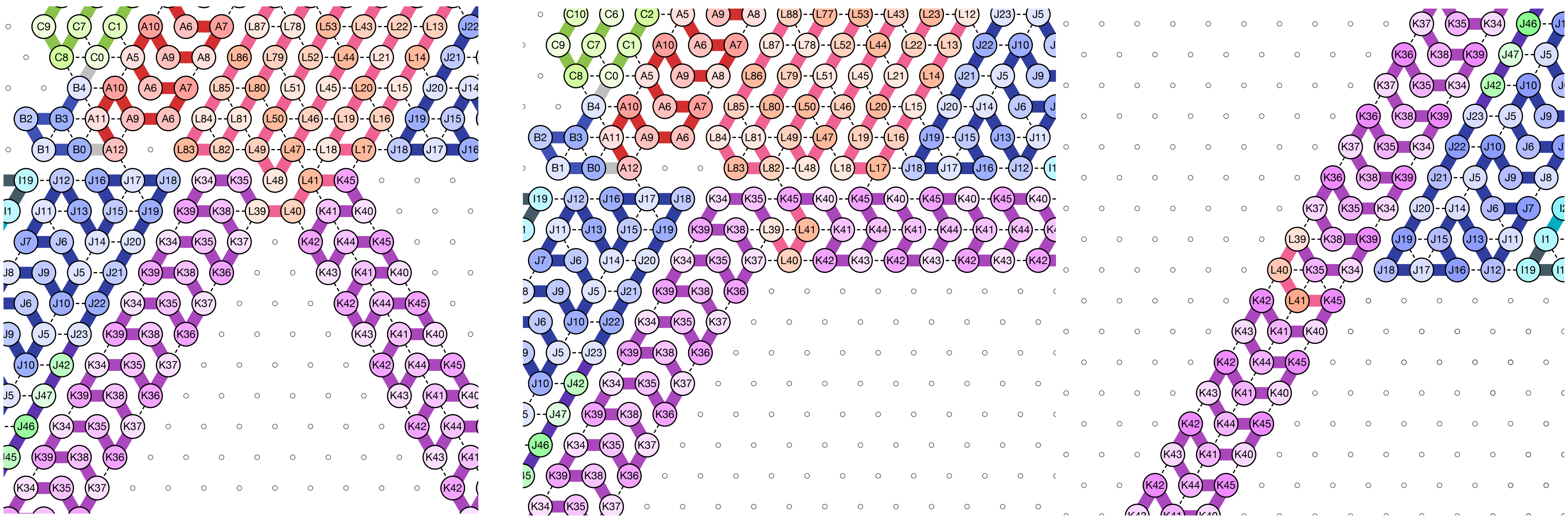


(d) The brick G ▶ Copy1.



(a) The brick G ◢◣ LineFeed.

Glider turns



(a) **G** bounces southeastward in presence of a spike encoding a 0 and folds into **G ⊵ Read0**.

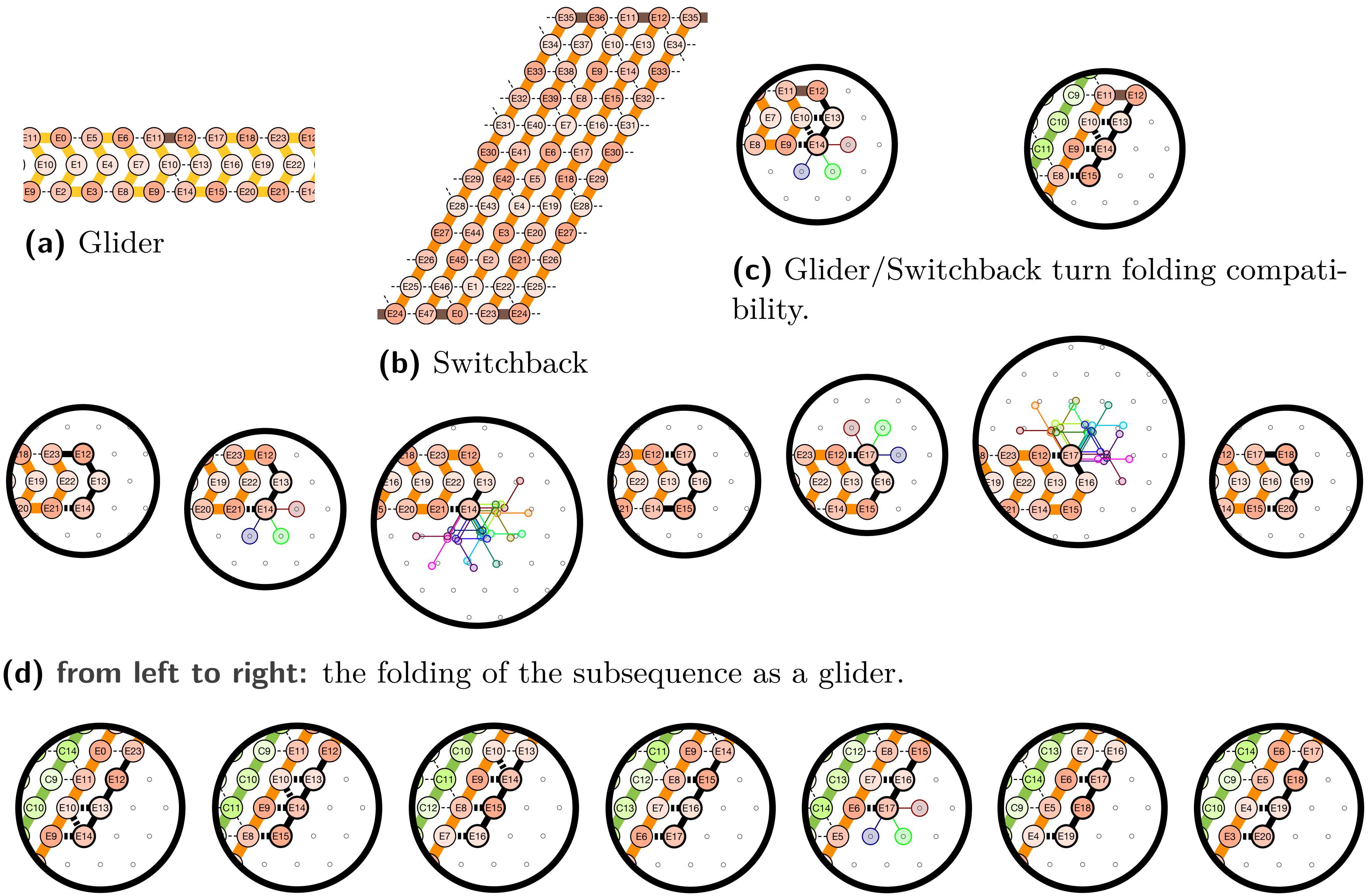
(b) **G** bounces eastward on a flat surface encoding a 1, and folds into **G ⊵ Read1**.

(c) **G** goes straight southwestward in absence of obstacle, and folds into **G ⋭ LineFeed**.

Some programming paradigms

- Switchback expanding in Gliders
- Offset
- Exponential coloring
- Socks

Switchback expanding into gliders



(a) Glider

(b) Switchback

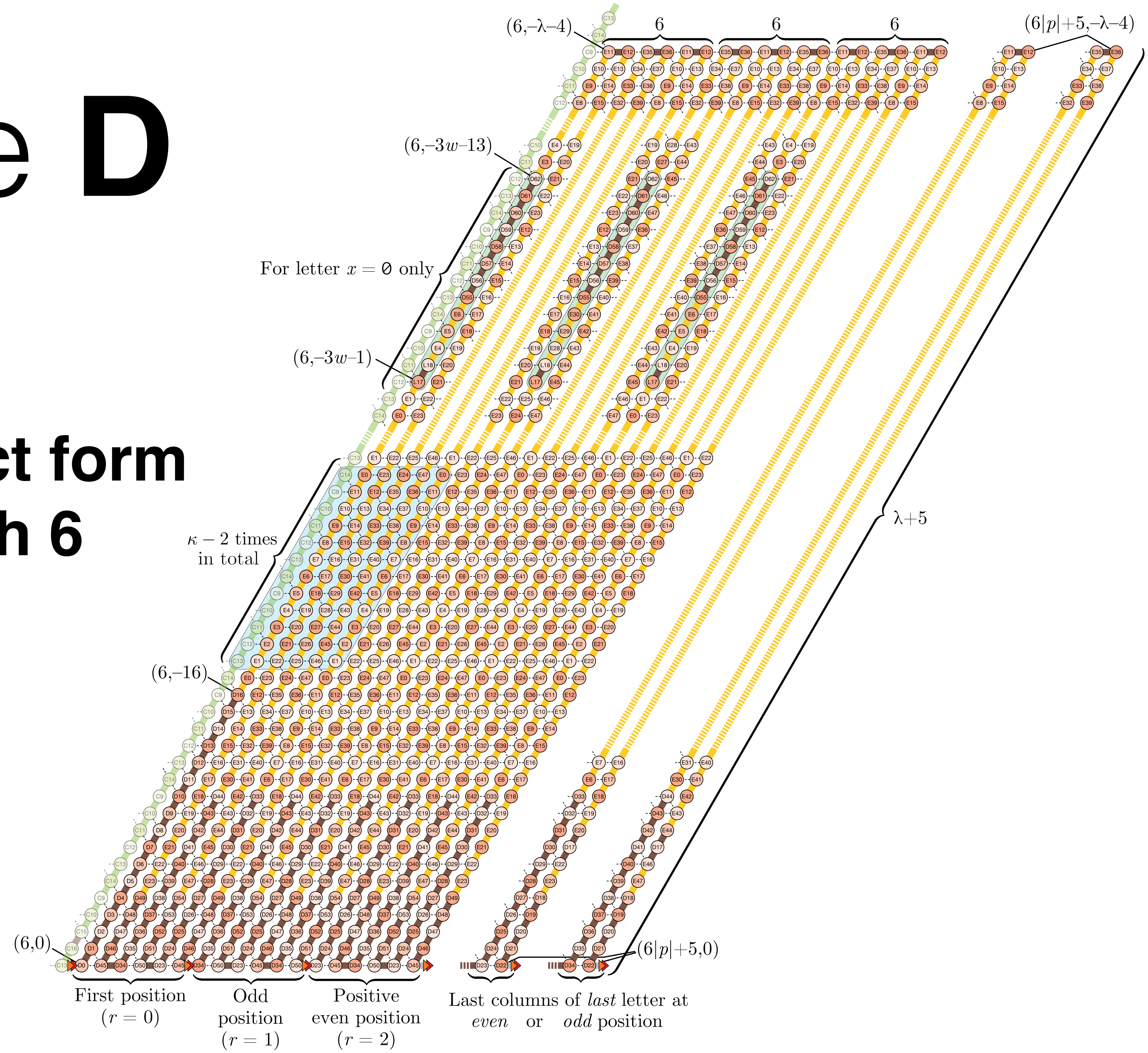
(c) Glider/Switchback turn folding compatibility.

(d) from left to right: the folding of the subsequence as a glider.

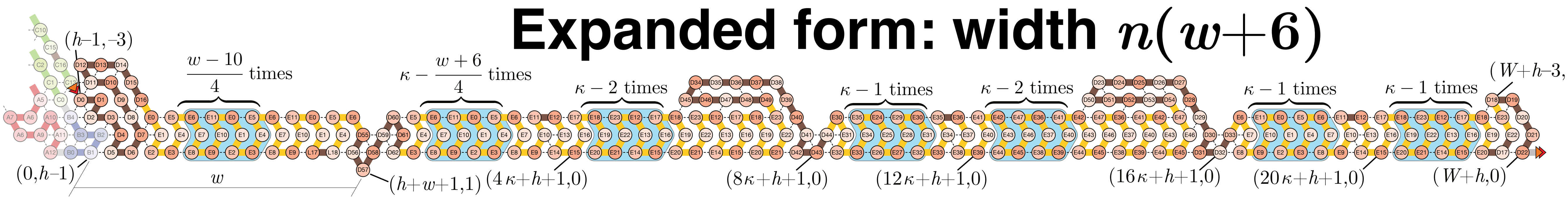
(e) from left to right: the folding of the subsequence as a switchback.

Module **D**

Compact form width 6



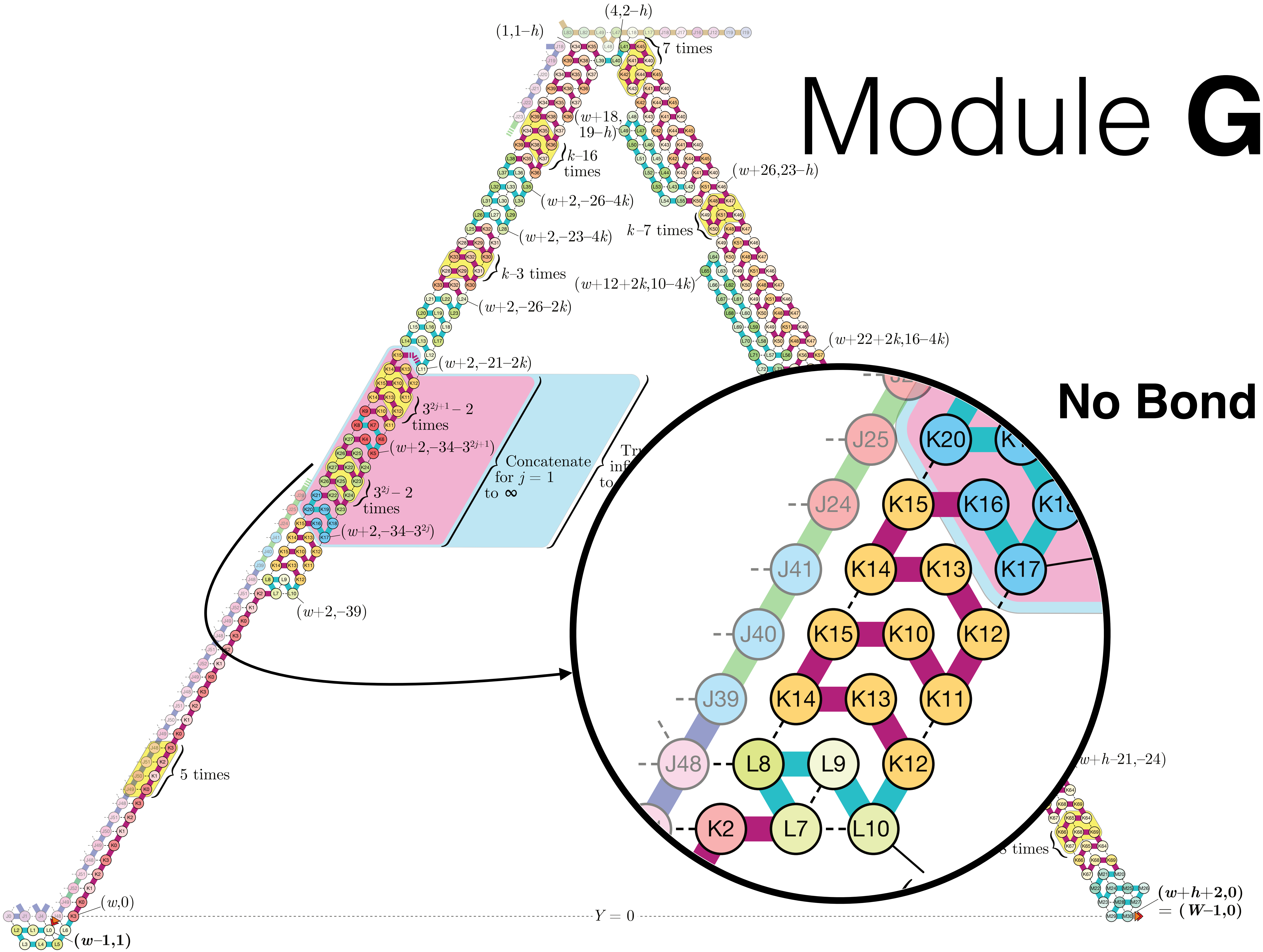
Expanded form: width $n(w+6)$



Exponential coloring

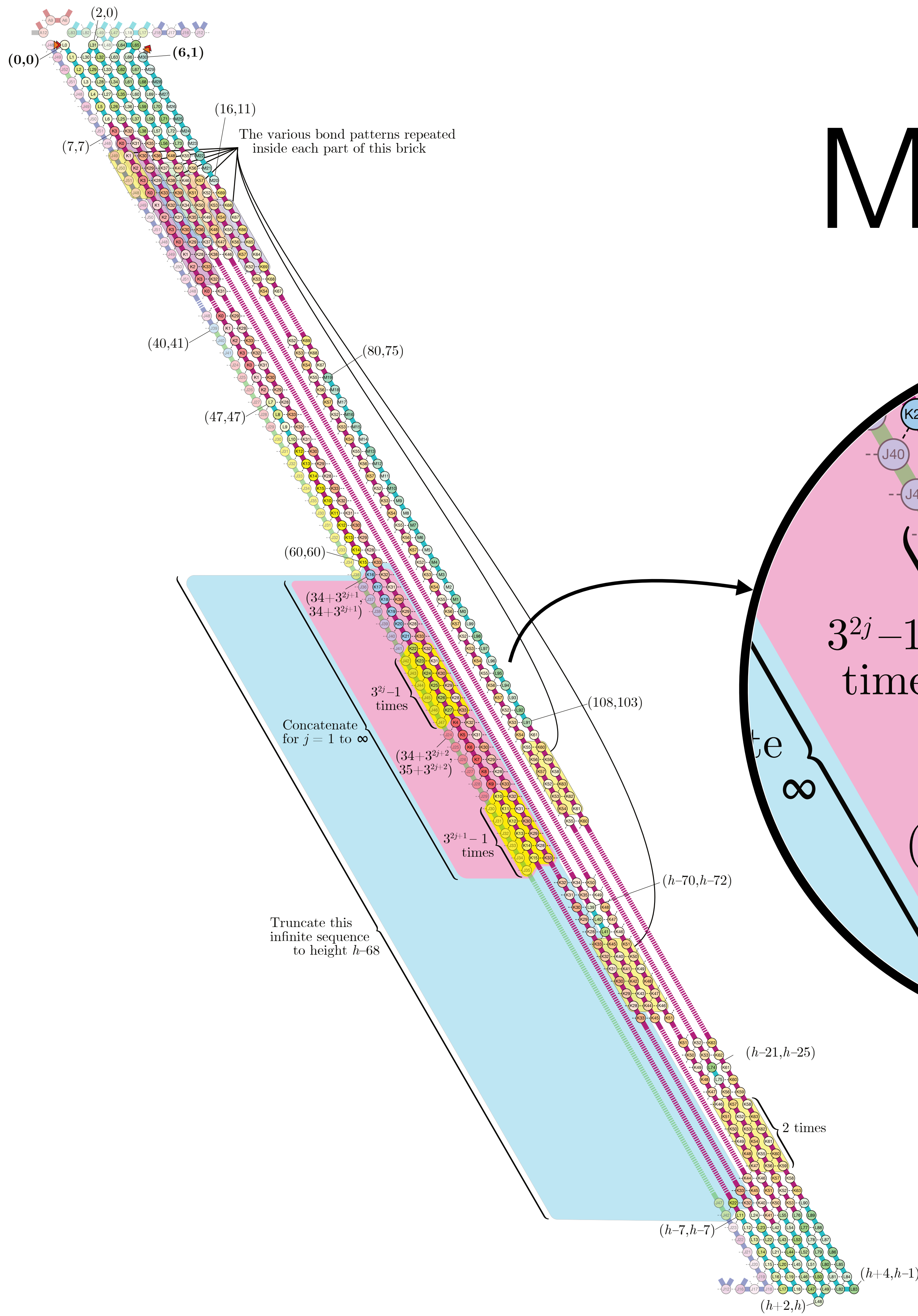
- Bonds everywhere if unshifted and then adopt switchback form
- Bonds nowhere if shifter and then adopt glider form

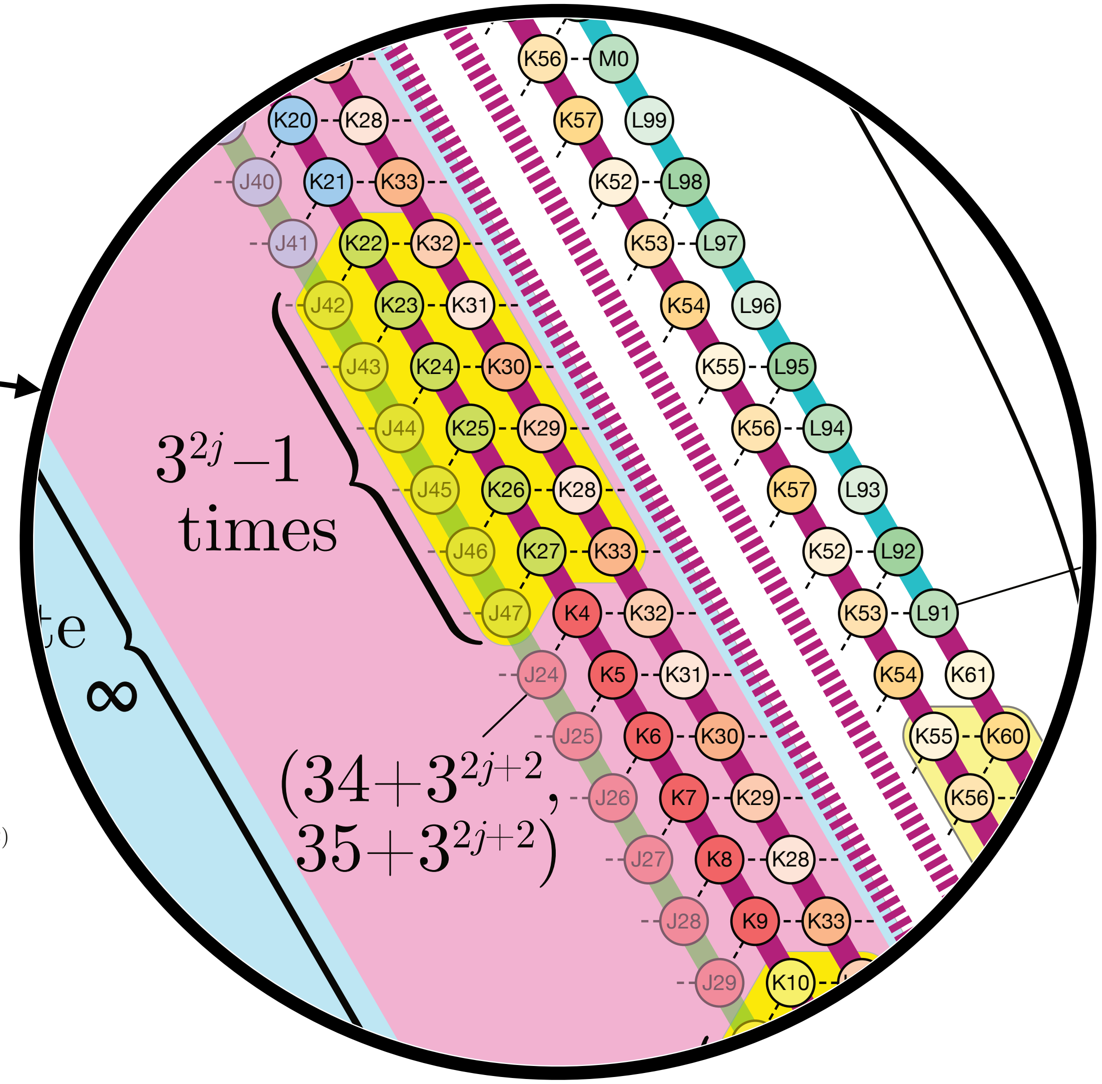
Module **G**



No Bond
(4,2–h)
(1,1–h)
7 times
(w+18, 19–h)
k–16 times
(w+2,–26–4k)
(w+26,23–h)
(w+2,–23–4k)
k–7 times
k–3 times
(w+12+2k,10–4k)
(w+2,–26–2k)
(w+22+2k,16–4k)
(w+2,–21–2k)
3^{2j+1} – 2 times
(w+2,–34–3^{2j+1})
Concatenate for j = 1 to ∞
3^{2j} – 2 times
(w+2,–34–3^{2j})
(w+2,–39)
5 times
(w,0)
Y = 0
(w–1,1)
(w+h–21,–24)
(w+h+2,0) = (W–1,0)

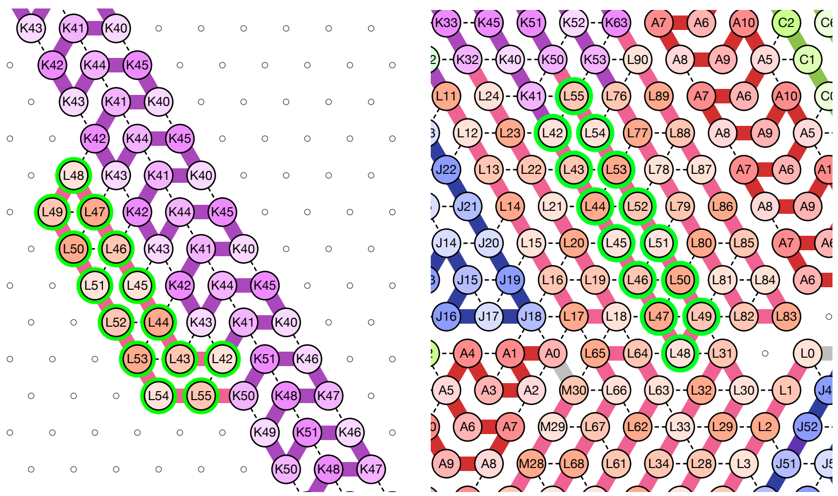
Module **G**



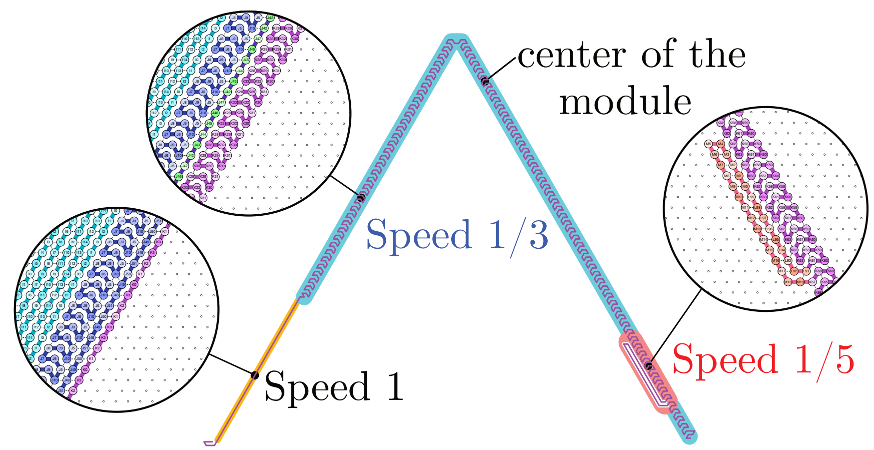


Bonds everywhere

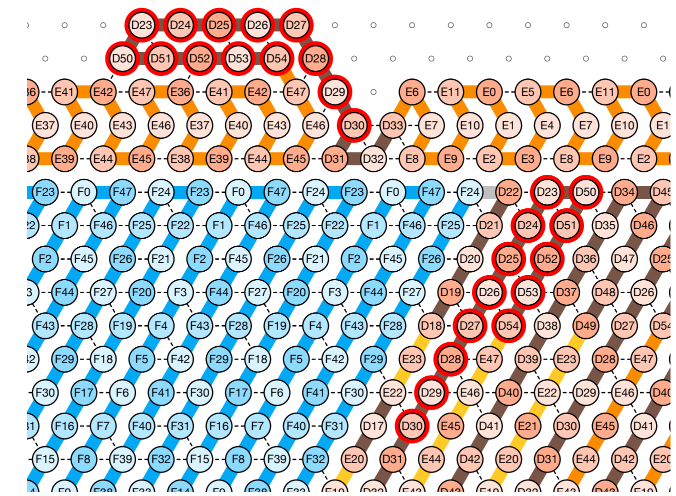
Socks



(a) Easier bond design



(b) Delaying gliders



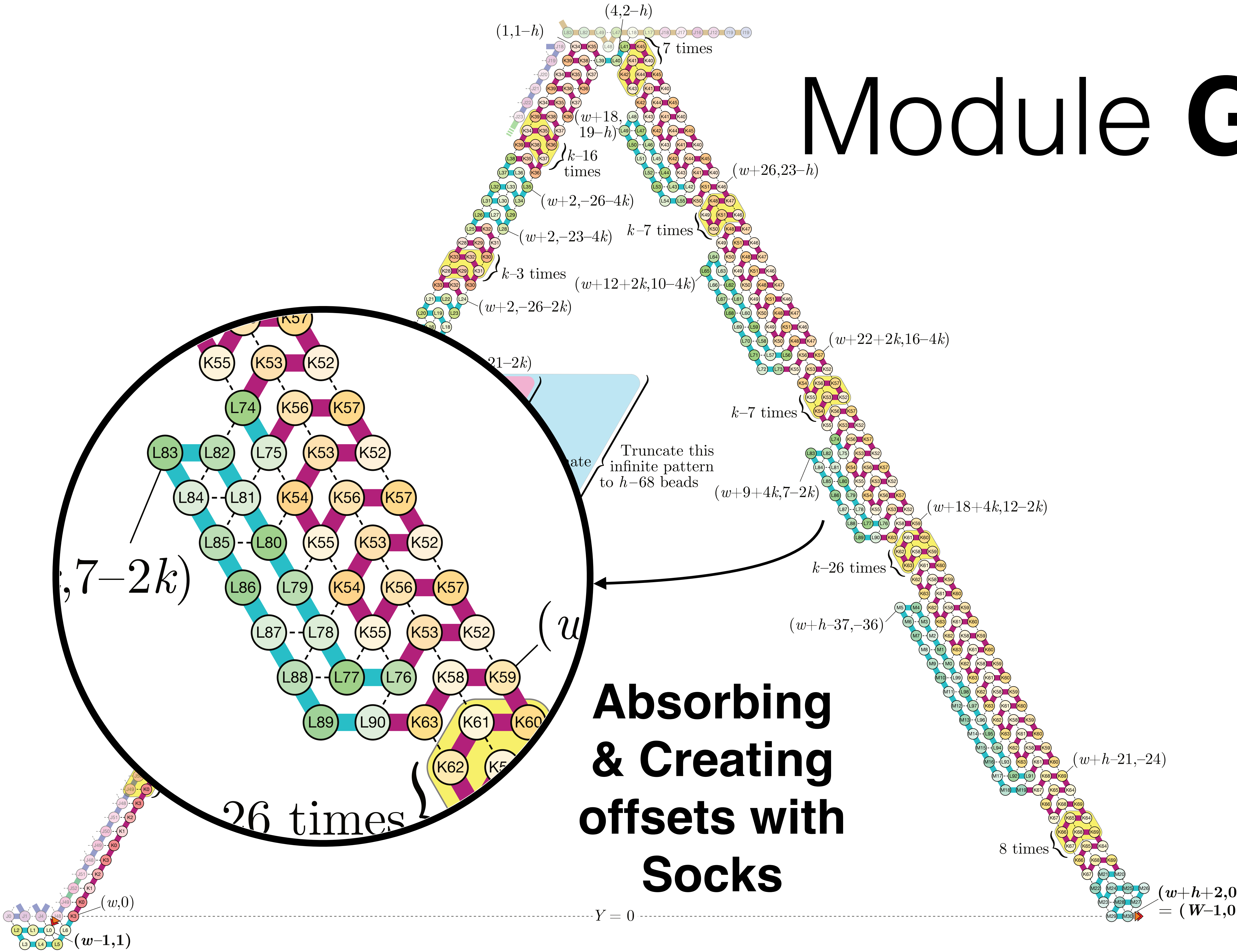
(c) Confinement

(a) Let fold parts in their natural forms, simplify the design

(b) Delaying and shifting to space out various functions

(c) Confinement to prevent unwanted interactions

Module **G**



Absorbing & Creating offsets with Socks

Proof of correctness

- Enumerate all possible environments for each brick
- Compute proof trees for each brick in all of its fixed environments
- Deal separately with the only three bricks having variable environments

Listing all environments

ZIG-UP

A	#0-98	#1286-1312	#-	#4995-4997
B	#99-103	#1313-	#-	#4998-5000
C	#104-159 #1314-1315			
$D_{\neq}$	#160-339	#340-384		
E	#1316-1392	#385-749		
F	#750-856	#1393-1401		
G	#857-1285	#1402-1853		

ZIG-DOWN

A	#1854-1874	#4745-4752	#2382-2578	#2745-2755	#2790-2797
B	#1875-1878	#2579-2580	#2756-		
C	#1879-1889 #2757-2758	#2798-2838 #4701-4702			
$D_{\neq}$	#1890-1913 #2581-2599 #2600-2602	#1914-1932	same as previous ones		
E	#1933-2011	#2603-2632	#2759-2789		
F	#2012-2041	#2633-2643			
G	#2042-2381	#2644-2744			

WRITE

$D_{\neq}$	#2839-2999	#4753-4786
E	#4703-4733	#3000-3749 #4787-4945
F	#3750-3781 #4946-4959	#4734-4744

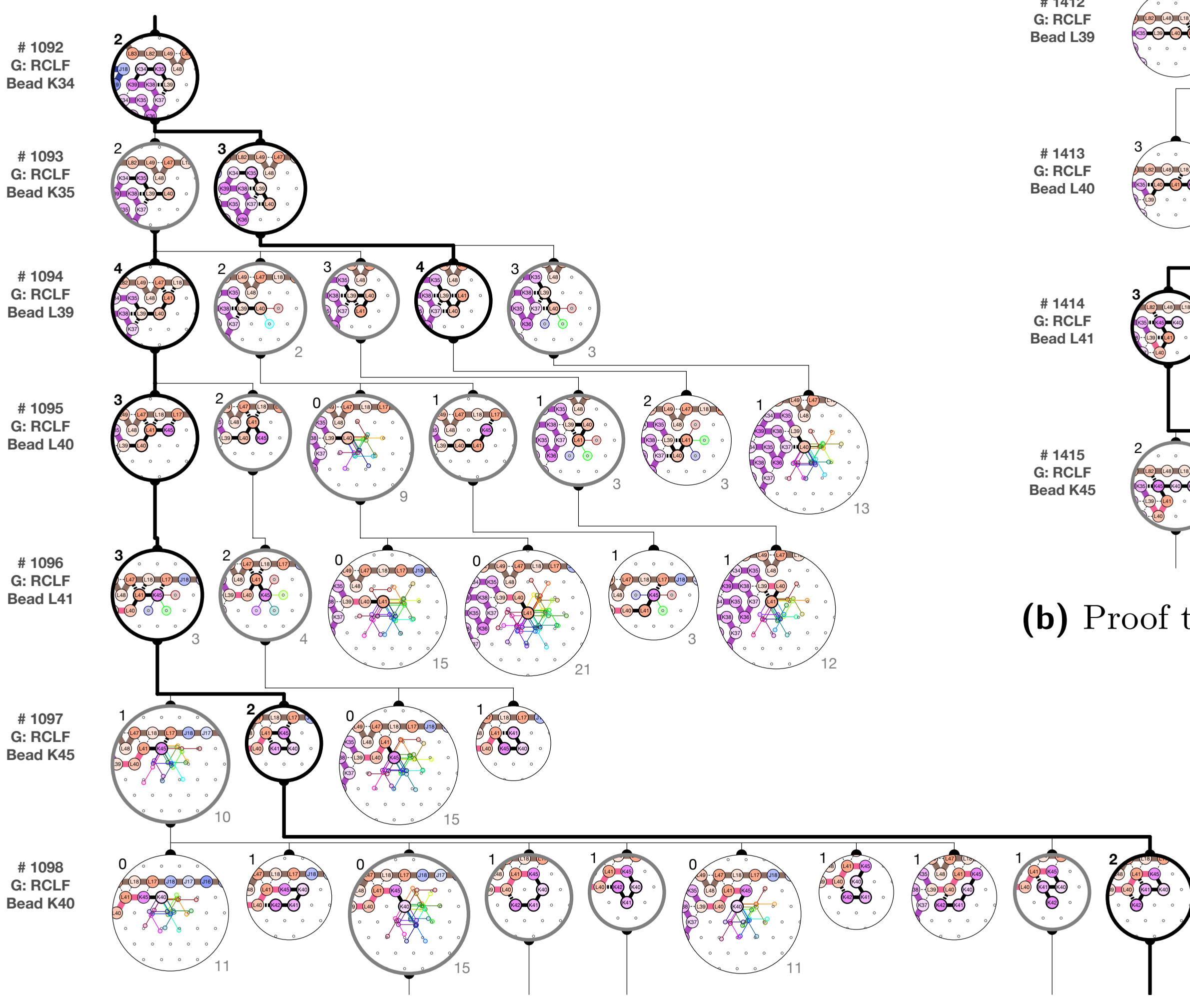
ZAG-WRITE

A	#4994-	#3806-3816 , #4059-4069 , #4215-4225 #4310-4320	
C	#3817-3827 , #4070-4080 , #4226-4236 #4321-4331 , #4459-4460 , #4475-4476 #4550-4551 , #4605-4606		
$D_{\neq}$	#3828-3851 #4237-4263 #4332-4355	#3939-3941 , #4434-4439 , #4525-4527 #4571-4573 , #4587-4589 , #4595-4597 #4618-4620	
E	#4081-4176 , #4461-4474 #4552-4570 , #4607-4617	#3852-3924 , #3942-4020 , #4264-4271 #4356-4428 , #4440-4458 , #4504-4519 #4528-4549 , #4574-4581 , #4590-4594 #4598-4604 , #4621-4644	
F	#3925-3938 , #4021-4034 , #4177-4190 #4272-4285 , #4429-4433 , #4520-4524 #4582-4586	#4645-4653	#4960-4967
G	#4968-4993	#3782-3805 , #4035-4058 , #4191-4214 #4286-4309	

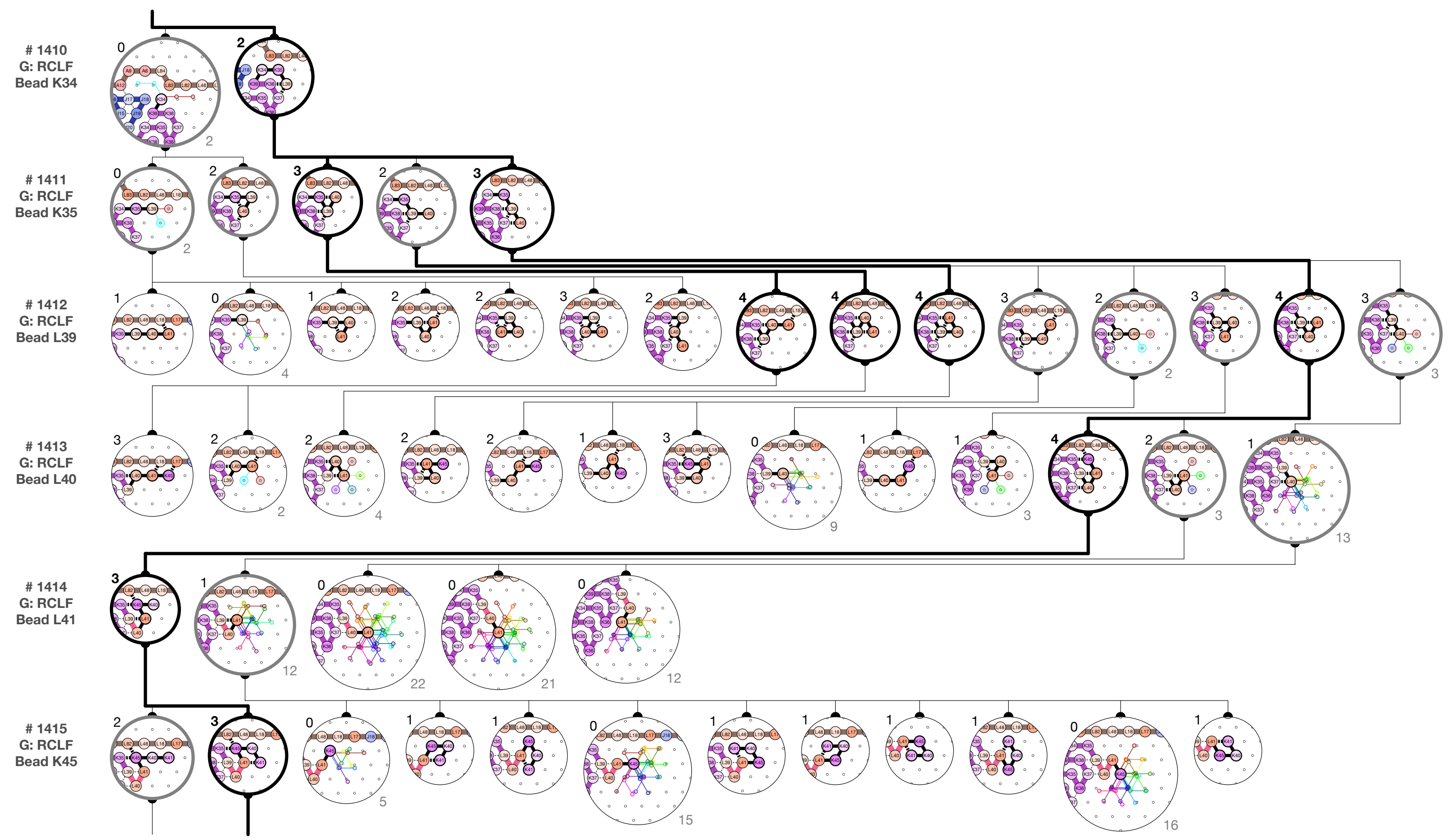
ZAG

A	see Zig-Down	see Zig-Down	
B	see Zig-Down	see Zig-Down	
C	see Zig-Down		
$D_{\neq}$	see Zig-Down	see Zig-Down	see Zig-Down
E	see Zig-Down	see Zig-Down	see Zig-Down
F	see Zig-Down	see Zig-Down	
G	#4654-4700	see Zig-Down	see Zig-Down

Example: Reading 0/1

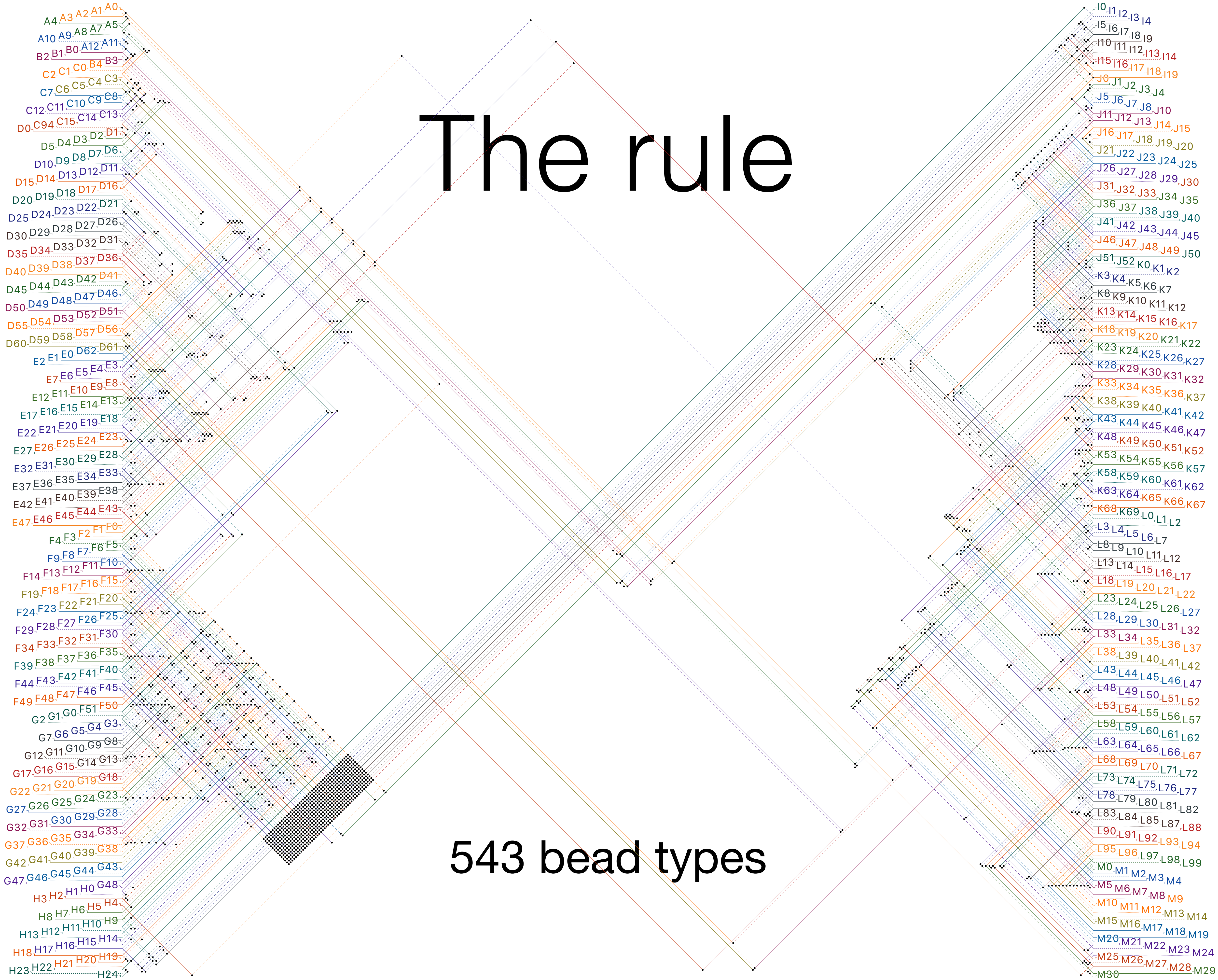


(a) Proof tree for the glider turn in G ▶ Read0.



(b) Proof tree for the glider turn in G ▶ Read1.

The rule



543 bead types