HW1 Molecular Programming

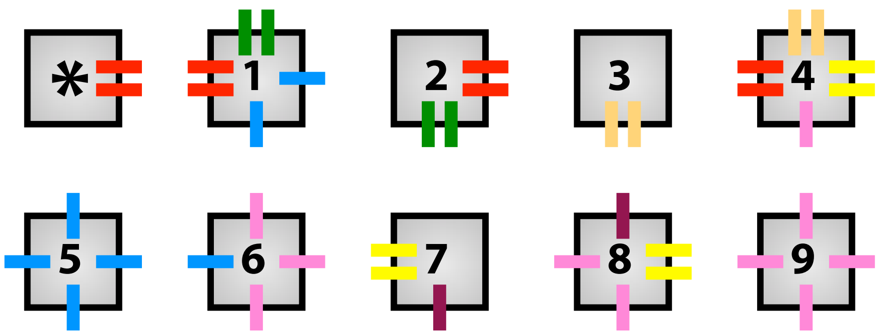
MPRI 2.11.1

22.10.2018 - **Due on Mon. 29/10 before 12:45**

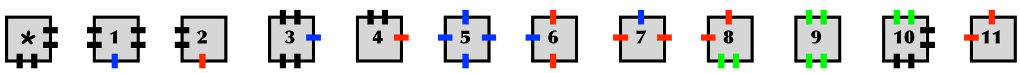
You are asked to complete the exercise marked with a [★] and to send me your solutions to:
`nicolas.schabanel@cnrs.fr`
as a PDF file named `HW1-Lastname.pdf` on **Mon. 29/10 before 12:45**.

■ **Exercise 1 (Algorithmic Self-Assembly).** Recall that the self-assembly process consists in, given a finite tileset (with infinitely many tiles of each type), starting from the seed tile (marked with a ★), glueing tiles with matching colors to the current aggregate so that each new tile is attached by at least *two* links to the aggregate (either on the same border or on two borders). Recall that a shape is *final* if no tiles can be attached to it anymore.

▶ ***Question* 1.1)** *What is the exact family of final shapes self-assembled by the following tileset? (No proof nor justification is asked.) Indicate the local order of assembly by drawing arrows over the tiles of a generic final shape. Which are the two competing tiles that decide the size of the resulting final shape?*



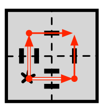
■ **Exercise 2.** What is the family of shapes built by this tileset at temperature $T^\circ = 2$? *(no justification asked)*



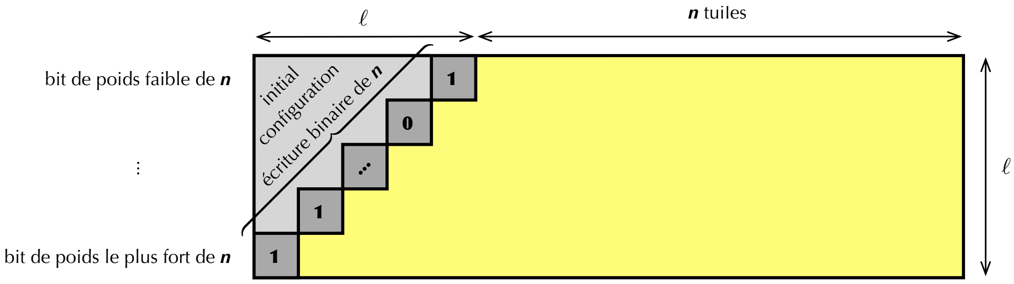
Indicate the assembly order with arrows[1] on a generic production. Is this a well-ordered tileset?

■ **Exercise 3 (Counter at $T^\circ = 2$ (★)).** Given an integer n, and an seed configuration consisting of an isosceles rectangle triangle isocèle of side $\ell = \lceil \log_2 n \rceil$ where the bits of n are encoded on the diagonal as shown in grey bellow.

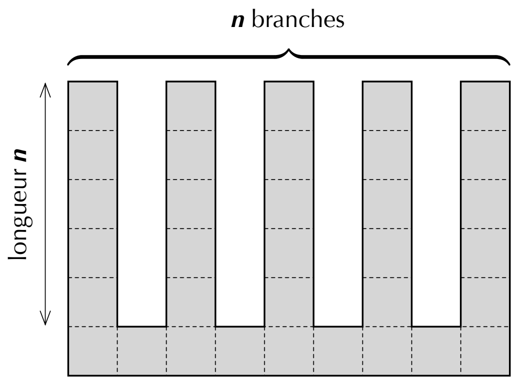
Propose a well-ordered (finite) tileset which assembles the yellow at $T^\circ = 2$ to realise a rectangle of size $\ell \times (n + \ell)$. Carefully indicate the position of the glue of strength 1 and 2

[1] Draw an arrow $(i, j) \rightarrow (i', j')$ iff tile (i, j) is attachedbefore tile (i', j'): for instance, 

on the diagonal of the seed configuration. Indicate the assembly order$^{(1)}$. What does the tiles encode?



■ **Exercise 4.** Propose a staged assembly scheme at temperature $T^\circ = 1$ of the shape family E of candelabrums with n branches of length n.



Describe the tiles, glues, their number, the number of stages and the number of different bechers needed. Give an illustration of the stages to build a generic production.

■ **Exercise 5.** Assume a random Poisson model where the random time X between two consecutive appearances of a tile of a given type τ at a given empty location follows an exponential law: $p(x) = c \cdot e^{-cx}$ where $c > 1$ is the concentration of the tiles of type τ. We want to prove the following theorem:

Theorem 1 (Adleman et al, 2001). *Consider an ordered tile system $\mathcal{T}$ that assembles deterministically a single shape S. Let $\prec$ be the partial order of the assembly, i.e. such that $(i,j) \prec (k,l)$ if the tile at position (i,j) is attached before the tile at (k,l) by $\mathcal{T}$. With very high probably, the assembly time of a shape S by $\mathcal{T}$ is:*

$$O(\gamma \times \mathrm{rank}(S))$$

where γ only depends on the concentrations and rank(S) *is the highest rank in the shape S (i.e. the length of the longest path in $\prec$).*

▶ ***Question 5.1)*** *Let X be an exponential random variable such that $p(X = x) = ce^{-cx}$ for all real $x \geqslant 0$, for some $c > 0$. Show that X is* memoryless, *i.e. for all $u, t \geqslant 0$,*

$$p(X = t + u | X \geqslant u) = p(X = t)$$

Let T be the assembly time of the shape S, i.e. the time at which the last tile of shape S is attached. We denote by $w(P)$ the random variable for the weight of a $\prec$-path P, defined as: $w(P) \sum_{(i,j)\in P} X_{i,j}$.

▶ ***Question* 5.2*)*** *Let $X_{i,j}$ be the independant exponential random variable for the time between two consecutive appearances of the tile to be attached at position (i, j) in S. Show that:*

$$T = \max_{\prec\text{-path } P} w(P)$$

▶ ***Question* 5.3*)*** *Let $X_1, \ldots, X_n$ be n independent exponential variables s.t. $p(X_i = x) = c_i e^{-c_i x}$ with $c_i > 1$. Show that there is γ which depends only of $\min_i c_i$ such that:*

$$\Pr\{X_1 + \cdots + X_n \geqslant \gamma \cdot n\} \leqslant 1/4^n$$

▷ Hint. *Note that $\mathbb{E}[e^{X_i}] < \infty$ and apply Markov inequality to $Z = e^{X_1+\cdots+X_n}$.*

▶ ***Question* 5.4*)*** *Conclude.*

■ **Exercise 6 (Minimum number of tile types).**

▶ ***Question* 6.1*)*** *Show by a careful case study that no tileset with < 5 tile types can assemble at $T^\circ = 2$ the $n \times n$-squares family for all $n \geqslant 2$.*

▶ ***Question* 6.2*)*** *Show by a careful case study that no tileset with < 6 tile types can assemble at $T^\circ = 2$ the $n \times m$-rectangles family for all $n, m \geqslant 2$.*