HW3 MPRI 2.11.1 Molecular Programming

23.10.2019 - **Due on Wed. 6/11 before 11:45**

You are asked to complete questions 1.1) and 1.2) and to send me your solutions to:
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as a PDF file named **HW3-Lastname.pdf** on **Wed. 6/11 before 11:45**.

■ **Exercise 1 (Window Movie Lemma).** We investigate the computation power of tile assembly at temperature $T^\circ = 1$. We allow *mismatches*, i.e. a tile can be added to the current aggregate as soon as it is attached by *at least one side* to the current aggregate for which the glues match (the other sides in contact can have mismatching glues). Unless specified explicitly otherwise, all assemblies take place at $T^\circ = 1$ in this exercise.

Let us first consider a (finite) tile set $\mathcal{T}$ which only assembles unidimensional segments of size $1 \times \ell$ for some $\ell \geqslant 1$ starting from its seed tile. Let $\tau = |\mathcal{T}|$ denote the number of tile types in $\mathcal{T}$ in all of the following. Recall that the *final productions* of a tileset $\mathcal{T}$ are the shapes corresponding to every possible assembly of tiles from $\mathcal{T}$ starting from the seed tile of $\mathcal{T}$ and where no more tile can be added.

▶ ***Question 1.1)*** *Show (and explicit) that there is a constant $k(\tau)$, which depends only on τ, such that if a segment of size $1 \times \ell$ with $\ell \geqslant k(\tau)$ is a final production of $\mathcal{T}$, then there is an integer $1 \leqslant i < k(\tau)$ such that all the segments $1 \times (\ell + n \cdot i)$ are also final productions of $\mathcal{T}$ for all $n \geqslant -1$. If so, we say that the tile set $\mathcal{T}$ is* pumpable.

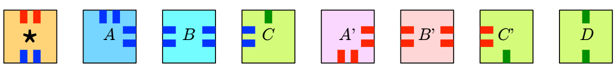
Let us now consider a (finite) tile set $\mathcal{T}$ whose final productions are 2-thick rectangles of size $2 \times \ell$ for some $\ell \geqslant 1$.

▶ ***Question 1.2)*** *Show (and explicit) that there is a constant $k_2(\tau)$, which depends only on τ, such that if a 2-thick rectangle of size $2 \times \ell$ with $\ell \geqslant k_2(\tau)$ is a final production of $\mathcal{T}$, then $\mathcal{T}$ is pumpable, i.e. that there is an integer $1 \leqslant i < k_2(\tau)$ such that all the 2-thick rectangles $2 \times (\ell + n \cdot i)$ are also final productions of $\mathcal{T}$ for all $n \geqslant -1$.*
▷ Hint. *Pay attention to the order in which the tiles are attached, make sure that the pumped structure can indeed self-assemble.*

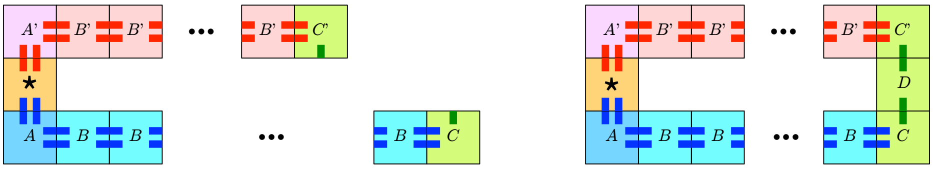
Let us now generalise and consider a (finite) tile set $\mathcal{T}$ whose final productions are q-thick rectangles of size $q \times \ell$ for some $\ell \geqslant 1$.

▶ ***Question 1.3)*** *Show (and explicit) that there is a constant $k_q(\tau)$, which depends only on τ, such that if a q-thick rectangle of size $q \times \ell$ with $\ell \geqslant k_q(\tau)$ is a final production of $\mathcal{T}$, then $\mathcal{T}$ is pumpable, i.e. that there is an integer $1 \leqslant i < k_q(\tau)$ such that all the q-thick rectangles $q \times (\ell + n \cdot i)$ are also final productions of $\mathcal{T}$ for all $n \geqslant -1$.*

Consider the following tile set $\mathcal{U} = \{\bigstar, A, B, C, A', B', C', D\}$ at $T^\circ = 2$ for which $\bigstar$ is the seed tile:



The final productions of $\mathcal{U}$ at $T^\circ = 2$ consist of two arms which are either 1) of different lengths and then don't touch eachother; or 2) of equal length and then there is a tile D that makes contact between them:



► ***Question* 1.4*)*** *Show that no tile set can simulate* intrinsically *at* $T^\circ = 1$*, the dynamics of* $\mathcal{U}$ *at* $T^\circ = 2$*.*
▷ Hint. *As a simplification, consider that in an intrinsic simulation, all megacell corresponding to an empty position in the simulated system must never be filled by more than* 30% *of tiles, and all megacell corresponding to a non-empty position in the simulated system must be filled at* 100% *by tiles. If you have time left: how would you waive these assumptions?*