HW2 Molecular Programming

MPRI 2.11.1

16.10.2019 - **Due on Wed. 23/10 before 12:45**

You are asked to complete the exercise marked with a [★] and to send me your solutions to:
`nicolas.schabanel@ens-lyon.fr`
as a PDF file named `HW2-Lastname.pdf` on **Wed. 23/10 before 12:45**.

■ **Exercise 1 (Exponential random variables & kTAM implementation).** Recall that an *exponential random variable* X with parameter $\lambda > 0$ is defined by: $(\forall x \geqslant 0)$ $\Pr\{X \geqslant x\} = e^{-\lambda x}$.

▶ ***Question 1.1)*** *Compute* $\mathbb{E}[X]$.
▷ Hint. *Recall that if X is a non-negative random variable, then* $\mathbb{E}[X] = \int_0^\infty \Pr\{X \geqslant x\}dx$.

▶ ***Question 1.2)*** *Show that the exponential distribution is* memoryless, *i.e. if X is exponentially distributed with parameter λ, then* $(\forall t, u \geqslant 0)$ $\Pr\{X \geqslant t+u \,|\, X \geqslant t\} = \Pr\{X \geqslant u\}$.

Let X and Y be two independent exponentially distributed random variables with respective parameters λ and μ.

▶ ***Question 1.3)*** *Show that* $\min(X,Y)$ *is also exponentially distributed. What is its parameter?*

▶ ***Question 1.4)*** *What is the probability that* $\min(X,Y) = X$*?*

▶ ***Question 1.5)*** *Same questions as the two above for n indenpendent exponentially dsitributed variables $X_1, \dots, X_n$ with parameters $\lambda_1, \dots, \lambda_n$.*

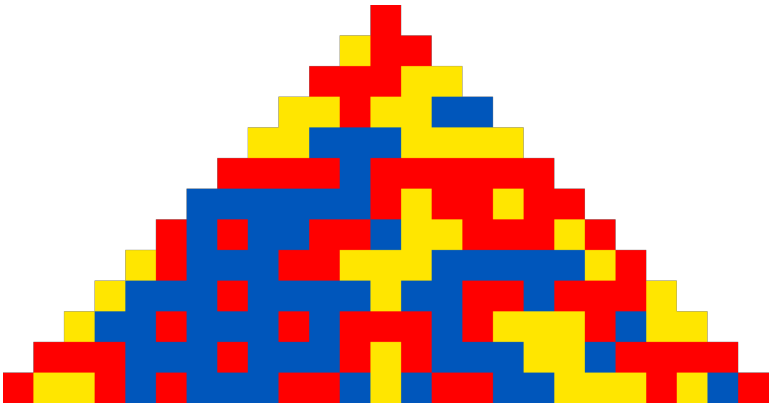
▶ ***Question 1.6)*** *Assume that a non-negative random variable X is given by its tail distribution* $F(x) = \Pr\{X \geqslant x\}$. *Show that X is identically distributed as $F^{-1}(U)$ where U is a uniform random variable in $[0,1]$.*

Describe how to sample an exponential random variable of rate λ.

▶ ***Question 1.7)*** *Propose an algorithm together with a data structure to implement the kTAM model with attachement rate $r_f = k_f[Strand] = k_f\, e^{-G_{mc}}$ and detachment rate $r_{s,b} = k_f\, e^{-b \cdot G_{se}}$ where b is the number of bonds made by the strand with the current agregate.*

Use parameters $k_f = 10^6/M/sec$, $G_{mc} = 12.9$ and $G_{se} = 6.5$ for the algorithmic phase.

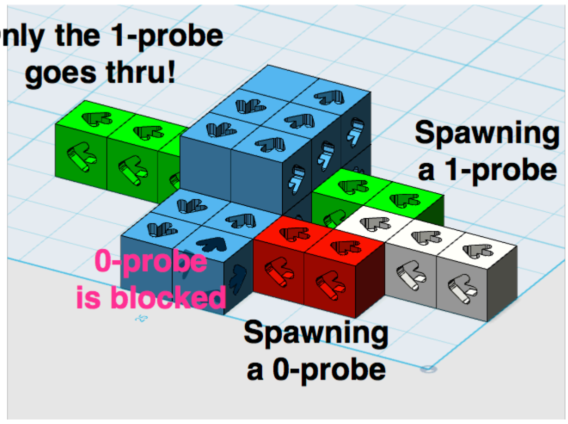
■ **Exercise 2 (Tileset for simulating cellular automata (★)).** A cellular automaton consists of a finite set of *states* Q, a function $f : Q^3 \to Q$, called the *rule*, and an initial configuration $c^0 \in Q^*$. The configuration at time $t+1$ is obtained from the configuration at time t as follows: $c_i^{t+1} = f(c_i^t, c_{i+1}^t, c_{i+2}^t)$ for $0 \leqslant i < |c^t| - 2$. The calculation stops at the first time T such that $|c^T| < 3$ and the result of the computation is c_0^T. A classic visualization of the computation of a cellular automaton consists of a pyramid where the bottom line is the initial configuration and time goes upwards. Here is an example:



for the rule $f(x,y,z) = \begin{cases} 🟨 & \text{if } \{x,y,z\} = \{🟥\} \text{ or } \{🟨,🟦\} \\ 🟦 & \text{if } \{x,y,z\} = \{🟨\} \text{ or } \{🟦,🟥\} \\ 🟥 & \text{if } \{x,y,z\} = \{🟦\} \text{ or } \{🟨,🟥\} \text{ or } \{🟨,🟦,🟥\} \end{cases}$

▶ ***Question 2.1)*** *Propose a finite tileset whose self-assembly simulates the computation of any Q-state cellular automata from any initial configuration and whose size is independent of the initial configuration length. Give a generic example of the execution of your assembly for generic computation steps. Give the number of variants of each tile type as a function of $|Q|$. Provide the procedure which selects the tiles used to simulate a given Q-state cellular automaton.*
▷ Hint. *Do you need upscaling? Consider reshaping the pyramid to simplify your design.*

■ **Exercise 3 (Probabilistic simulation Turing Machine at $T^\circ = 1$ in 2D).** Recall that in 3D, for any single-tape binary-alphabet Turing machine M, there is a tile set which simulates M using a clever trick to encode 0s and 1s. These are encoded with bridges and read using two probes where only one go through the bridge:



▶ ***Question 3.1)*** *By adjusting the concentrations (and thus the rate at which the different tiles attached), describe a tile set together with concentrations for each tile type, that simulates a given single-tape binary-alphabet Turing machine M with an arbitrary small error ε for each symbol read in 2D at temperature $T^\circ = 1$.*