HW2 M2IF CR07 Molecular Programming

2023.10.06 - Due on Fri. 10.13 before 13:30

You are asked to complete the exercise marked with a [★] and to send me your solutions to:
nicolas.schabanel@ens-lyon.fr
as a PDF file named **HW2-Lastname.pdf** on **Fri. 10.13 before 13:30**.

[★] Exercise 1 (Scale the wall). Recall that a tile assembly system $\mathcal{T} = (T, \sigma, \tau)$ consists of a tile set T, a seed tile $\sigma \in T$ and a temperature $\tau \in \mathbb{N}$. Consider the situation in Fig. 1 consisting of a wall of height h.

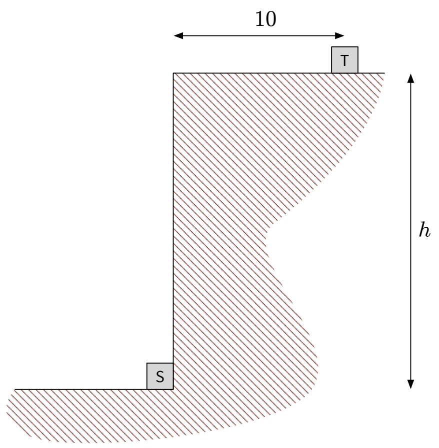

Figure 1: A wall of height h.

► **Question 1.1)** *Can you find a tile assembly system $\mathcal{T}$ for the abstract Tile Assembly Model (aTAM) where the rules are as follows?*

- *The seed tile is placed at position $S = (0, 0)$*
- *For all $h \in \mathbb{N}$, every terminal assembly of $\mathcal{T}$ should place a tile at the target position $T = (10, h)$ and be of finite size*
- *$\mathcal{T}$ may not place tiles to the right and below the cut of the plane shown in Figure 1.*
- *You may give an infinite sequence of glues such that the h-prefix of that sequence will appear on the wall, to help the tiles 'climb up'.*

Answer. ▷ First notice that there is a simple probabilistic solution to this problem that will always place the tile at position T but will be finite only almost surely (if the concentrations of B and C tiles are at least as large as the concentration of A), see Fig. 2.

However, there is no solution for the deterministic case. Indeed, let us assume that there is a deterministic tileset that solves the problem for all $h > 0$. Then, let us show that it can assemble an infinite shape. We will proceed incrementally by building a sequence of shapes of strictly increasing heights as illustrated in Fig 3. The key is consider the assembly just before a tile is attached to the right of the vertical line L of the wall. Let

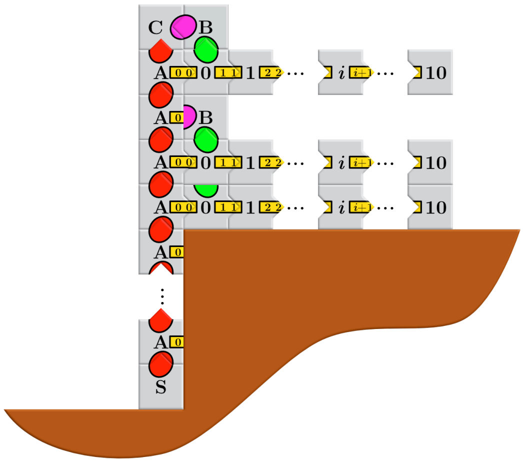

Figure 2: A probabilistic solution.

A_1 be the assembly just before that time. Let h_1 be the height of the highest tile just to the left of L in A_1. A_1 is also a valid assembly for the case where $h = h1$. Thus, as the tileset is a solution for $h = h_1$, any extension of A_1 must ultimately solve the problem for $h = h_1$. It must then indeed at some point place a tile to the right of L, strictly above h_1. Let us consider A_2 the assembly just before that and $h_2 > h_1$ the height of the highest tile just to the left of L in A_2. Repeating the process we conclude that the tileset necessarily contains an infinite assembly that never crosses L and thus does not solve the problem.

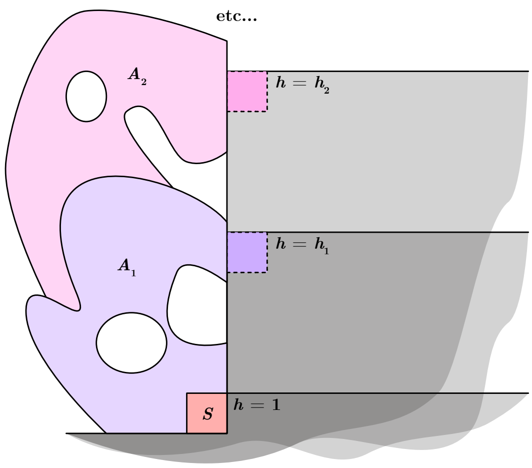

Figure 3: No deterministic solution.

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