Universality in assembly models

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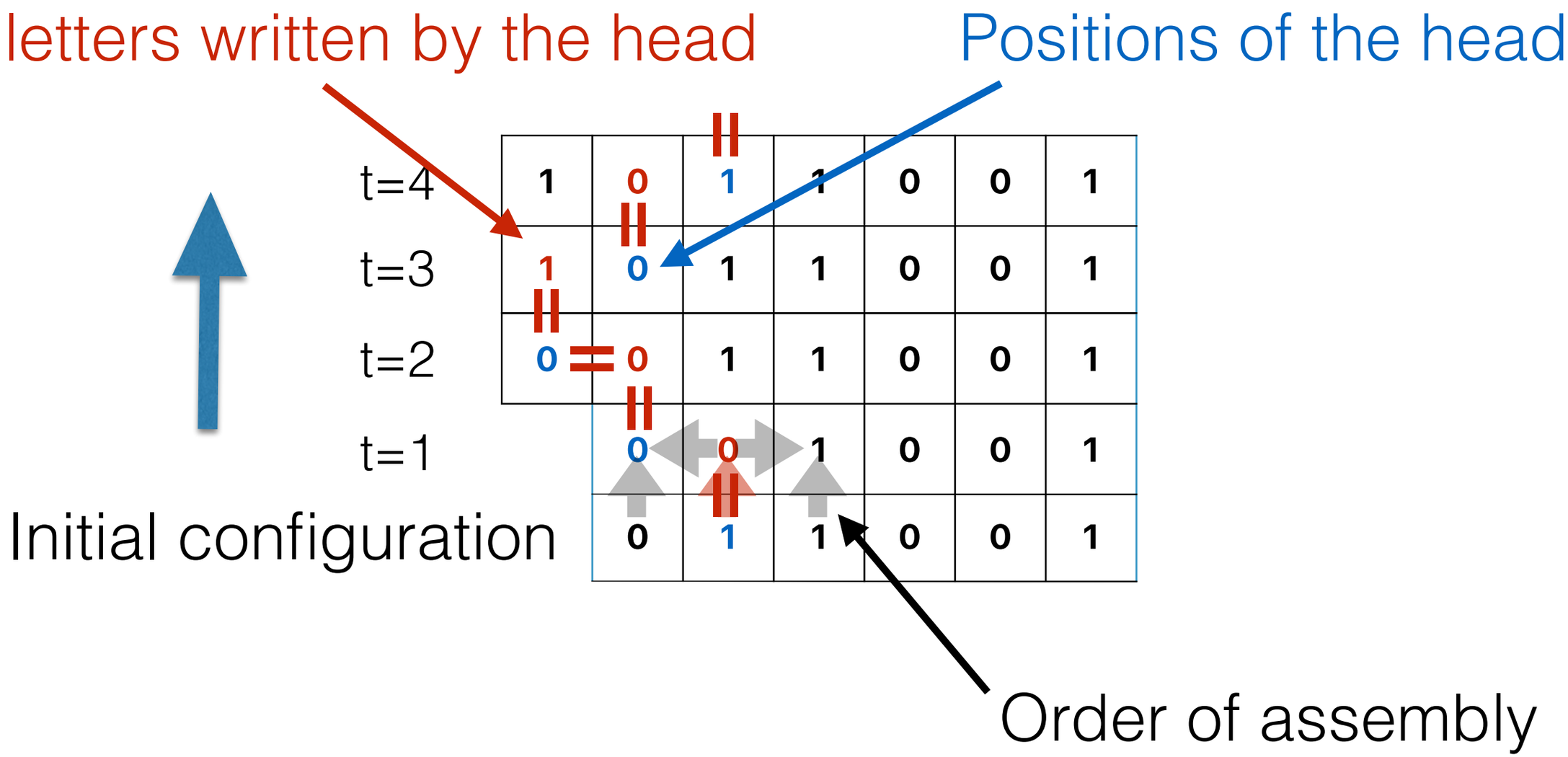
Universality in algorithmic self-assembly

- Can we decide if an assembly terminates?
- Can we design a tile system for a given shape?
- Do I need more than one tileset?
- Do I need more than one tile?
- Do I need more than one molecule?

Algorithmic self-assembly simulates any Turing machine at T°2 in 2D



Algorithmic self-assembly simulates any Turing machine at T°2 in 2D

Position and state of the head

Current tape right end

t=4	0	0	1,Halt	1	★
t=3	0	0	0,q'''	1	★
t=2	0	0	0	0,q''	★
t=1	0	0	1,q'	★	
t=0	0	1,q	1	★	

Transitions:

$(0,q''') \longmapsto (1,\text{Halt},\bullet)$

$(0,q'') \longmapsto (1,q''',\leftarrow)$

$(1,q') \longmapsto (0,q'',\rightarrow)$

$(1,q) \longmapsto (0,q',\rightarrow)$

Algorithmic self-assembly simulates any Turing machine at T°2 in 2D

t+1	a_{-L}	…	a_{-2}	a_{-1},q'	b	a_1	…	a_R	0	★
t	a_{-L}	…	a_{-2}	a_{-1}	a,q	a_1	…	a_R	★	

Tiles for (a,q) ⟼ (b,q',←)

Algorithmic self-assembly simulates any Turing machine at T°2 in 2D

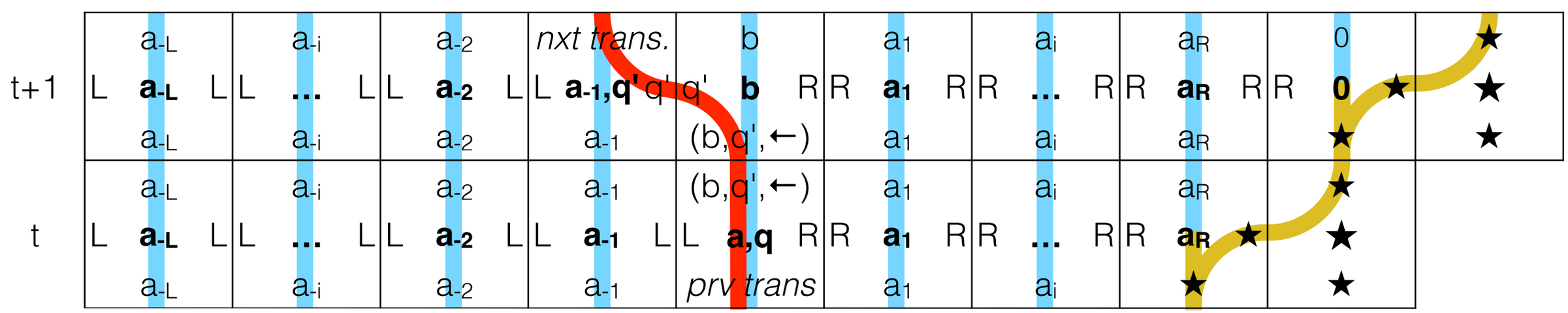
1) Organize the information flow passing through the glues on the sides

		a_{-L}			a_{-i}			a_{-2}			*nxt trans.*			b			a_1			a_i			a_R			0			★
t+1	L	**a_{-L}**	L	L	**…**	L	L	**a_{-2}**	L	L	**a_{-1},q'**	q'	q'	**b**	R	R	**a_1**	R	R	**…**	R	R	**a_R**	R	R	**0**	★		★
		a_{-L}			a_{-i}			a_{-2}			a_{-1}			(b,q',←)			a_1			a_i			a_R			★			★
		a_{-L}			a_{-i}			a_{-2}			a_{-1}			(b,q',←)			a_1			a_i			a_R			★			
t	L	**a_{-L}**	L	L	**…**	L	L	**a_{-2}**	L	L	**a_{-1}**	L	L	**a,q**	R	R	**a_1**	R	R	**…**	R	R	**a_R**	★		★			
		a_{-L}			a_{-i}			a_{-2}			a_{-1}			*prv trans*			a_1			a_i			★			★			

Tiles for (a,q) ⟼ (b,q',←)

Algorithmic self-assembly simulates any Turing machine at T°2 in 2D

1) Organize the information flow passing through the glues on the sides

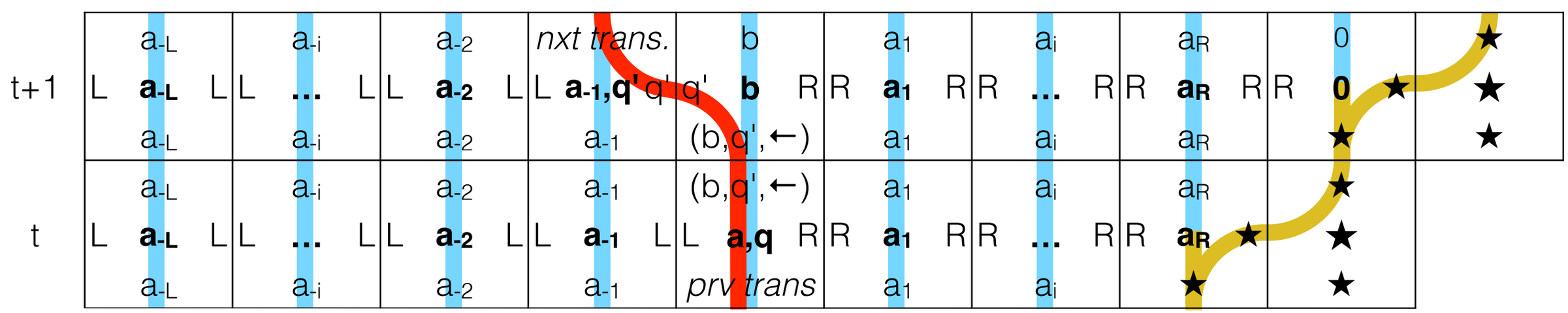


Tiles for (a,q) ⟼ (b,q',←)

Algorithmic self-assembly simulates any Turing machine at T°2 in 2D

1) Organize the information flow passing through the glues on the sides

2) Identify each type of tiles



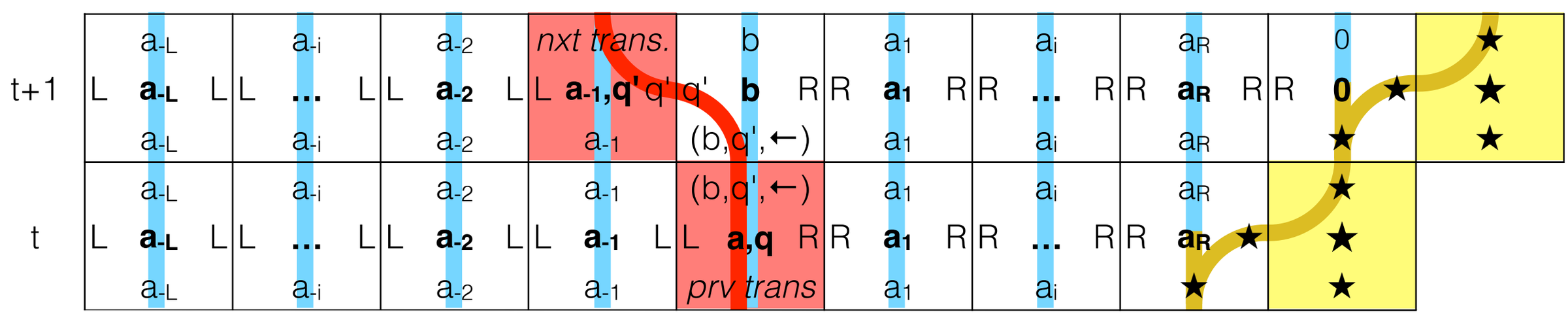
Tiles for (a,q) ⟼ (b,q',←)

Algorithmic self-assembly simulates any Turing machine at T°2 in 2D

1) Organize the information flow passing through the glues on the sides

2) Identify each type of tiles

The skeleton



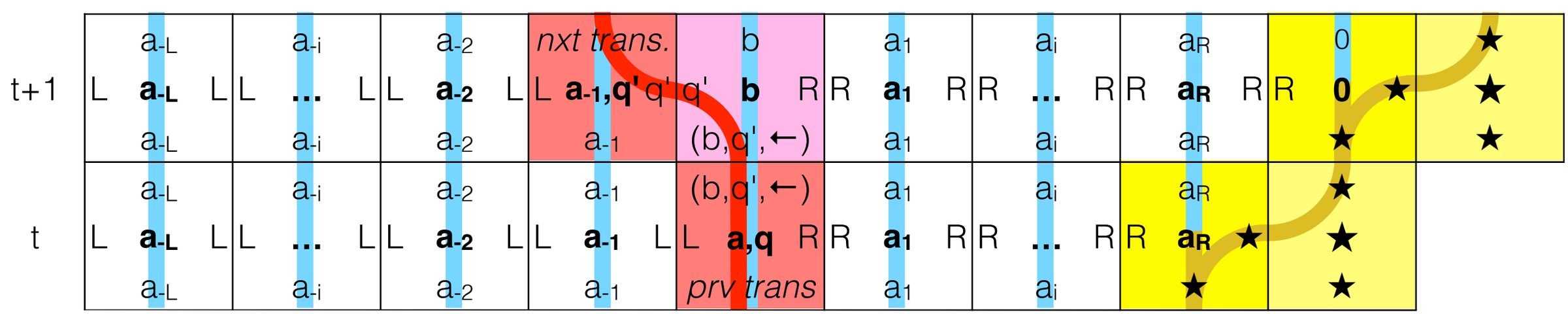
Tiles for (a,q) ⟼ (b,q',←)

Algorithmic self-assembly simulates any Turing machine at T°2 in 2D

1) Organize the information flow passing through the glues on the sides

2) Identify each type of tiles

The carriers



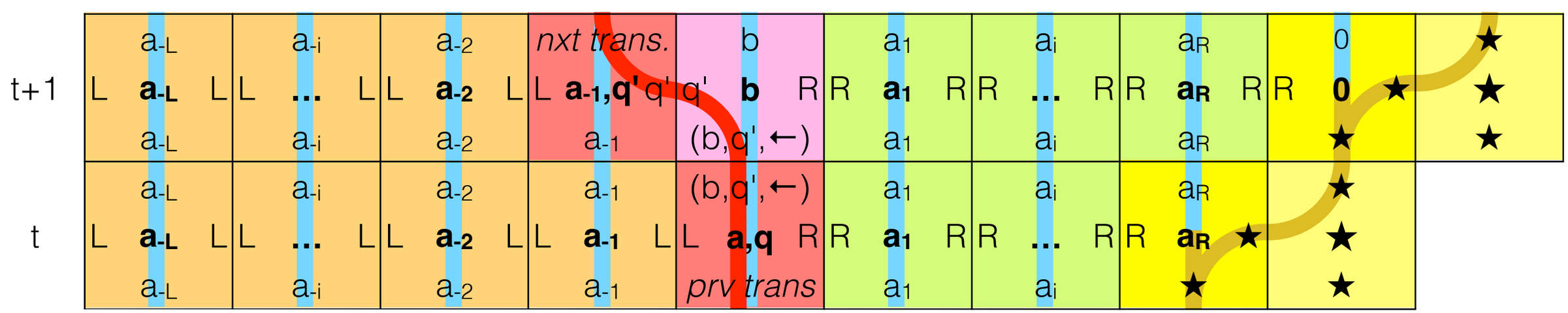
Tiles for (a,q) ⟼ (b,q',←)

Algorithmic self-assembly simulates any Turing machine at T°2 in 2D

1) Organize the information flow passing through the glues on the sides

2) Identify each type of tiles

The filling tiles



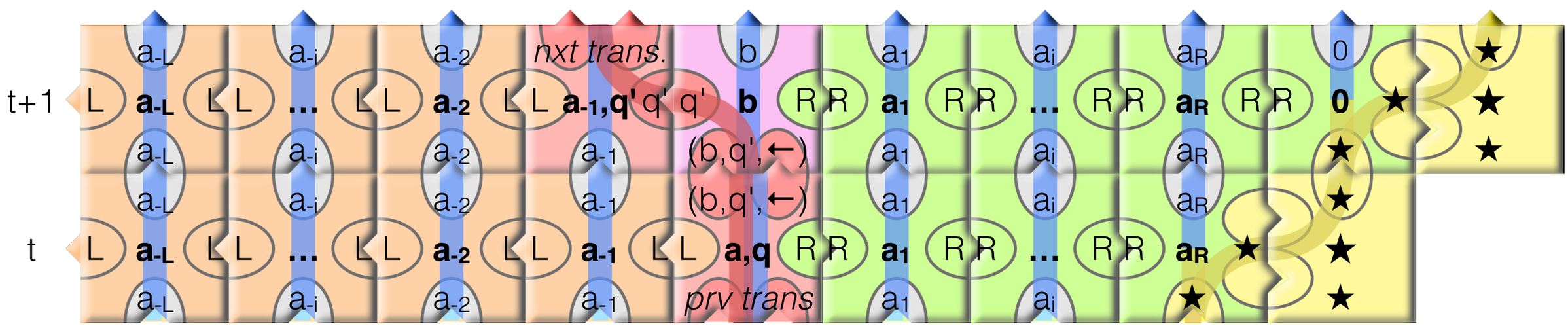
Tiles for (a,q) ⟼ (b,q',←)

Algorithmic self-assembly simulates any Turing machine at T°2 in 2D

1) Organize the information flow passing through the glues on the sides

2) Identify each type of tiles

3) Set the order, glue strength & colors to match the flow

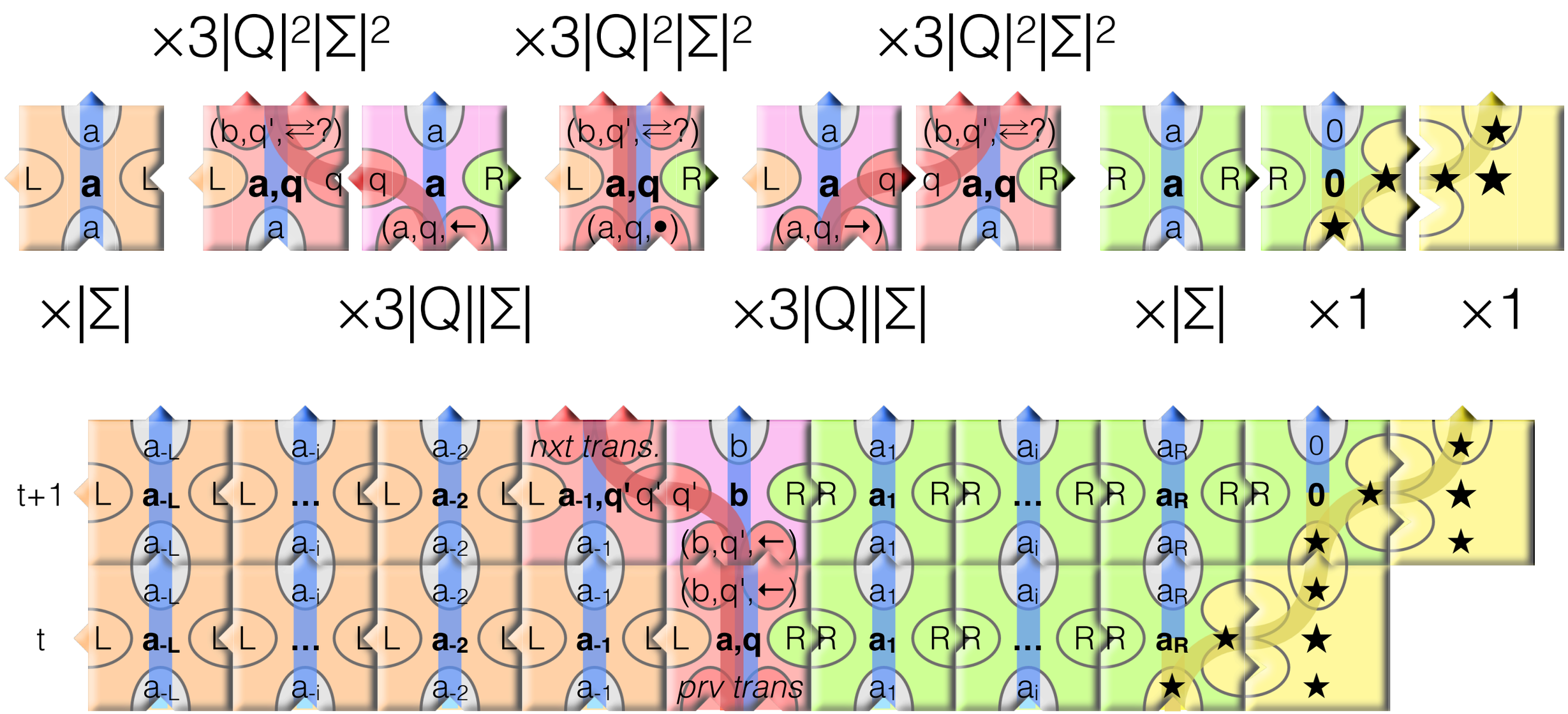


Tiles for (a,q) ⟼ (b,q',←)

The tileset

with size $O(|Q|^2|\Sigma|^2)$

The tiles for each transition $(a,q) \longmapsto (b,q',\rightleftarrows?)$



Encoding the input as a seed

- **Solution 1: hardcoding**
  Uses **n** tiles for n bits

A **0** B	B **0** C	C **1** C	C **1** D	D **0** E	E **1** F	F **1** G	G **1** H	H **0** I	I **0** J	J **1** K	K **1** ★

- Each tile requires **4 log n bits** to encode its glues, thus this encoding uses **4n log n bits** in total
- Can we do better, with less tiles ?

Kolmogorov complexity

- Given a universal Turing machine U:

$$K_U(x) = \min \{ |p| : U(p,\varepsilon) = x \}$$

is the size of the smallest program in U that outputs x

- **Fact.** $\forall U, \exists A$ s.t. $\forall x$, $K_U(x) \leq |x| + A$

  Proof. A is the size of the program "print" in U.

- **Theorem.** $K_U(x)$ **is independent of U**, indeed:

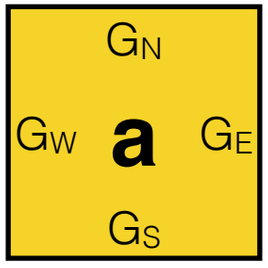
  $\forall U,U' \exists A,B$ s.t. $\forall x$, $K_{U'}(x) - A \leq K_U(x) \leq K_{U'}(x) + B$

  Proof. A and B are the sizes of the programs that execute U and U' respectively in U' and U.

Lower bounding the required number of tiles to encode the seed

- The Kolmogorov complexity **K(x)** is the size of the smallest program that outputs **x**

- Bit size of the encoding with **T** tiles is **$\leq 4\ T \log_2 T$**



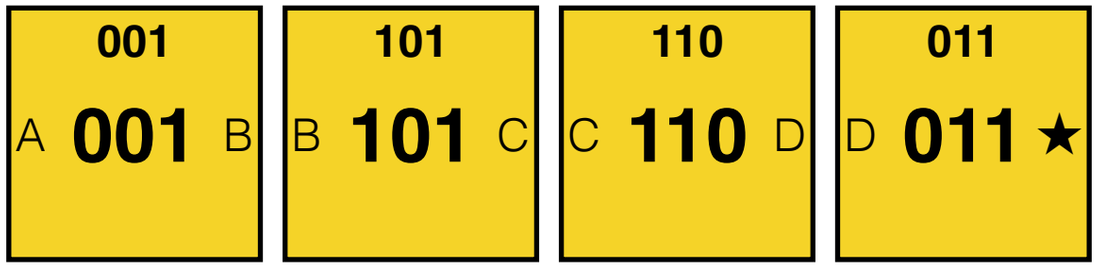
- A tileset that self-assembles x is a program that outputs x,

Thus: **$4T \log T \geq K(x)$**, I.e. **$T \geq K(x) / 4 \log K(x)$**

Unpacking a binary string

- Cut string **x** in **n / b** chunks of **b** bits and uncompress it

- Example: **x = 001 101 110 011** and **b = 3**

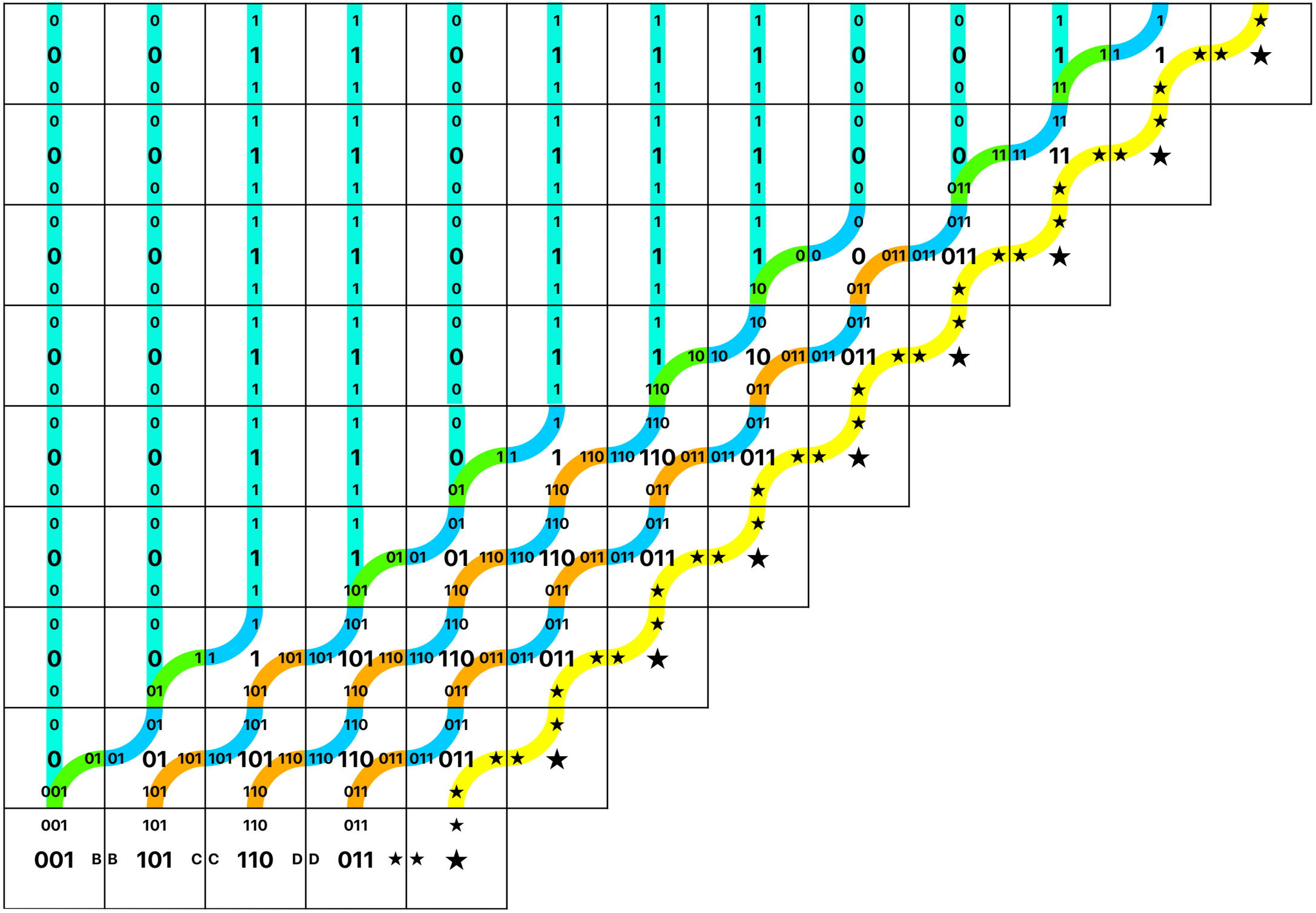


n / b tiles

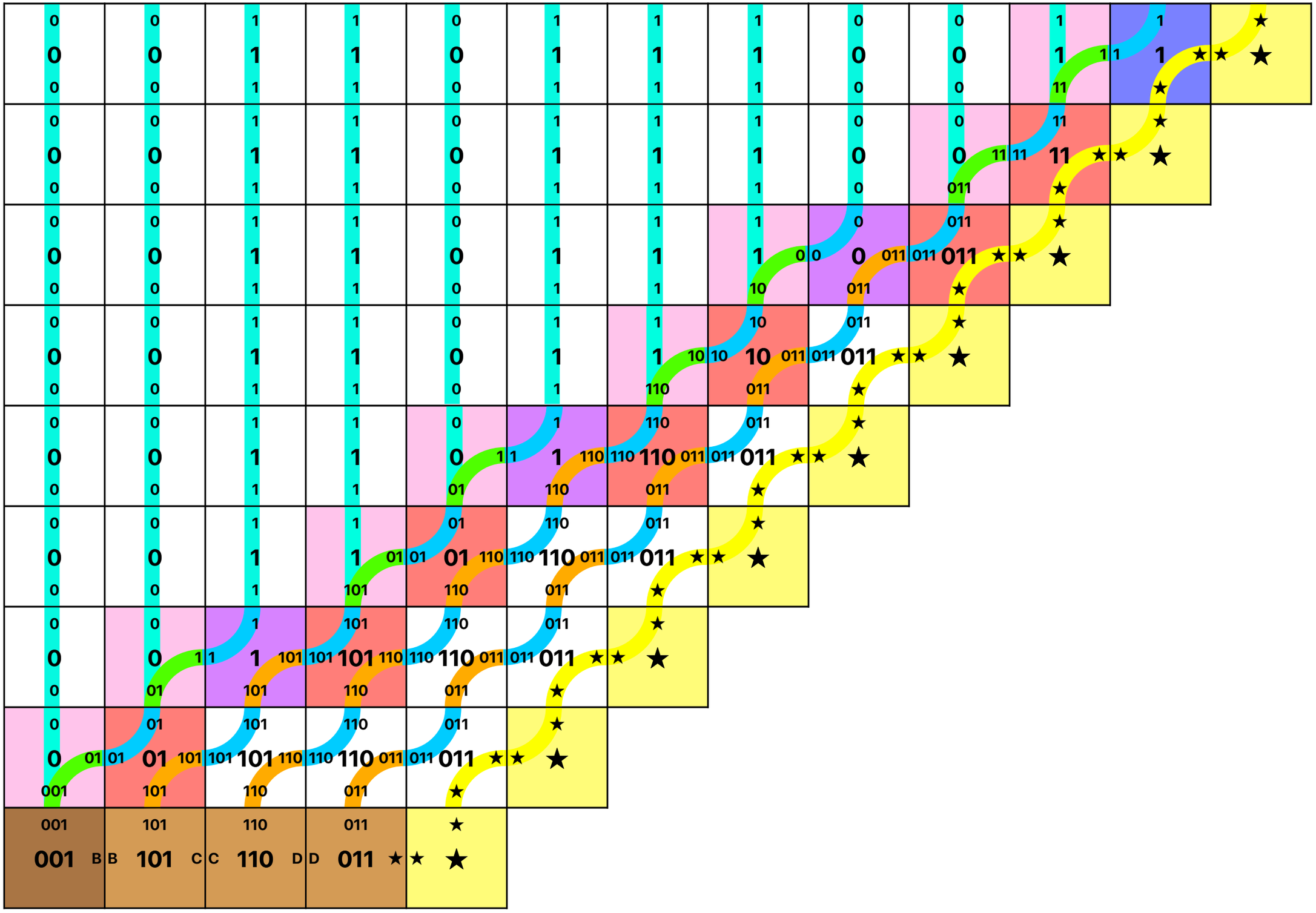
Unpacking a binary string

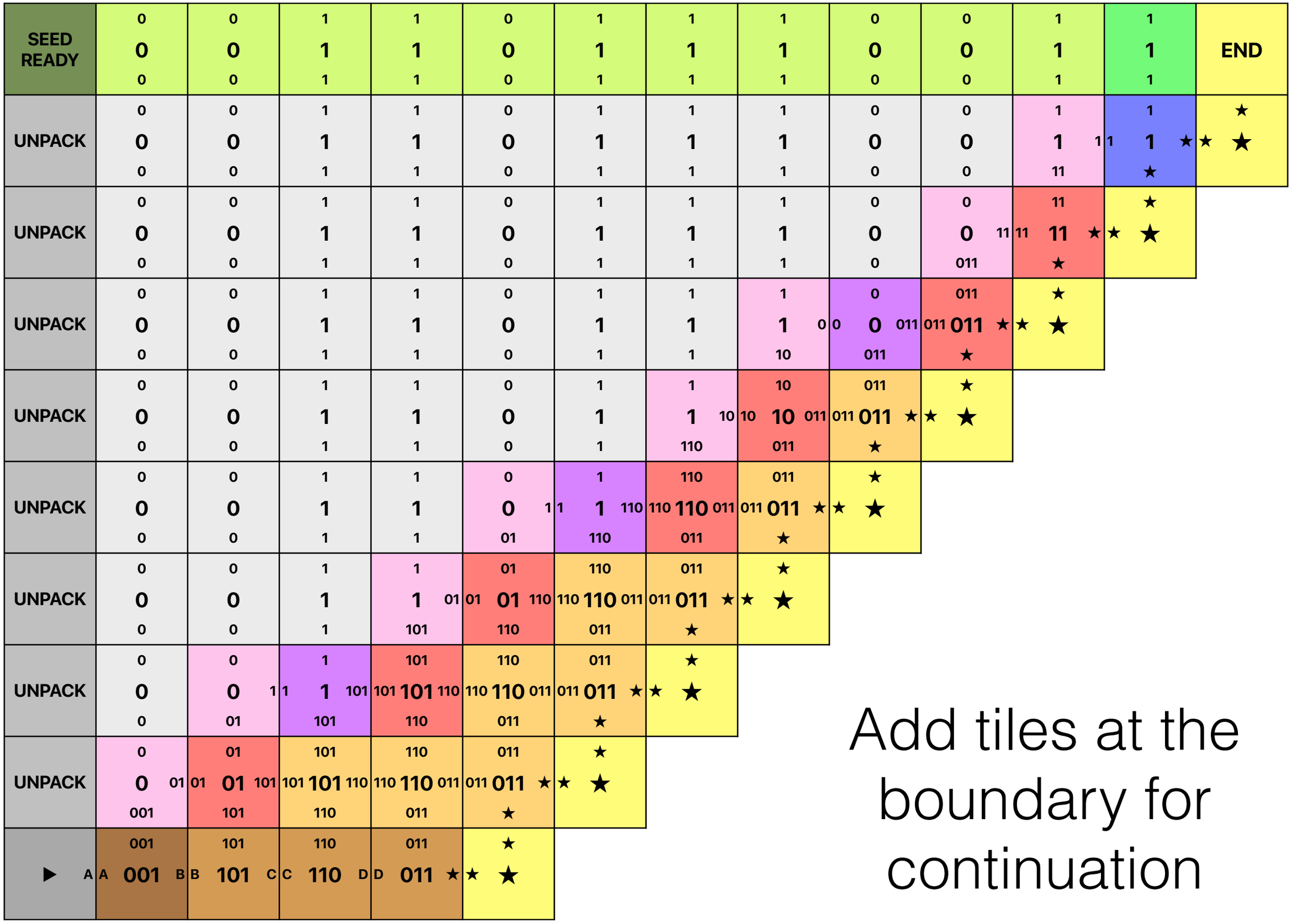
x unpacked	0	0	1	1	0	1	1	1	0	0	1	1	★
	0	0	1	1	0	1	1	1	0	0	11	★	
	0	0	1	1	0	1	1	1	0	011	★		
	0	0	1	1	0	1	1	10	011	★			
	0	0	1	1	0	1	110	011	★				
	0	0	1	1	01	110	011	★					
	0	0	1	101	110	011	★						
	0	01	101	110	011	★							
x packed	001	101	110	011	★								

Determine the flow of information

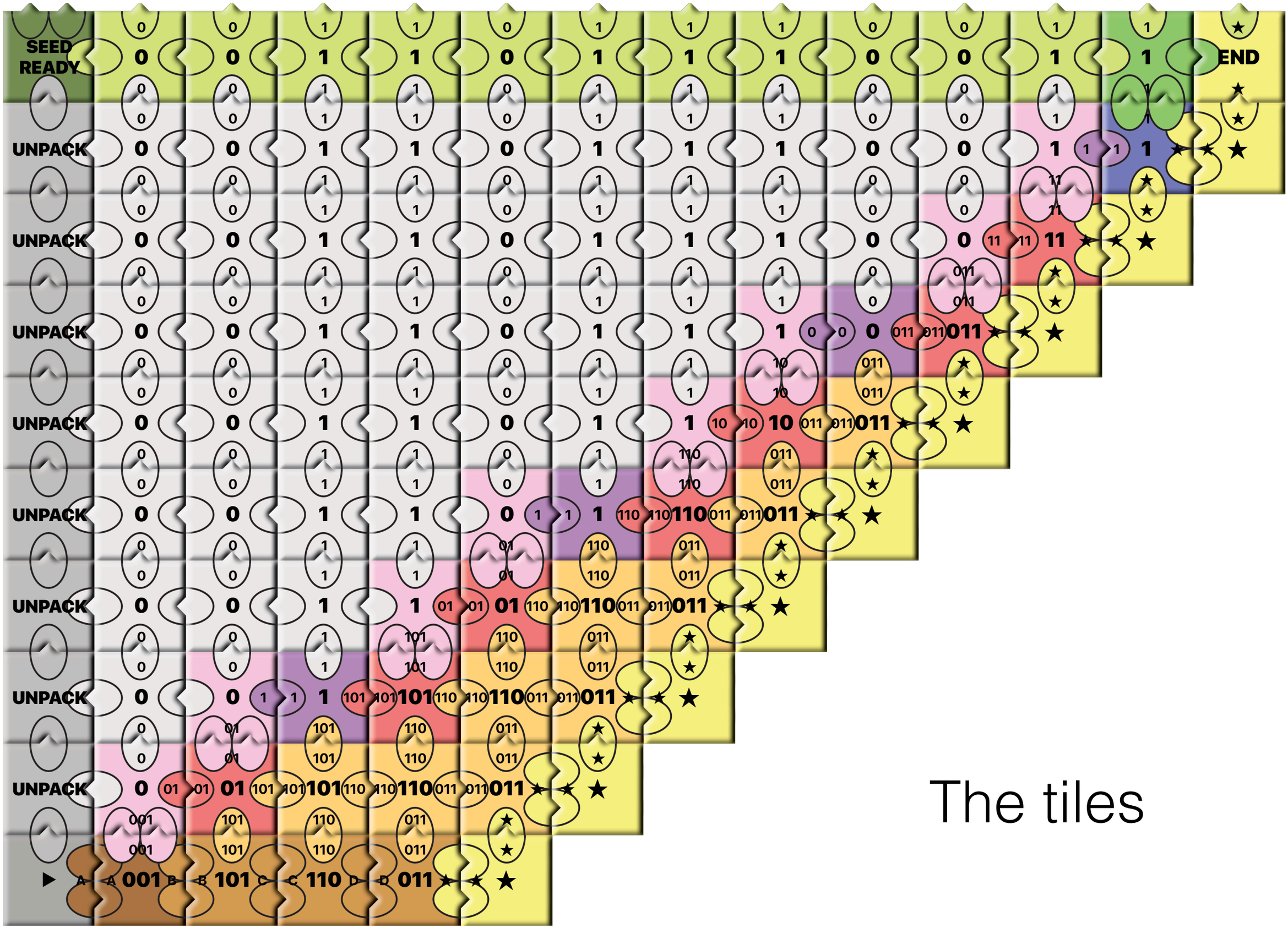


Determine the various tile types

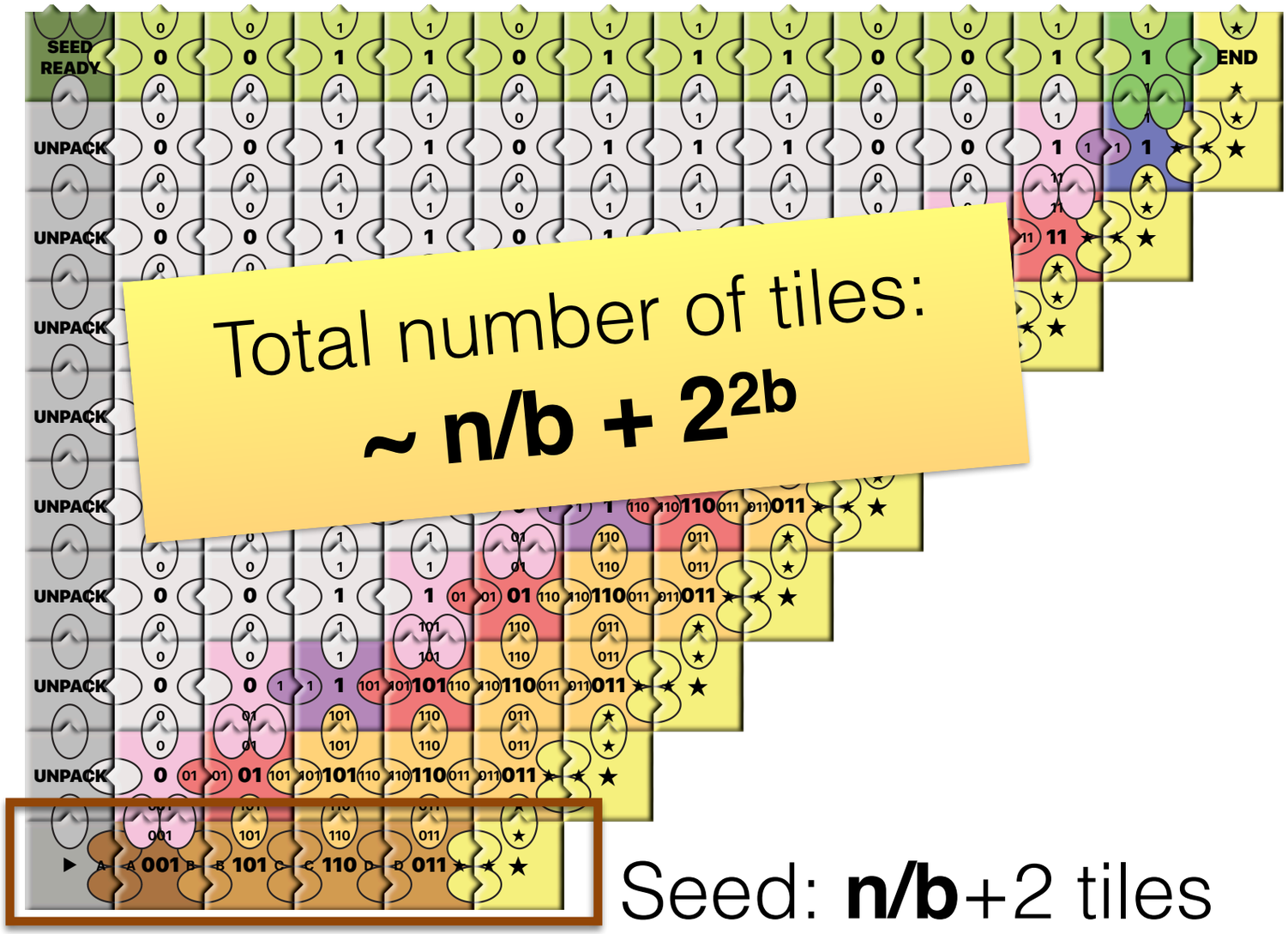




Add tiles at the boundary for continuation


SEED
READY
END
UNPACK
UNPACK
UNPACK
UNPACK
UNPACK
UNPACK
UNPACK
UNPACK
UNPACK
A A 001 B B 101 C C 110 D D 011

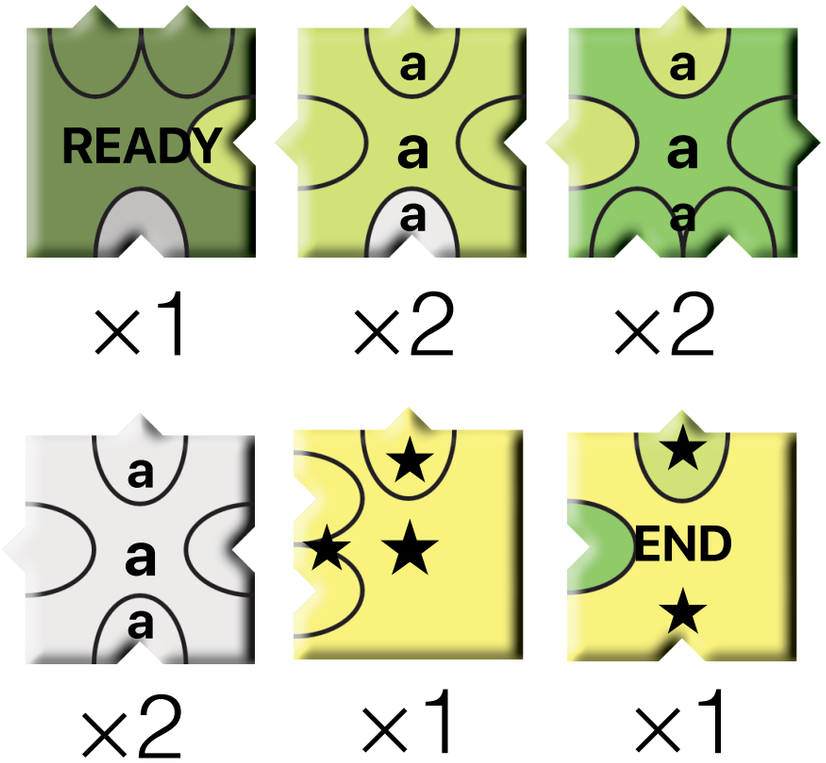
The tiles



Total number of tiles:

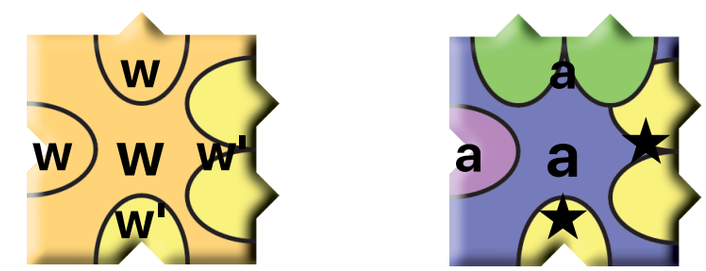
~ n/b + 2^{2b}

Seed: **n/b**+2 tiles



×1 ×2 ×2

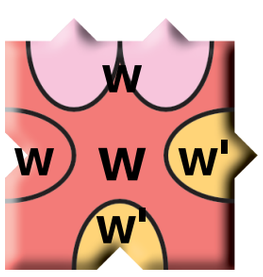
×2 ×1 ×1



×2^{2b} ×2

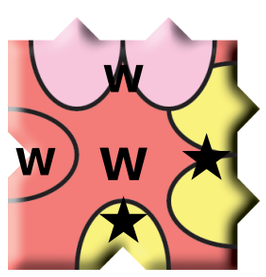
$2 \le |w| \le b$

$|w'| = b$



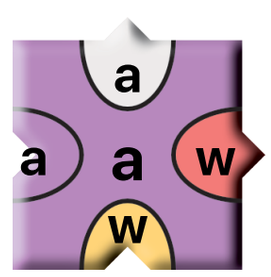
×2^{2b}

$2 \le |w| \le b$



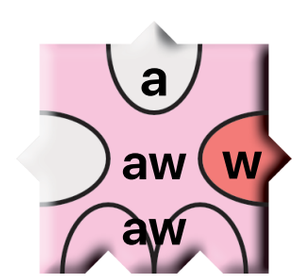
×2^{b}

$|w|=b$

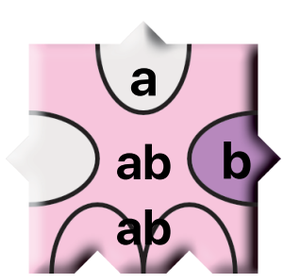


×2^{b+1}

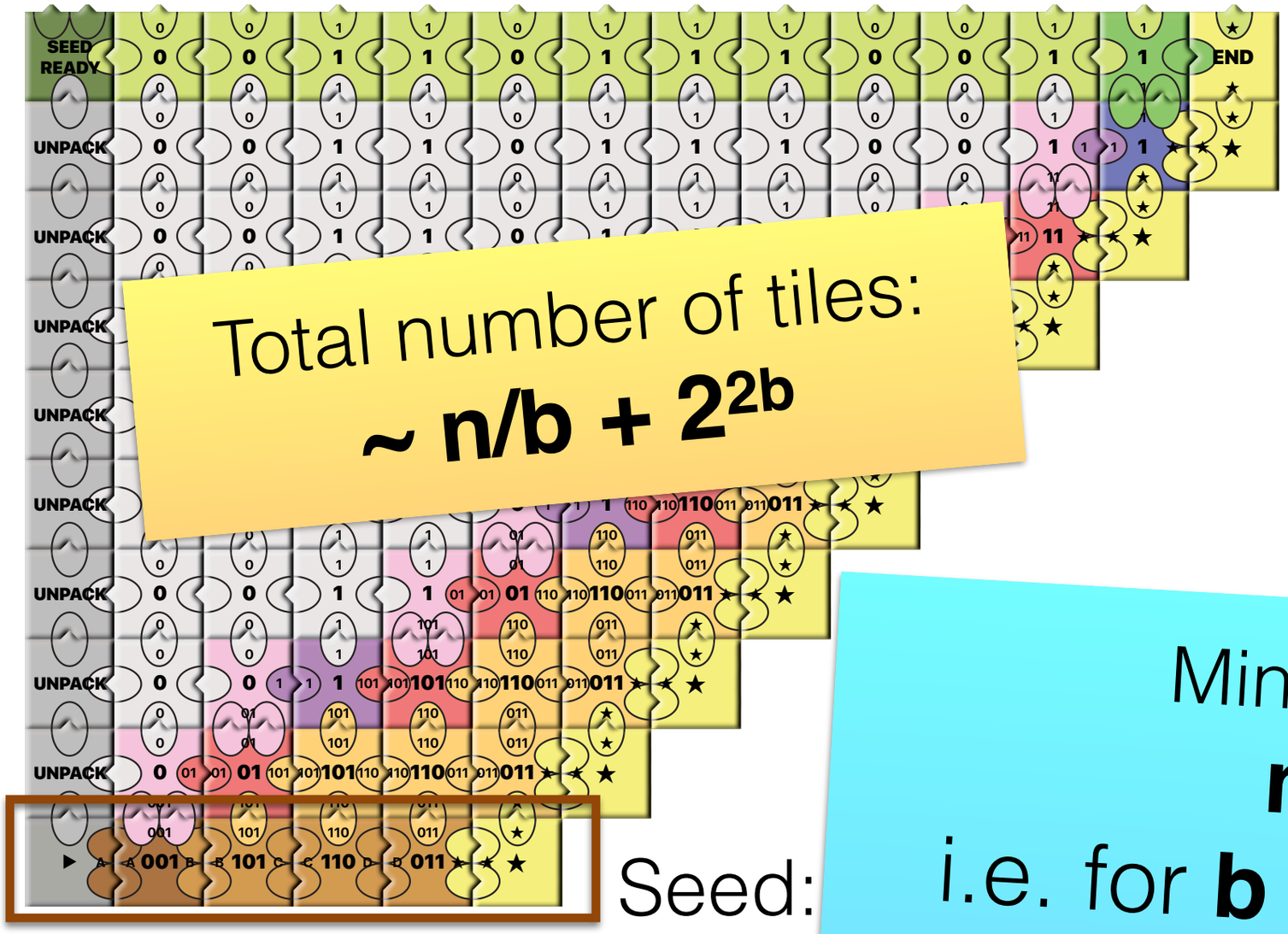
$2 \le |w| < b$



×2^{b}

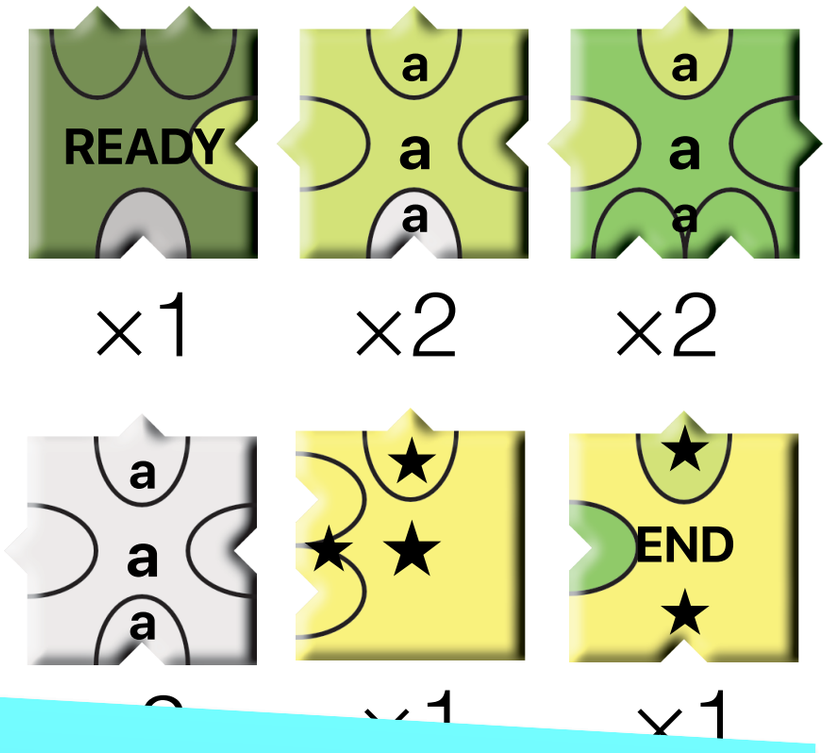


×4



Total number of tiles:

~ n/b + 2^{2b}



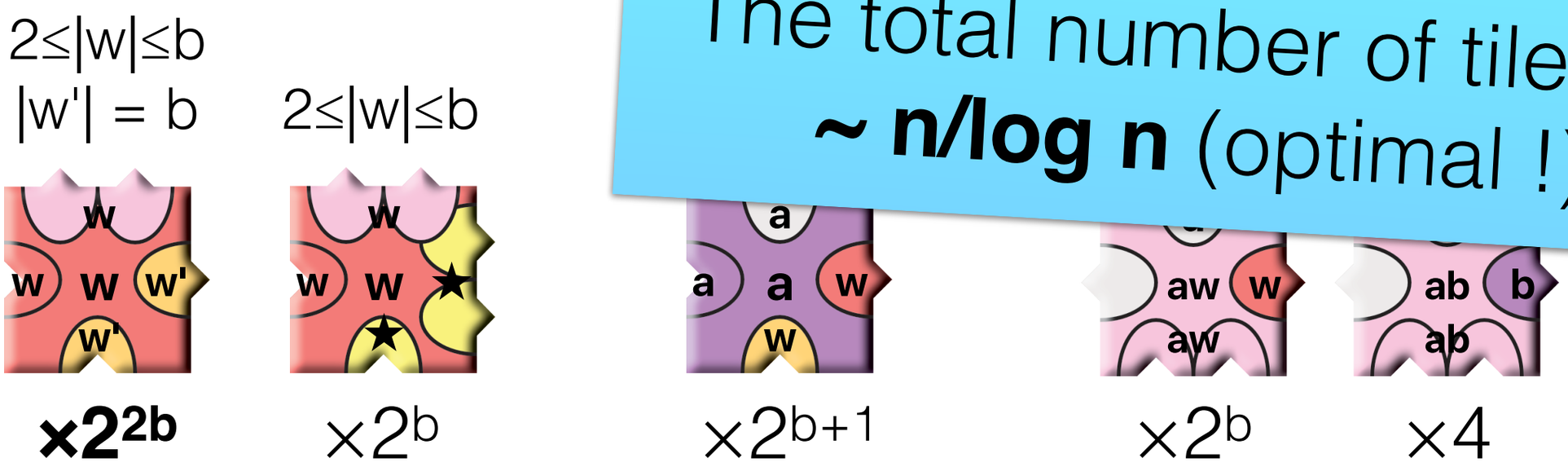
Minimized when

$n/b = 2^{2b}$

i.e. for **$b = \log(n/\log n)/2$**

The total number of tiles is:

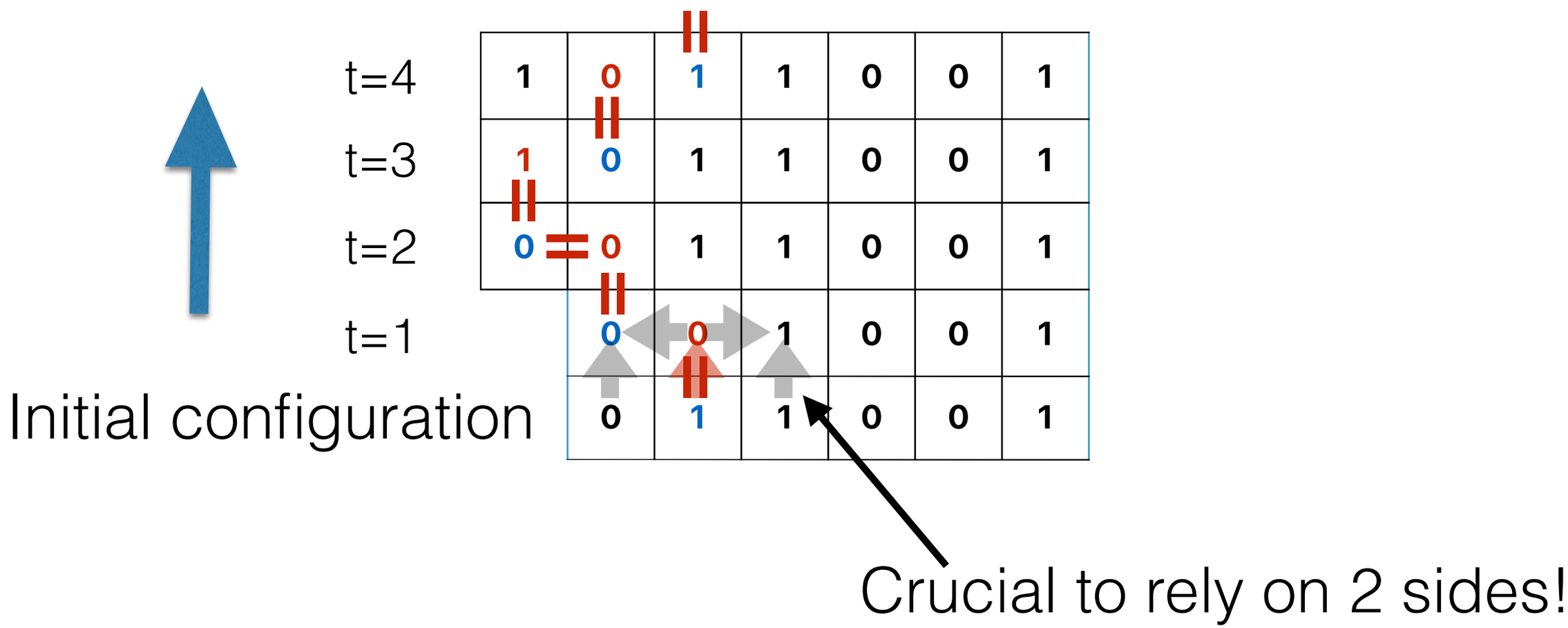
~ n/log n (optimal !)



What about $T^\circ=1$?

Does algorithmic self-assembly simulate Turing machine at **T°1** in 2D?

- How to read and propagate the position of the head?



Does algorithmic self-assembly simulate Turing machine at **T°1** in 2D?

- **Theorem.** [Meunier, Regnault, 2015]
  Any deterministic T°1 tile set can be pumped outside a fixed radius in 2D

- **Corollary.** No T°1 tilesystem is Turing complete in 2D

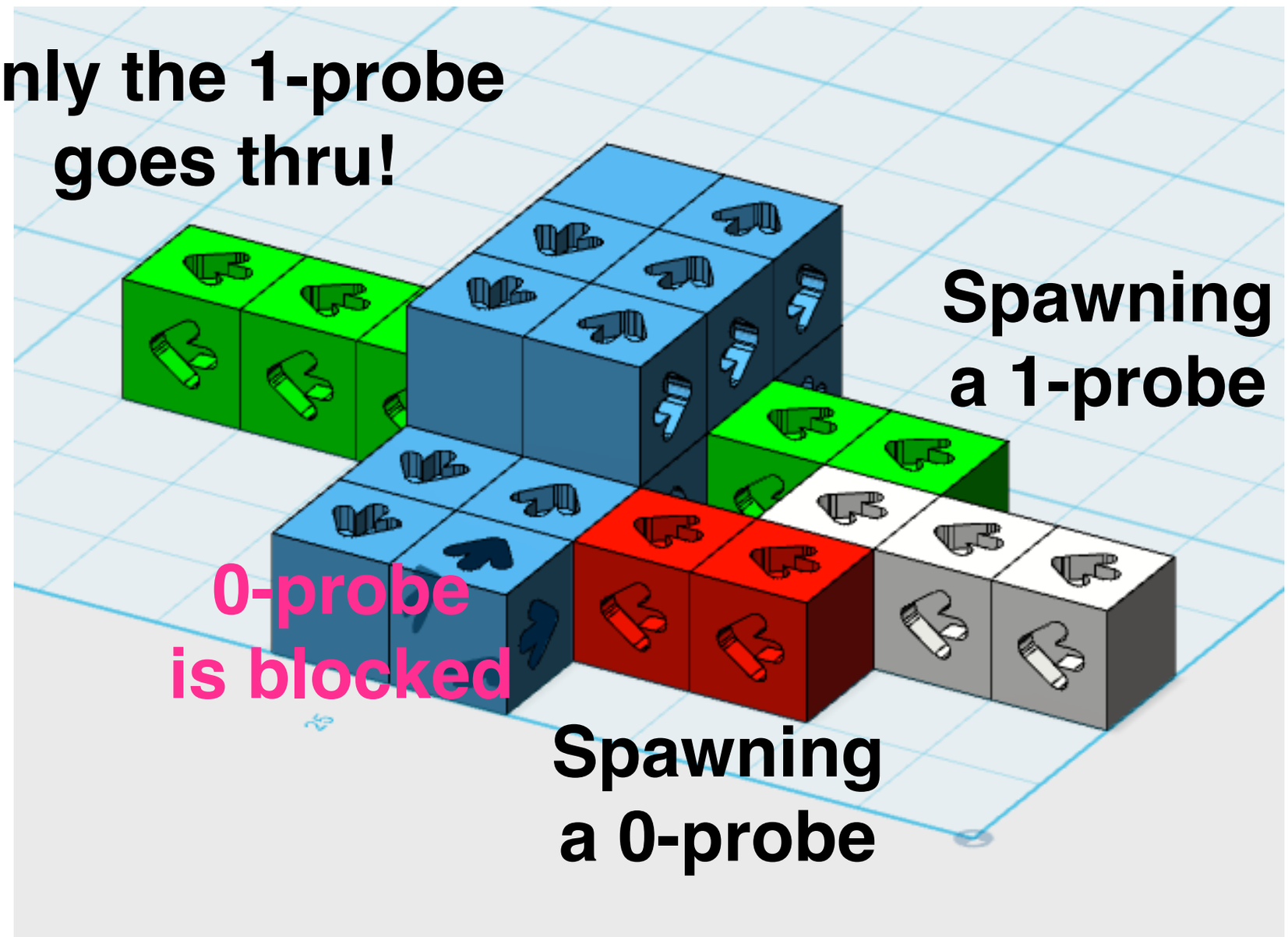
Algorithmic self-assembly simulates any Turing machine at T°1 in 3D

- **Theorem.** *[Cook, Fu, Schweller, 2011]* There is a T°1 tile set that simulates any Turing machine with 2-layers in 3D.

- *Key beautiful idea:* Blocking probe crystal

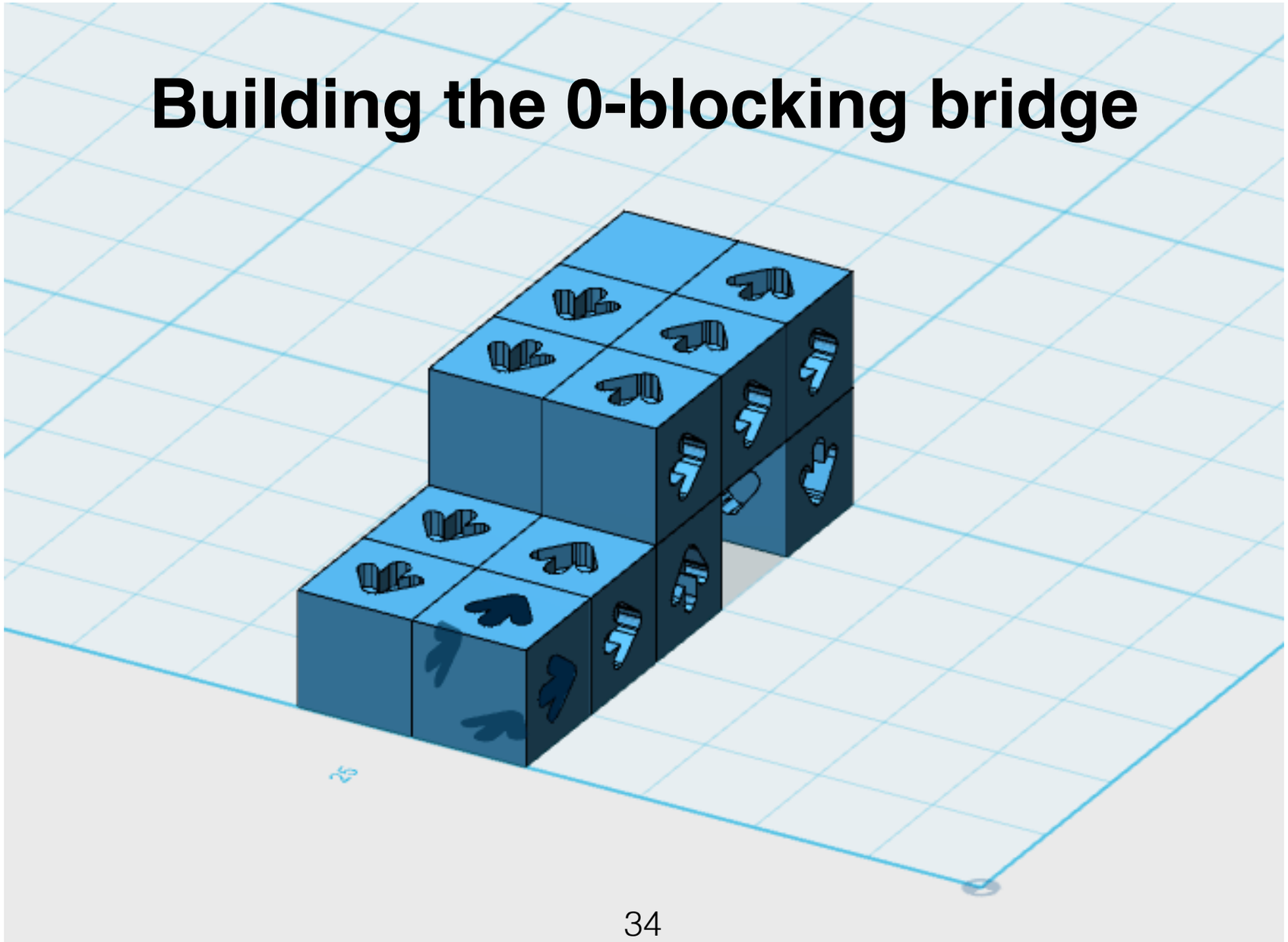
Algorithmic self-assembly simulates any Turing machine at T°1 in 3D

- *Key beautiful idea:* Blocking probe crystal



Algorithmic self-assembly simulates any Turing machine at T°1 in 3D

- *Key beautiful idea:* Blocking probe crystal



Algorithmic self-assembly simulates any Turing machine at T°1 in 3D

- *Key beautiful idea:* Blocking probe crystal

We can thus read and write 0 and 1 on the "next tape" !

Crystals are Turing complete !!!

An experimental realization of a universal computer

Conclusion

- A lot of new models
- A new kind of geometric algorithmic
- A lot of implication in biology and bio-engineering
- Lots of extensions: mixed dynamics, errors,…
  More on fixing errors at the last lecture