

Intrinsic Universality in tile assembly

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Universal Tileset at $T^{\circ 2}$

A universal tile set to build any (assemblable) shape

Is there a universal tileset at $T^\circ=2$?

- Yes! Up to rescaling

The tile assembly model is intrinsically universal

David Doty*

Jack H. Lutz†

Scott M. Summers¶

Matthew J. Patitz‡

Damien Woods||

Robert T. Schweller§

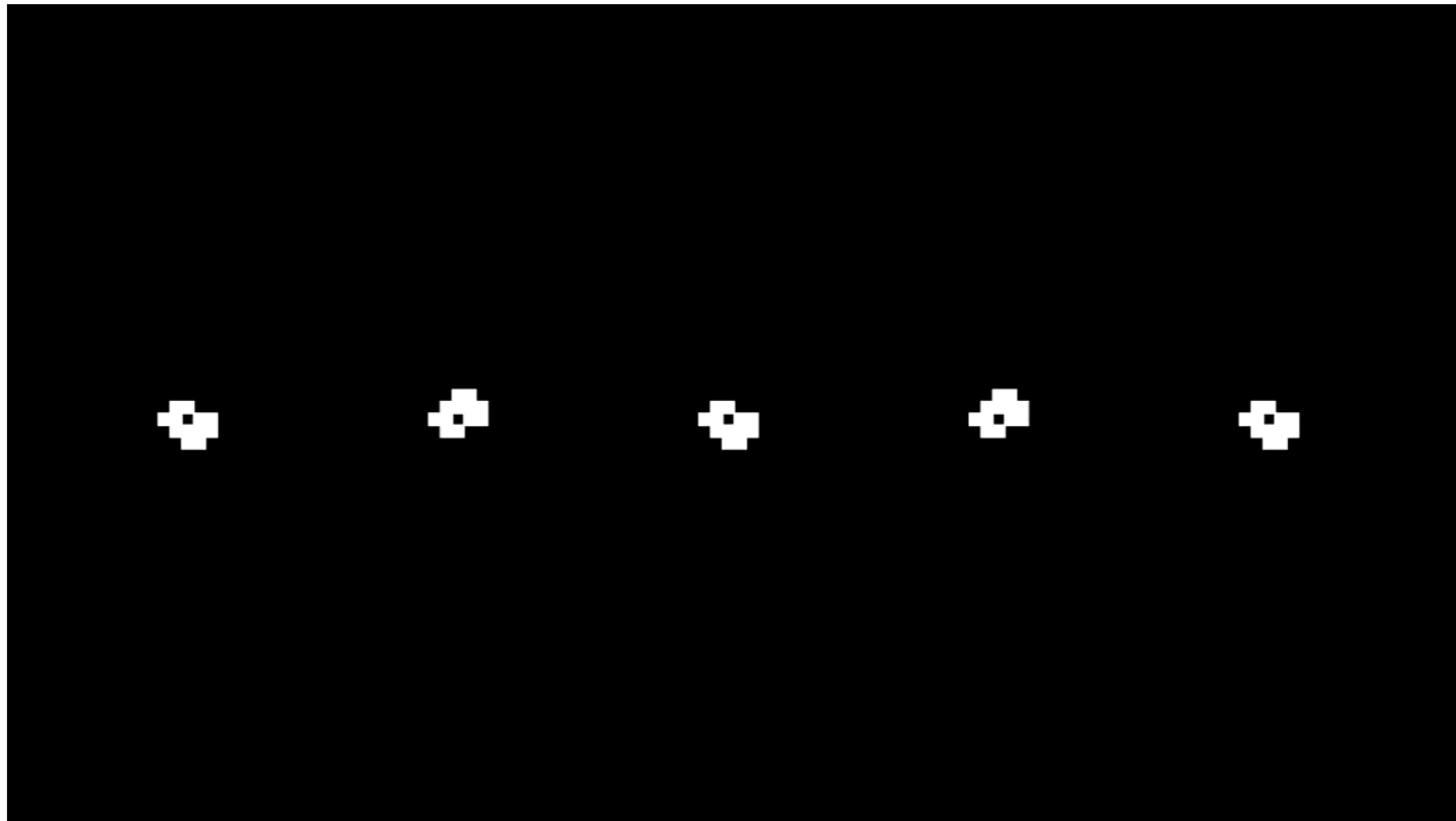
Abstract

We prove that the abstract Tile Assembly Model (aTAM) of nanoscale self-assembly is *intrinsically* universal. This means that there is a single tile assembly system \mathcal{U} that, with proper initialization, simulates any tile assembly system \mathcal{T} . The simulation is “intrinsic” in the sense that the self-assembly process carried out by \mathcal{U} is exactly that carried out by \mathcal{T} , with each tile of \mathcal{T} represented by an $m \times m$ “supertile” of \mathcal{U} . Our construction works for the full aTAM at any temperature, and it faithfully simulates the deterministic or nondeterministic behavior of each \mathcal{T} .

Our construction succeeds by solving an analog of the cell differentiation problem in developmental biology: Each supertile of \mathcal{U} , starting with those in the seed assembly “genome” of the simulated system \mathcal{T} . At each location of a potential assembly of \mathcal{U} , a decision is made whether and how to expand that location to generate a supertile and, if so, which tile of \mathcal{T} it will be. This is done using asynchronous communication and coordination between the tiles of \mathcal{U} to generate a global outcome(s).

Is there a universal tileset at $T^\circ=2$?

- Rescaling : ***intrinsic** simulation*

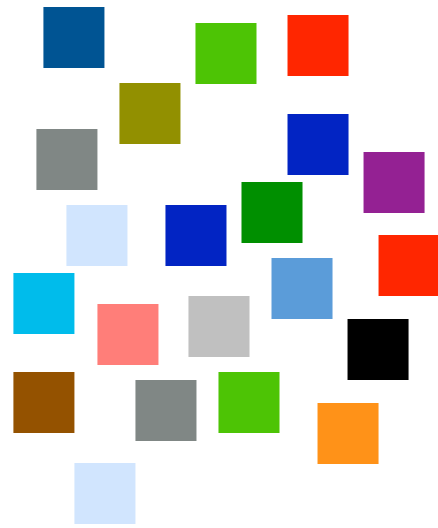


Brice Due 2006

The Game of Life self-simulating itself intrinsically:
Smaller cells simulate macro-cells

Comparing tile assembly models

Is there a set of **intrinsically universal tiles** that can **simulate any tile set**?

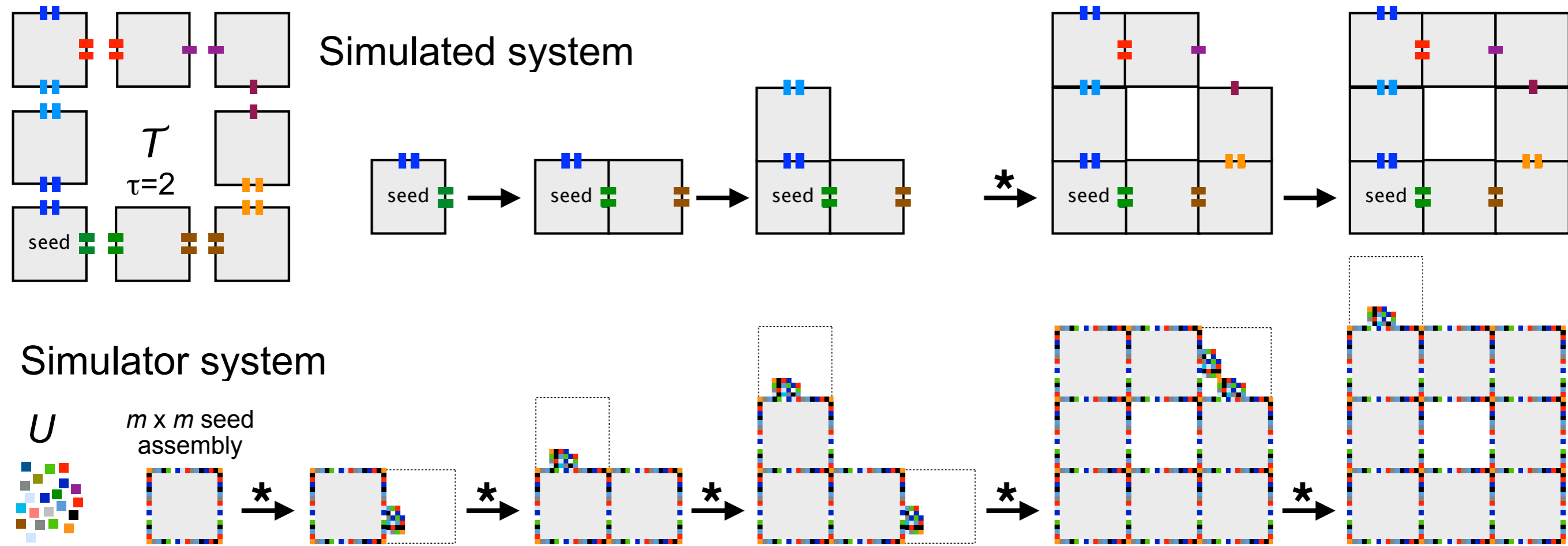


What does “act like” mean?

- What is it that tile assembly systems do?
 - Make shapes and patterns
 - Carry out a crystal-like growth process (dynamics)
- Let define **simulate** using these criteria that are intrinsic to the model

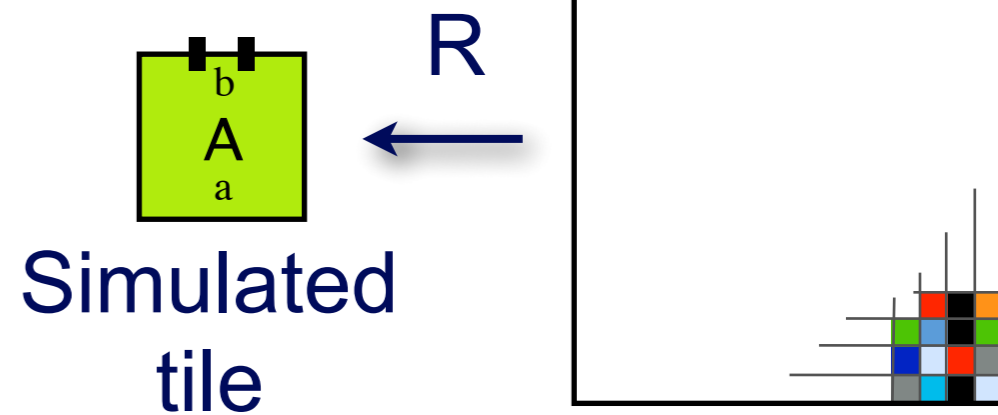
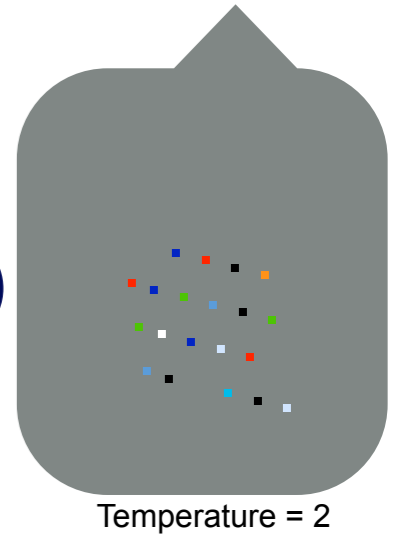
Simulation

- For (any) simulated tile assembly system \mathcal{T}
 - $\mathcal{T} = (\text{tileset } T, \text{ seed assembly } \sigma, \text{ temperature } \tau)$
- Tile assembly system \mathcal{U} simulates \mathcal{T} if:
 - **Tiles** from \mathcal{T} are represented by **$m \times m$ supertiles** in \mathcal{U}
 - Assemblies produced by \mathcal{U} **represent exactly** assemblies produced by \mathcal{T} (via a representation function $R : \text{Blocks of tiles from } U \rightarrow \text{tiles from } T$)
 - **Dynamics are equivalent** in \mathcal{U} and \mathcal{T} , ignoring $m \times m$ scaling

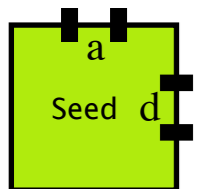


Simulation definition

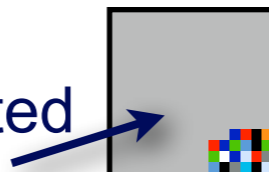
Universal
(simulator)
tile set



T



Preassembled
seed structure
(encodes simulated
TAS)

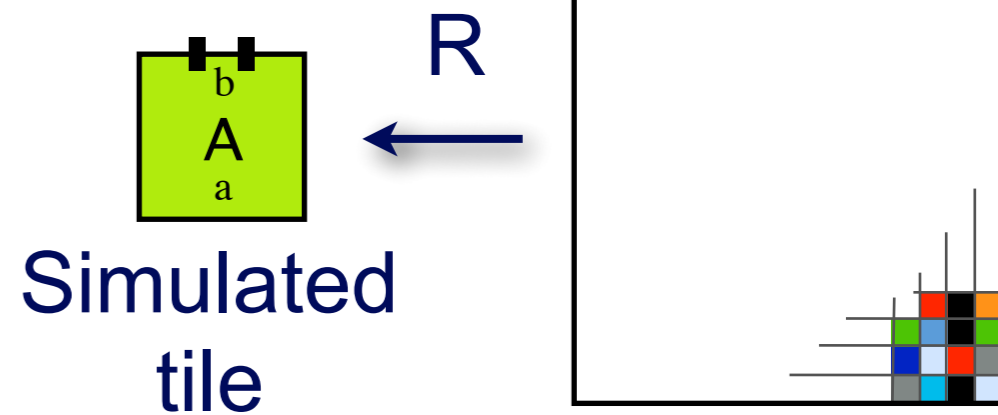
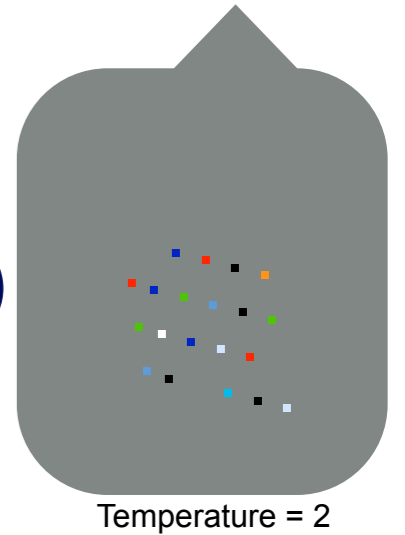


U

- Green tiles are simulated by supertiles
- For each assembly sequence in the simulated tile system, there is an assembly sequence in the simulator, and vice-versa

Simulation definition

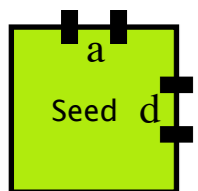
Universal
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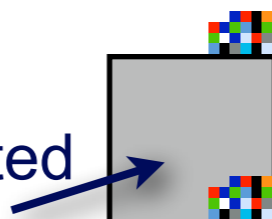
Simulator
supertile

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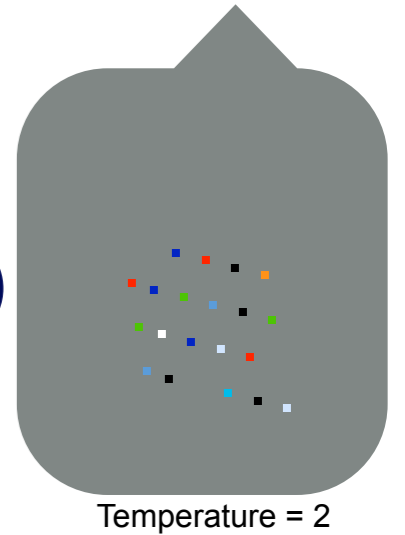
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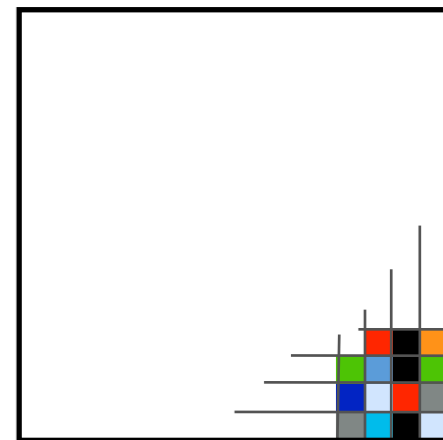
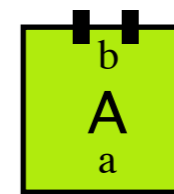
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Simulation definition

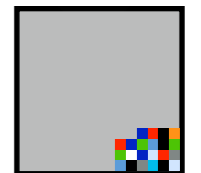
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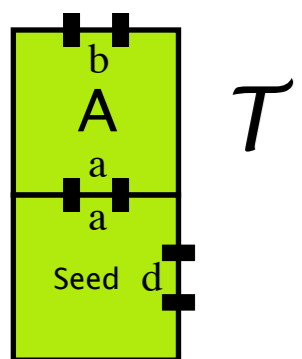
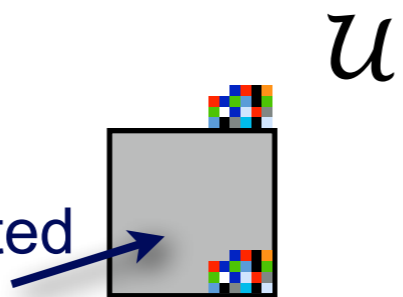
Simulated
tile



Simulator
supertile



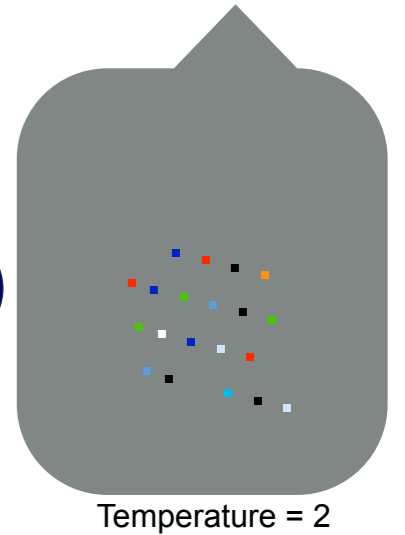
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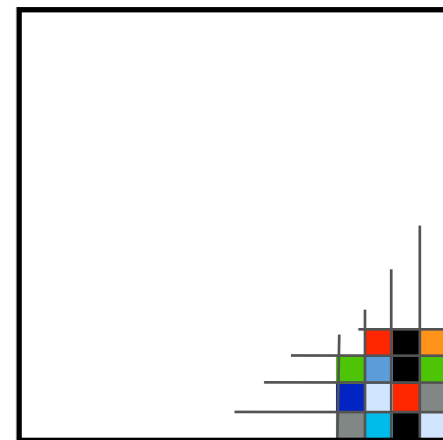
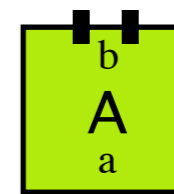
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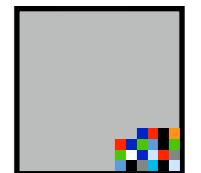
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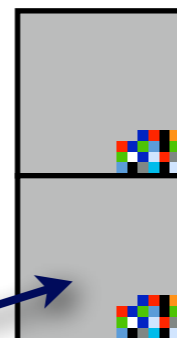
Simulated
tile



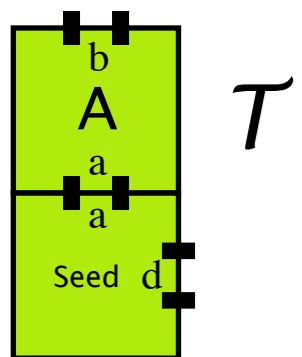
Simulator
supertile



Preassembled
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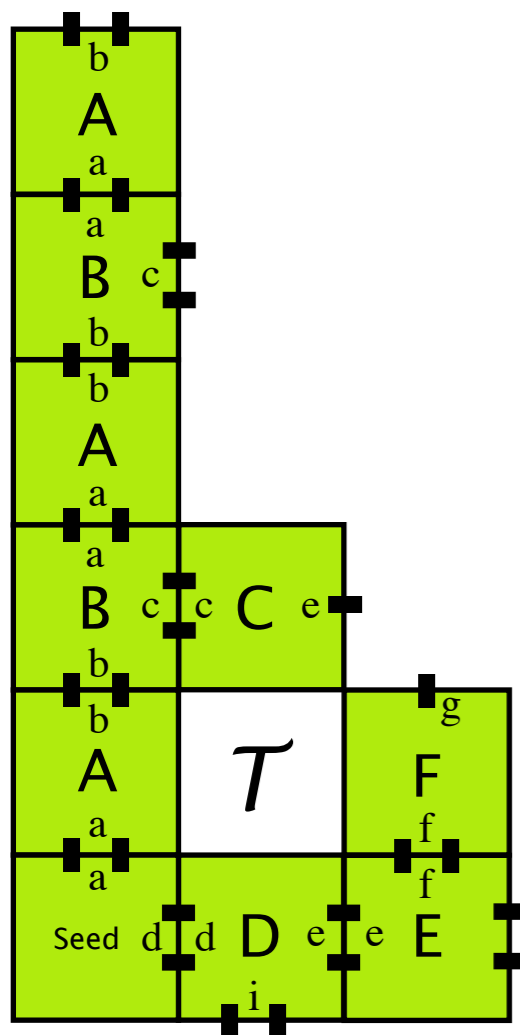


\mathcal{U}

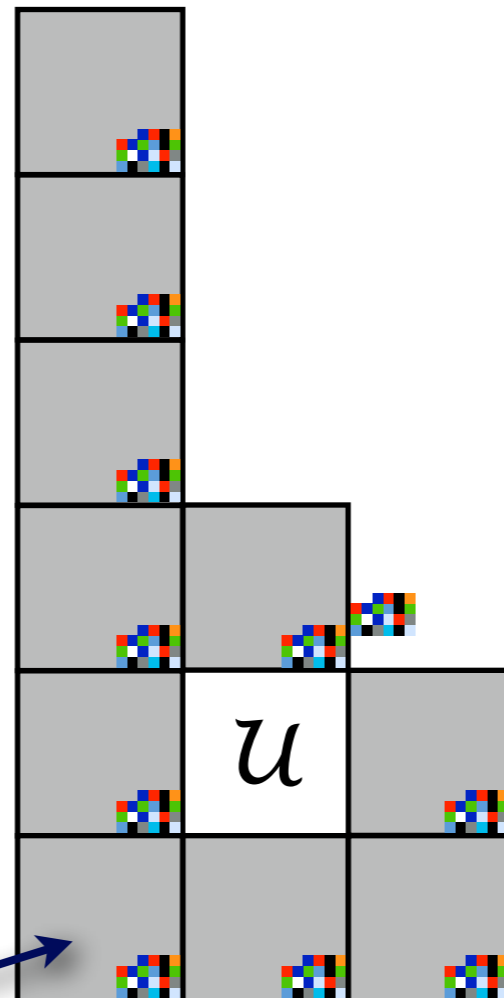


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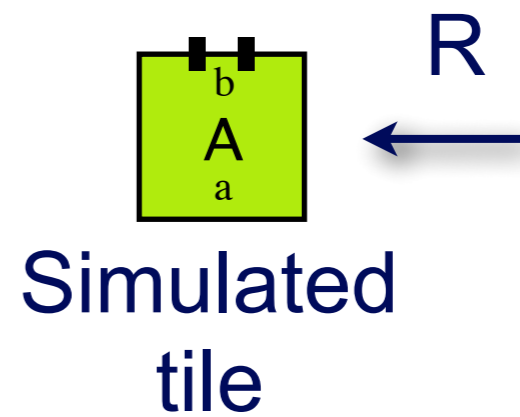
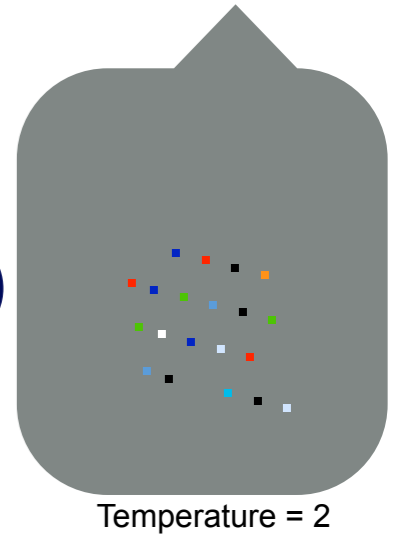
Simulation definition



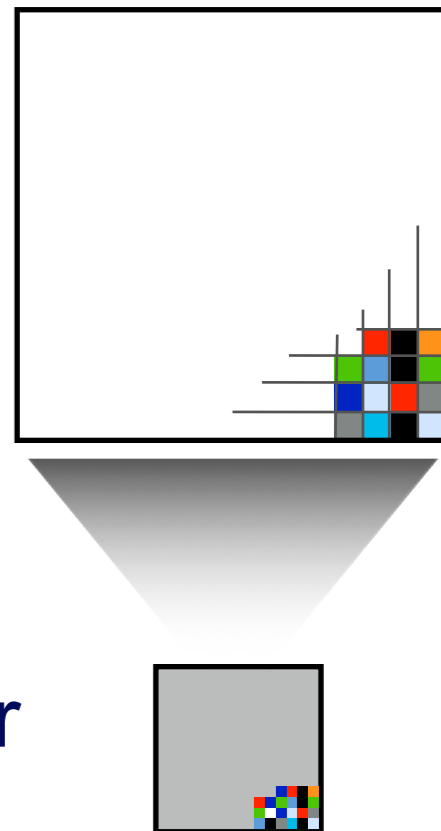
Preassembled seed structure (encodes simulated TAS)



Universal (simulator) tile set



Simulator supertile

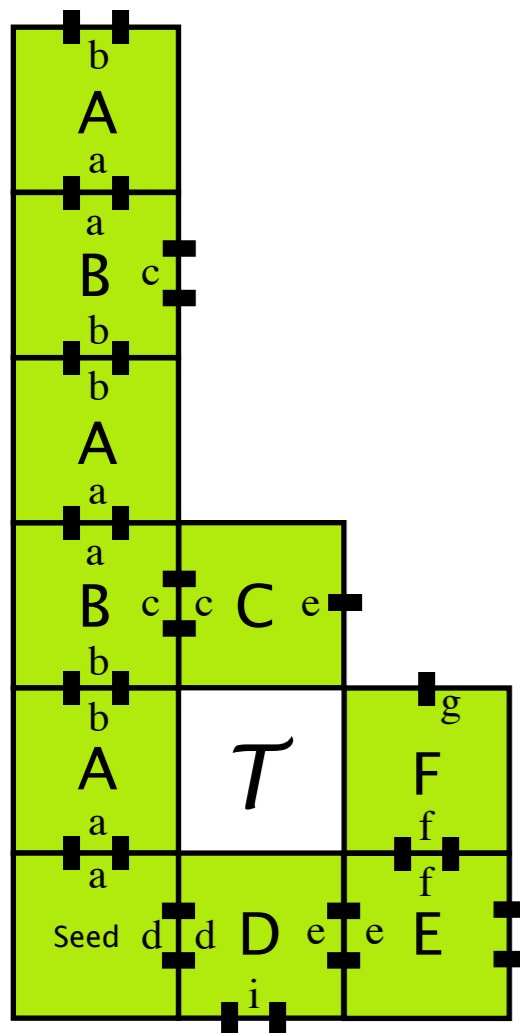
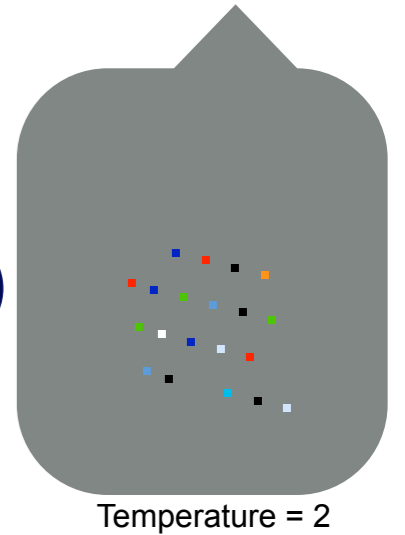


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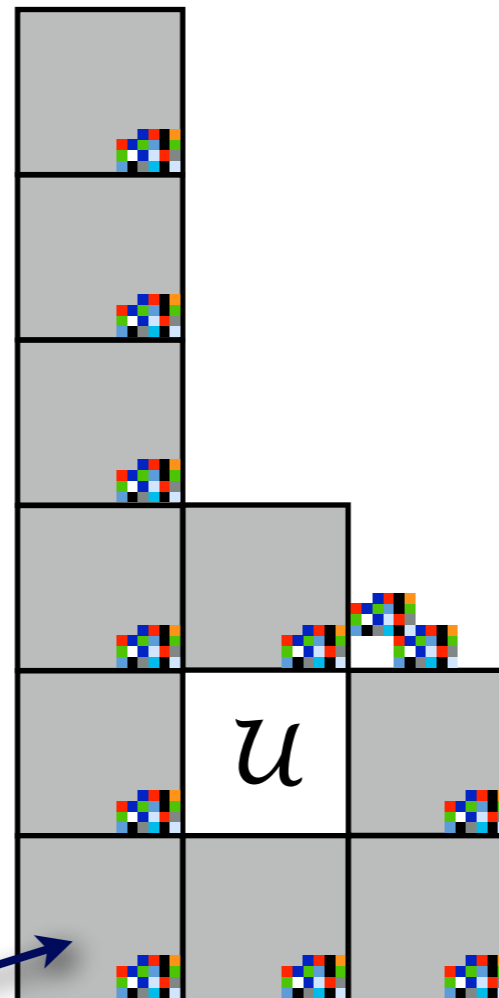
Simulation definition

Ignoring $m \times m$ scaling, production & dynamics are equivalent in the simulated system and simulator

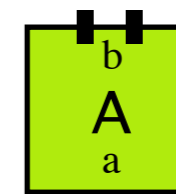
Universal
(simulator)
tile set



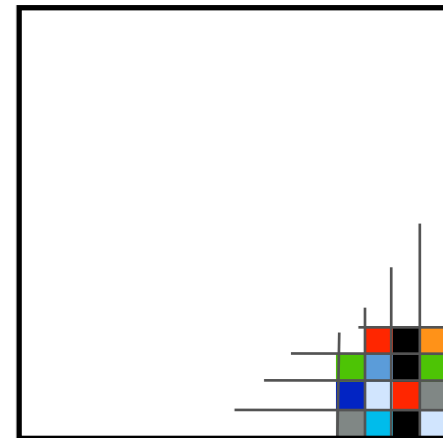
Preassembled
seed structure
(encodes simulated
TAS)



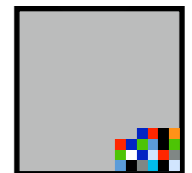
Simulated
tile



R ←



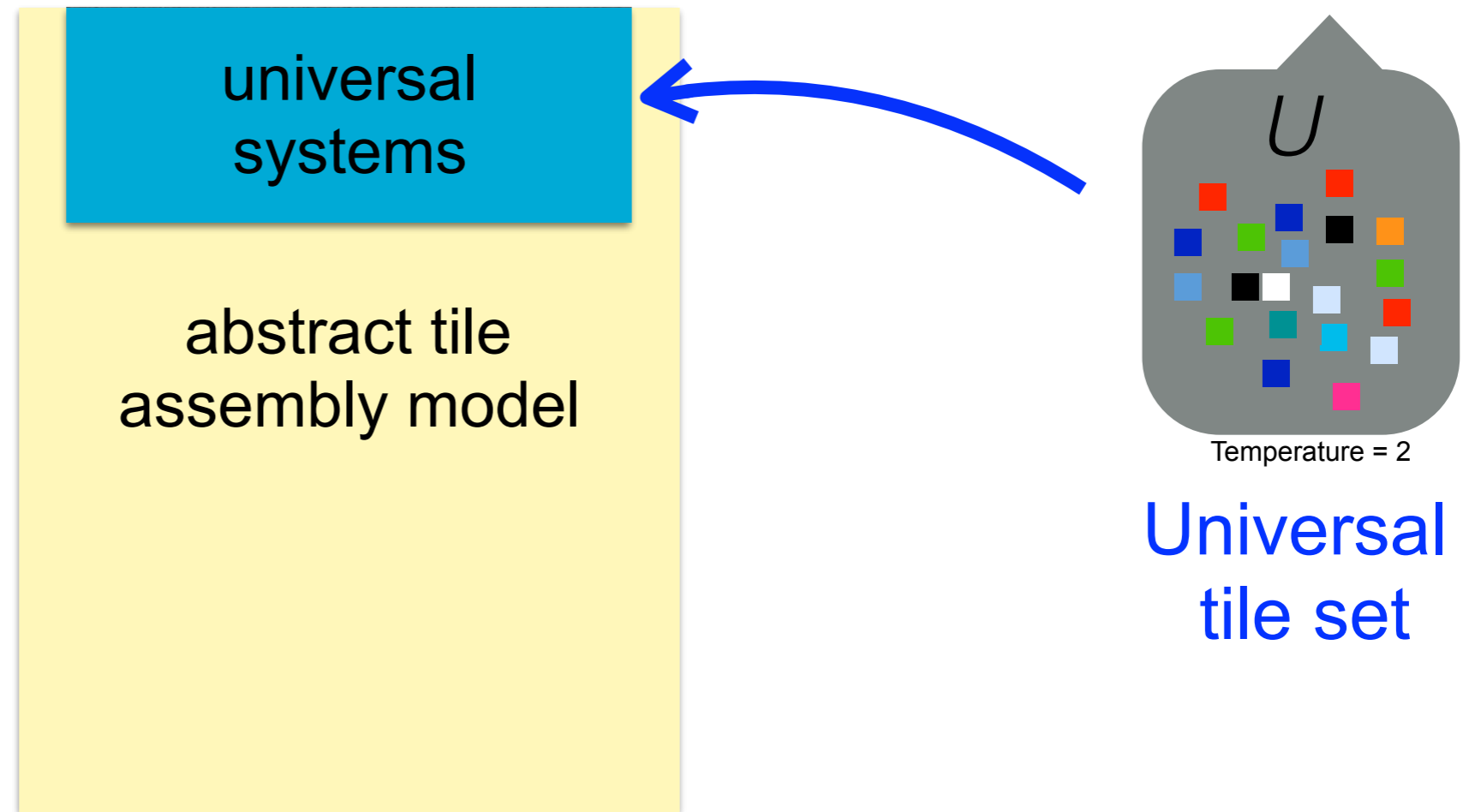
Simulator
supertile



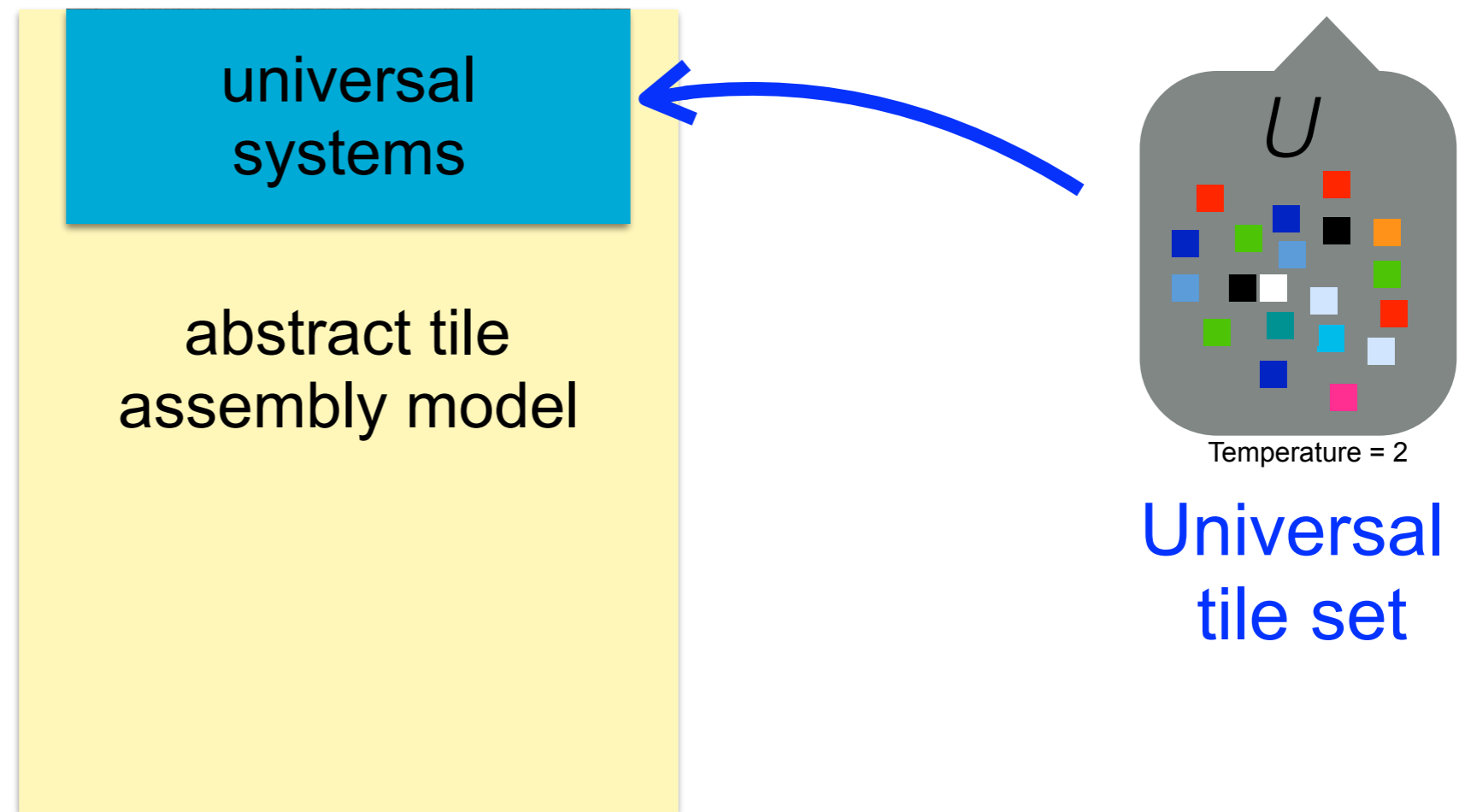
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etc.

Is the abstract tile assembly model intrinsically universal?



Is the abstract tile assembly model intrinsically universal? **Yes!**

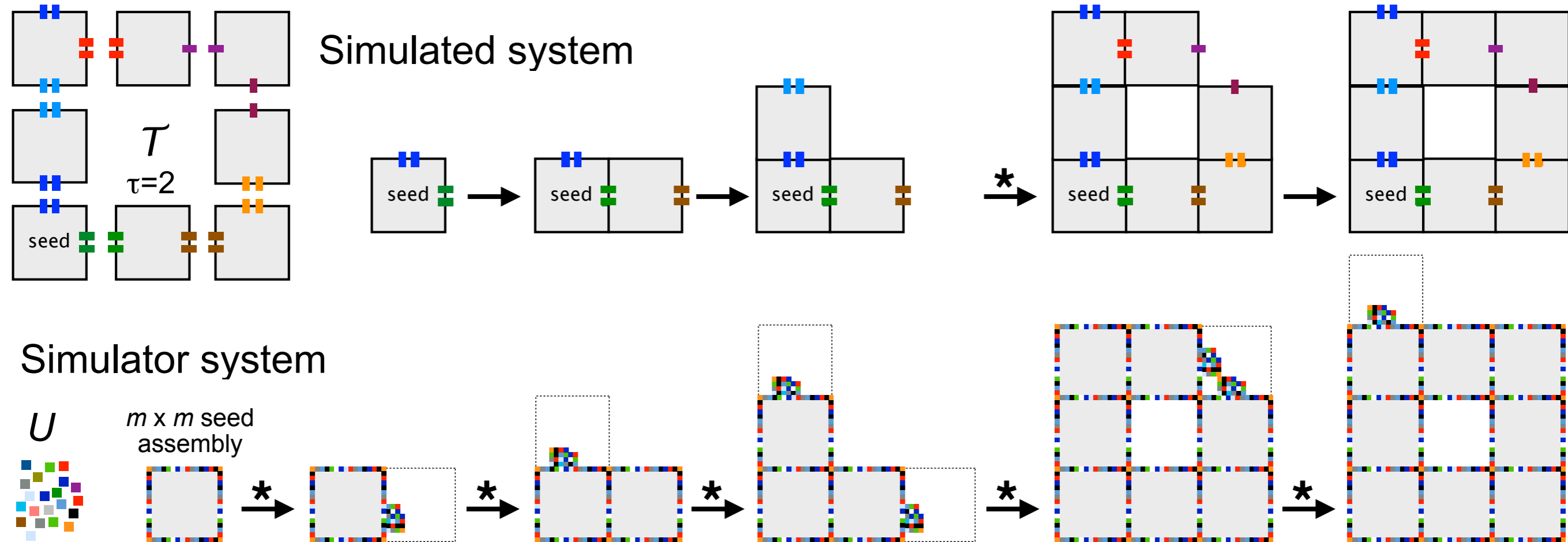


Theorem: There is a single intrinsically universal tile set U that simulates *any* tile assembly system

Doty, Lutz, Patitz, Schweller, Summers, Woods. FOCS 2012

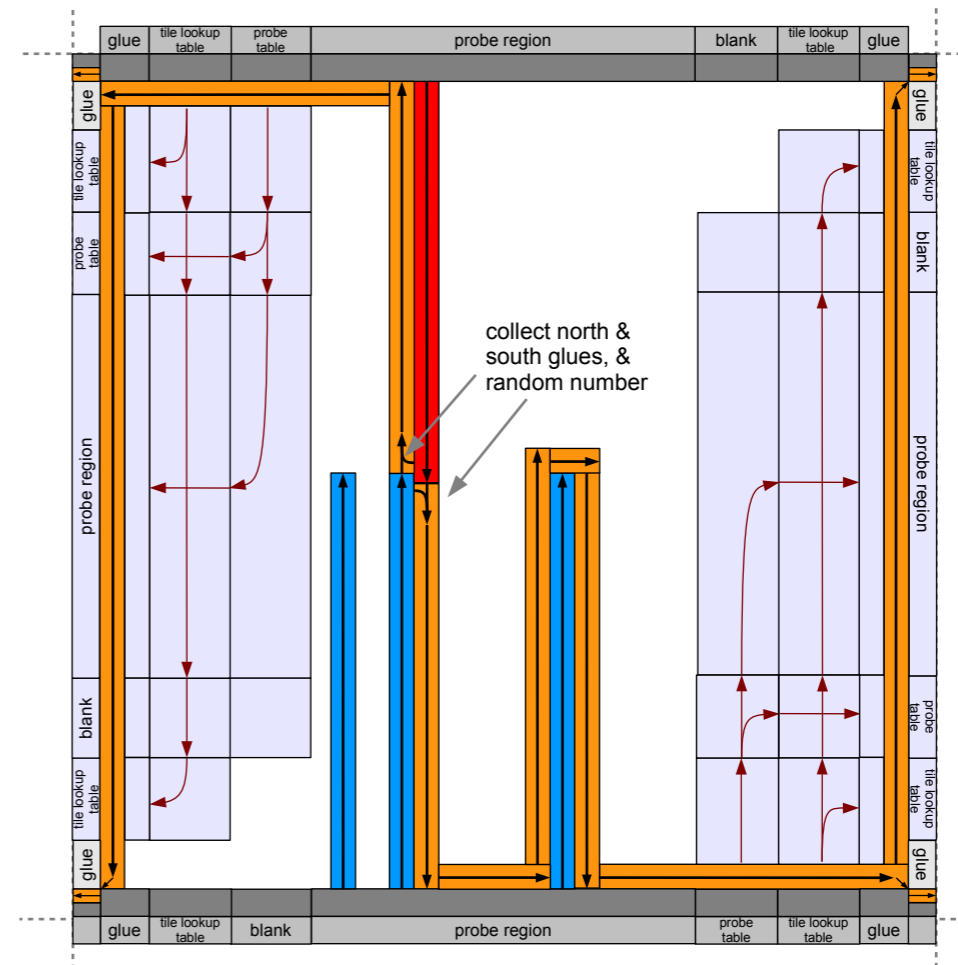
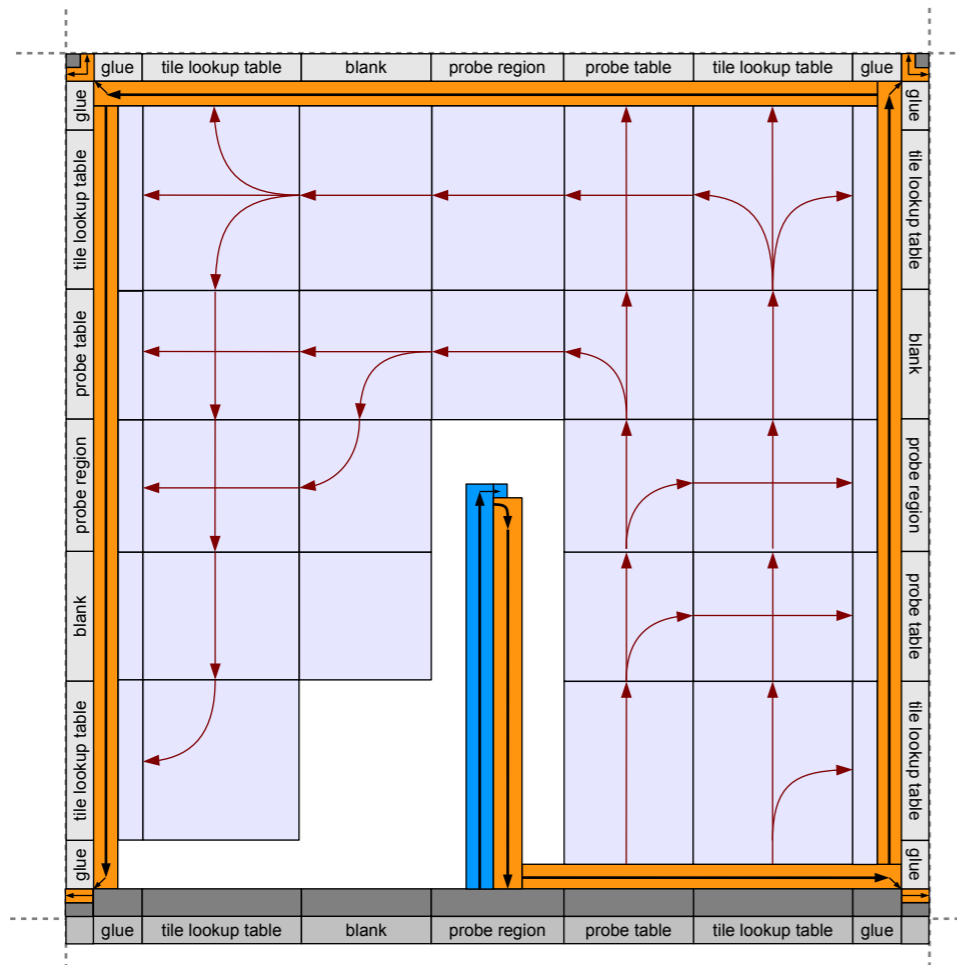
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Is there a universal tileset at $T^\circ=2$?

- Rescaling : *intrinsic simulation*

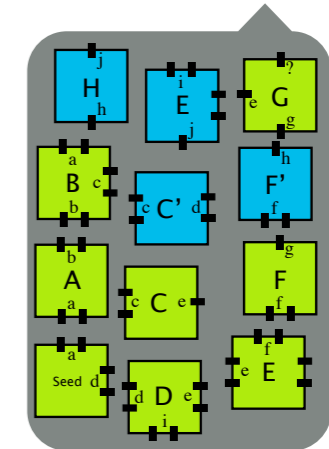


Doty Lutz
Patitz
Schweller
Summers
Woods 2012

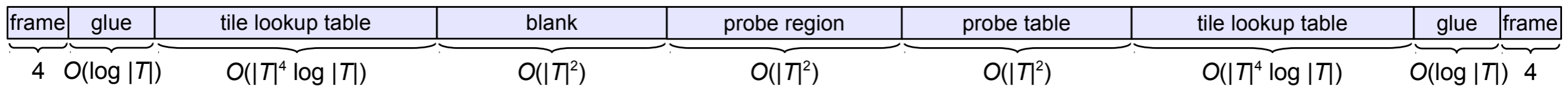
Examples of macro-tiles

Superside

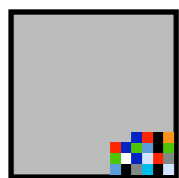
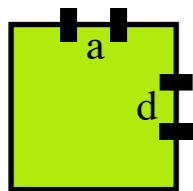
Encoding of the entire simulated **tile assembly system** written down using tiles from the simulator U



Temperature = 2

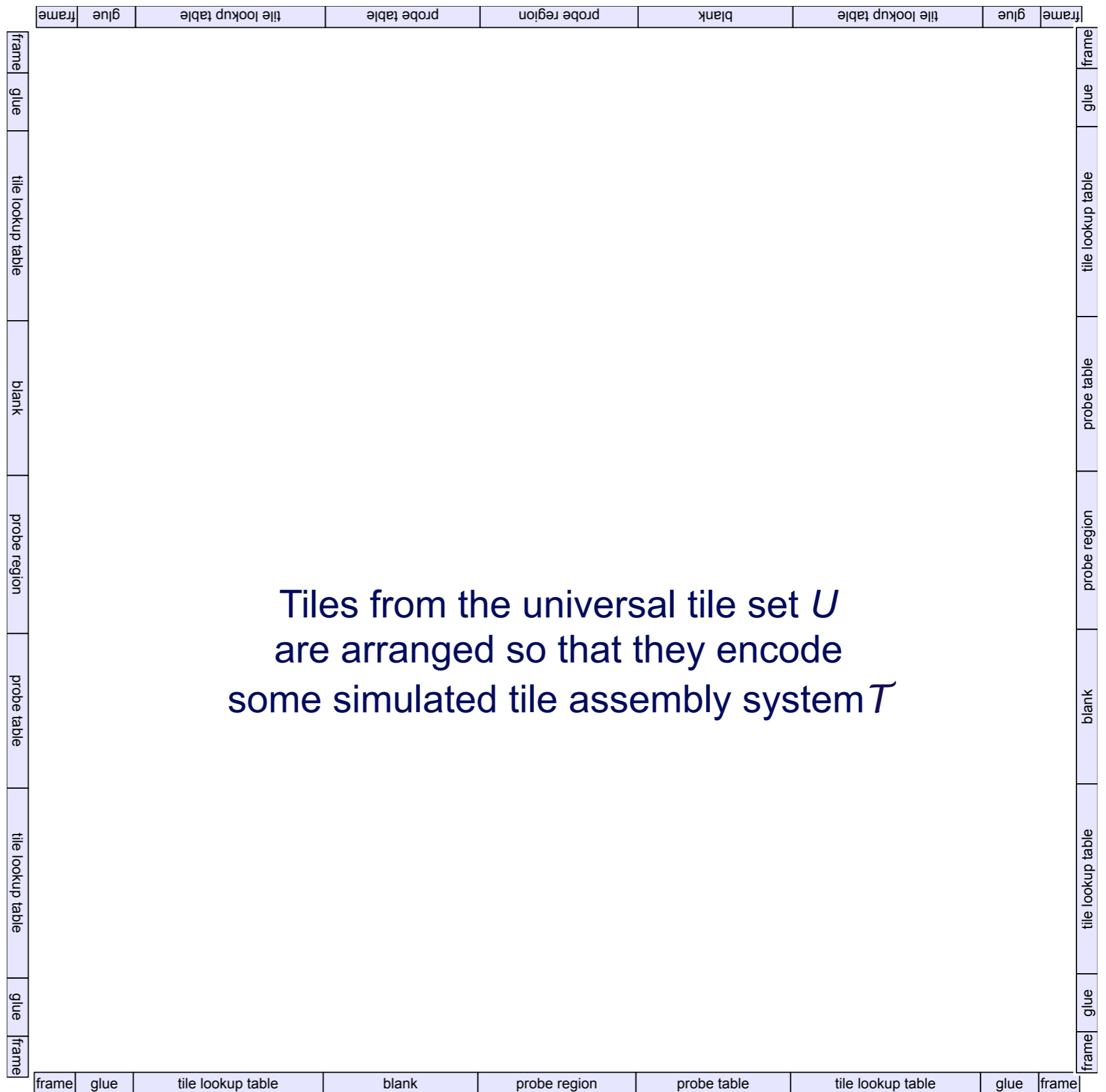


$|T|$ is number of tiles in the simulated tileset T .

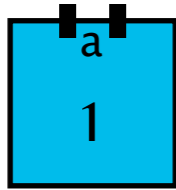


Preassembled
seed supertile

Tiles from the universal tile set U
are arranged so that they encode
some simulated tile assembly system \mathcal{T}

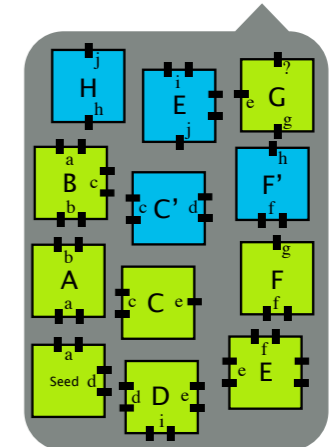


Superside

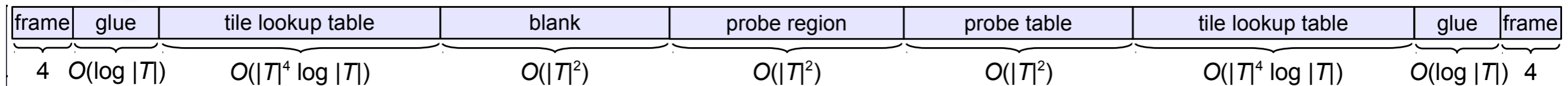


Encoded glue of
this superside
(e.g. “a”)

Encoding of the
entire simulated **tile
assembly system**
written down using
tiles from the
simulator U



Temperature = 2

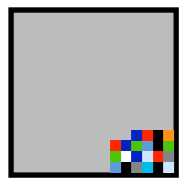
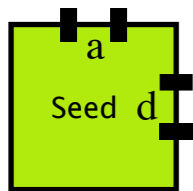


Encoding of tile type 1 from T

$|T|$ is number of tiles in the simulated tileset T .

glue
tile

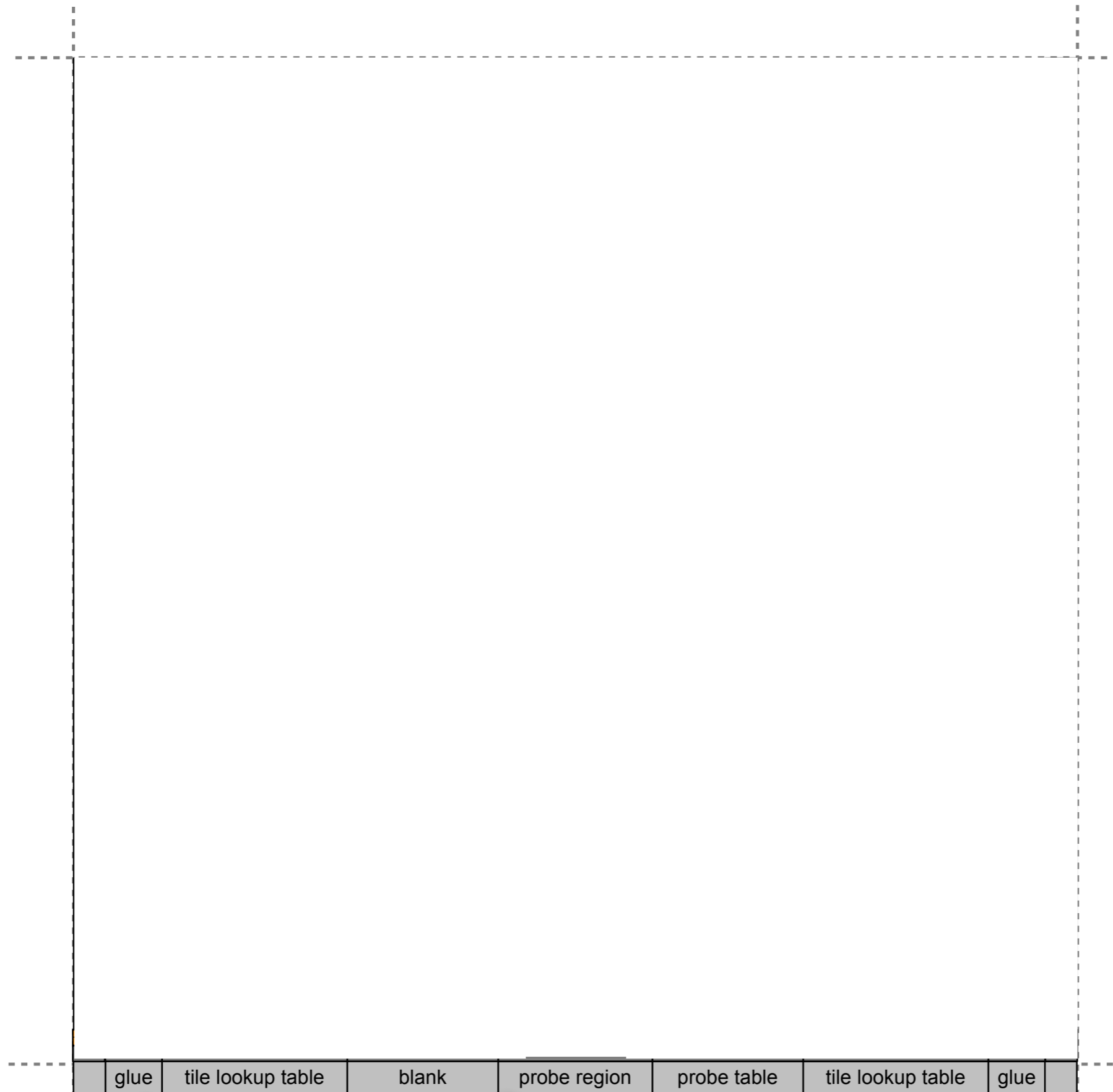
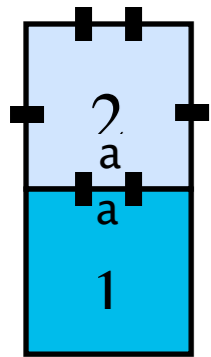
glue
fra



Preassembled
seed supertile

One-sided binding with a single strength- τ south superside

Goal: place a description of T and "tile 2" around the 4 super-edges



Glue "a" is encoded here

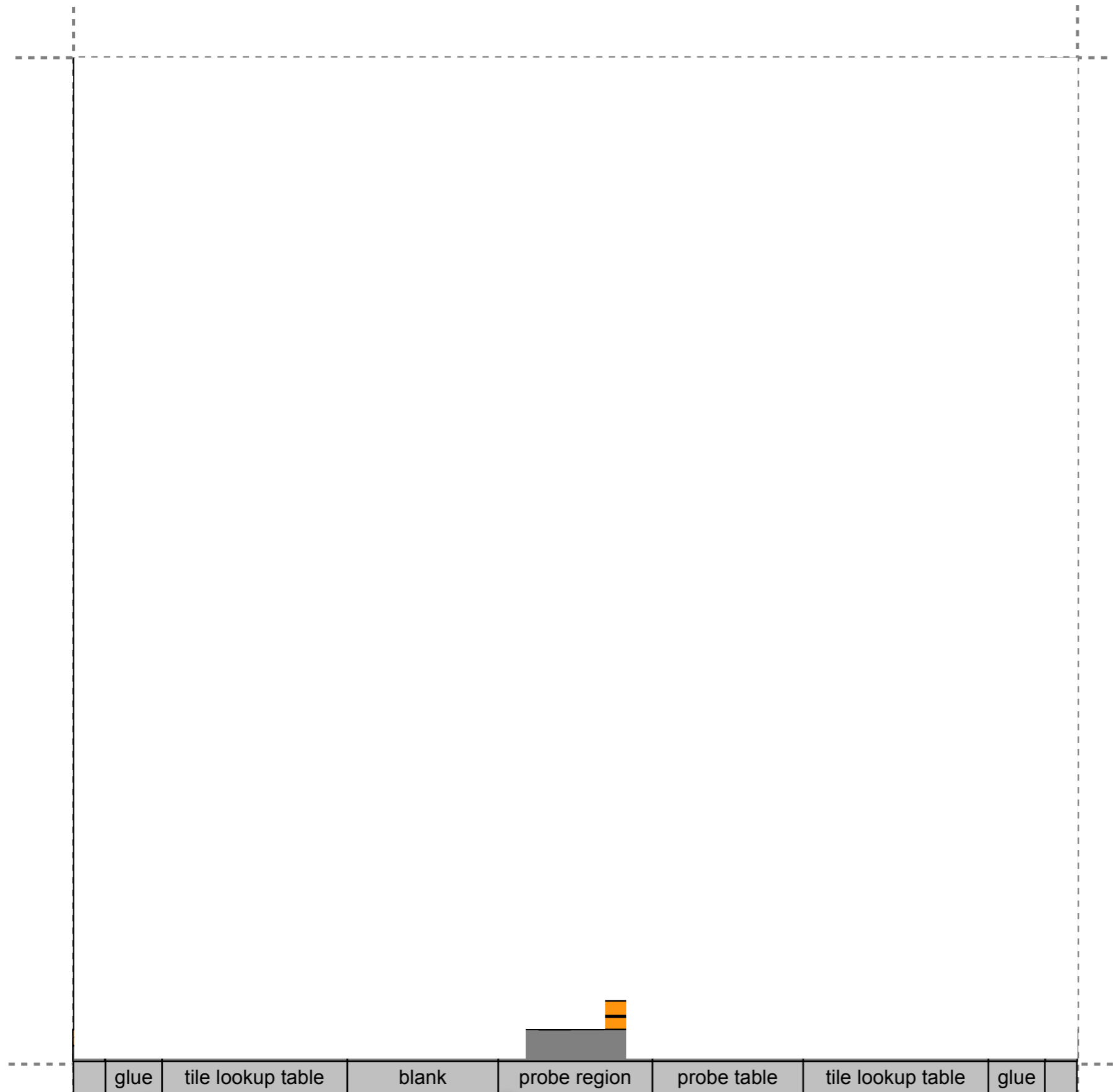
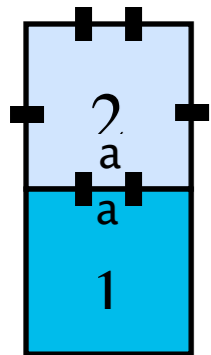
Supertile 1

Tile set T is encoded here

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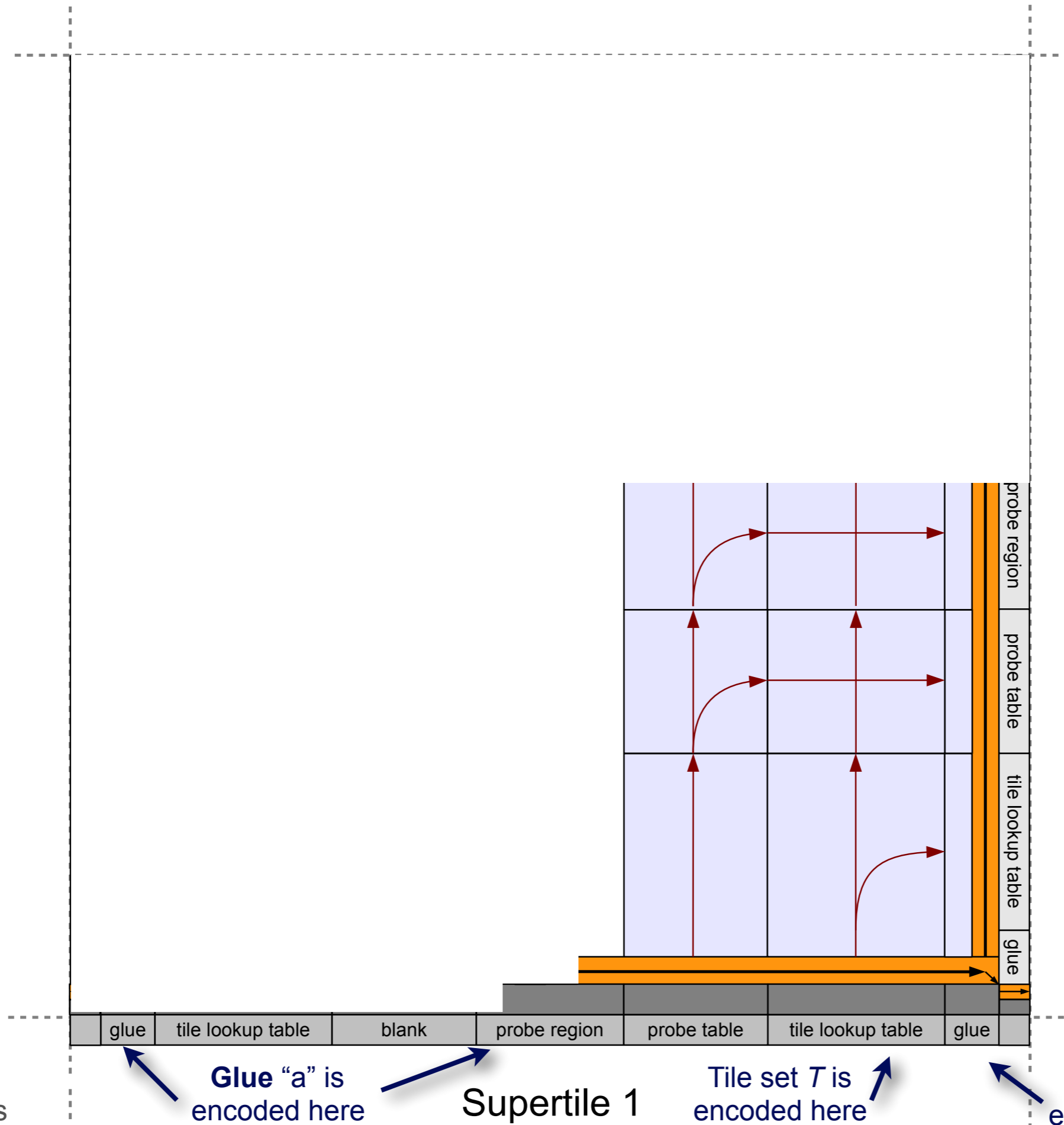
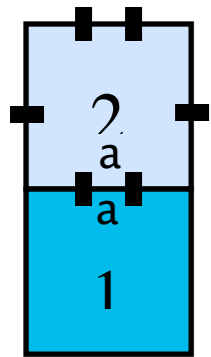
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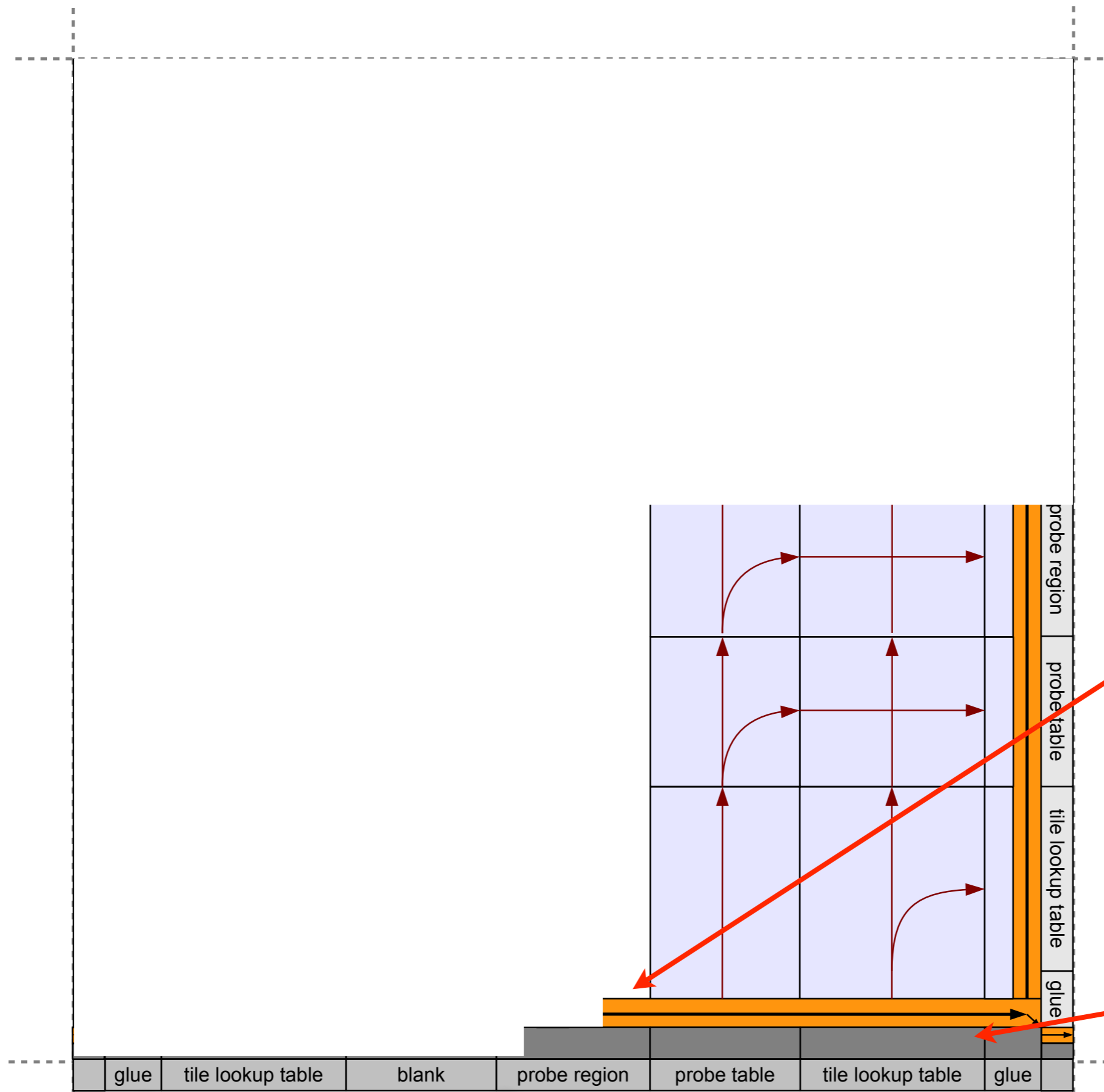
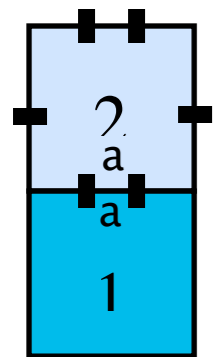
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One-sided binding with a single strength- τ south superside

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One-sided binding with a single strength- τ south superside



Goal: place a description of T and "tile 2" around the 4 super-edges

crawler encodes glue of south superside

"Genome" is copied

"Genome" is read

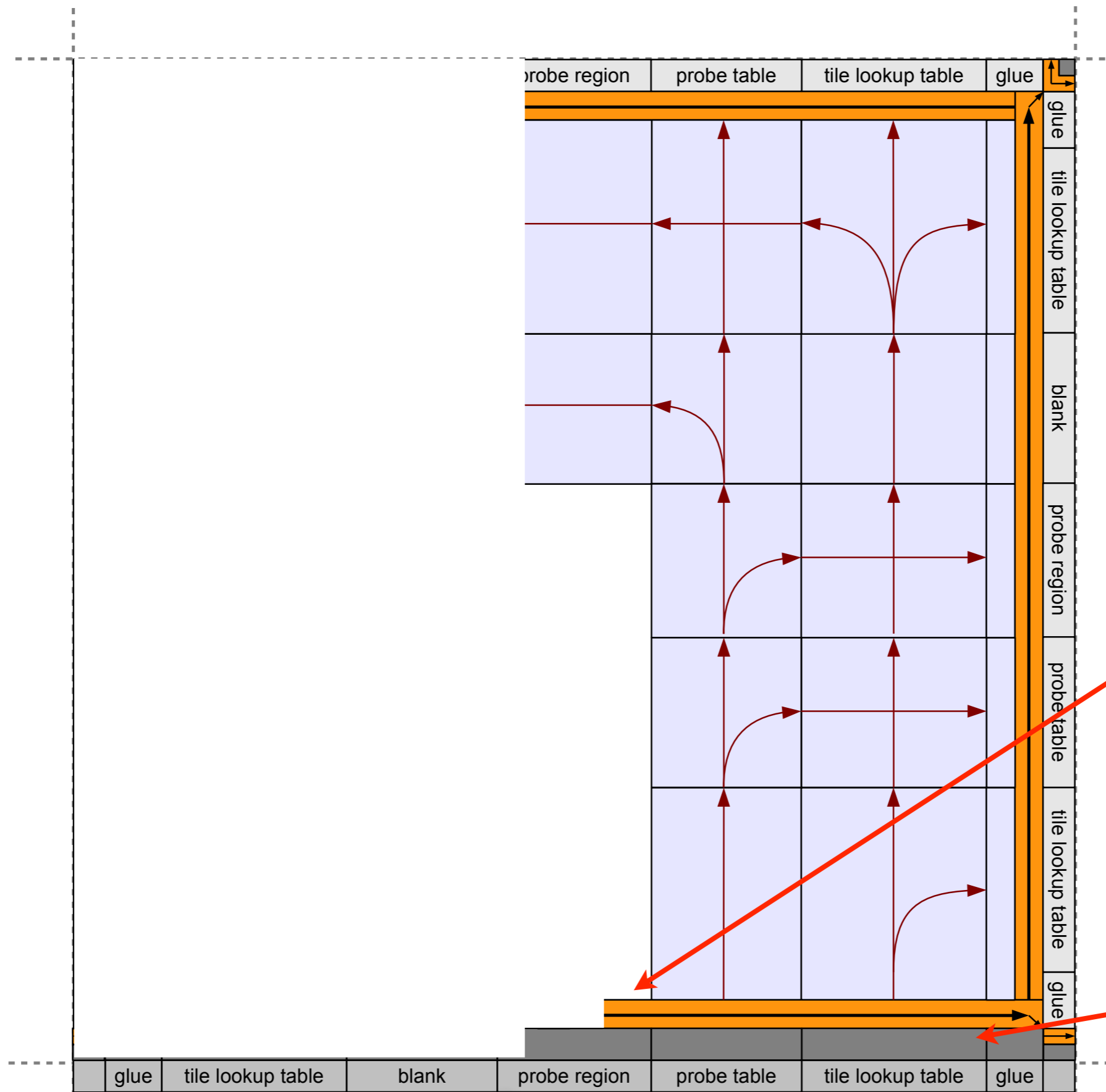
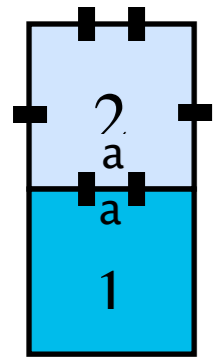
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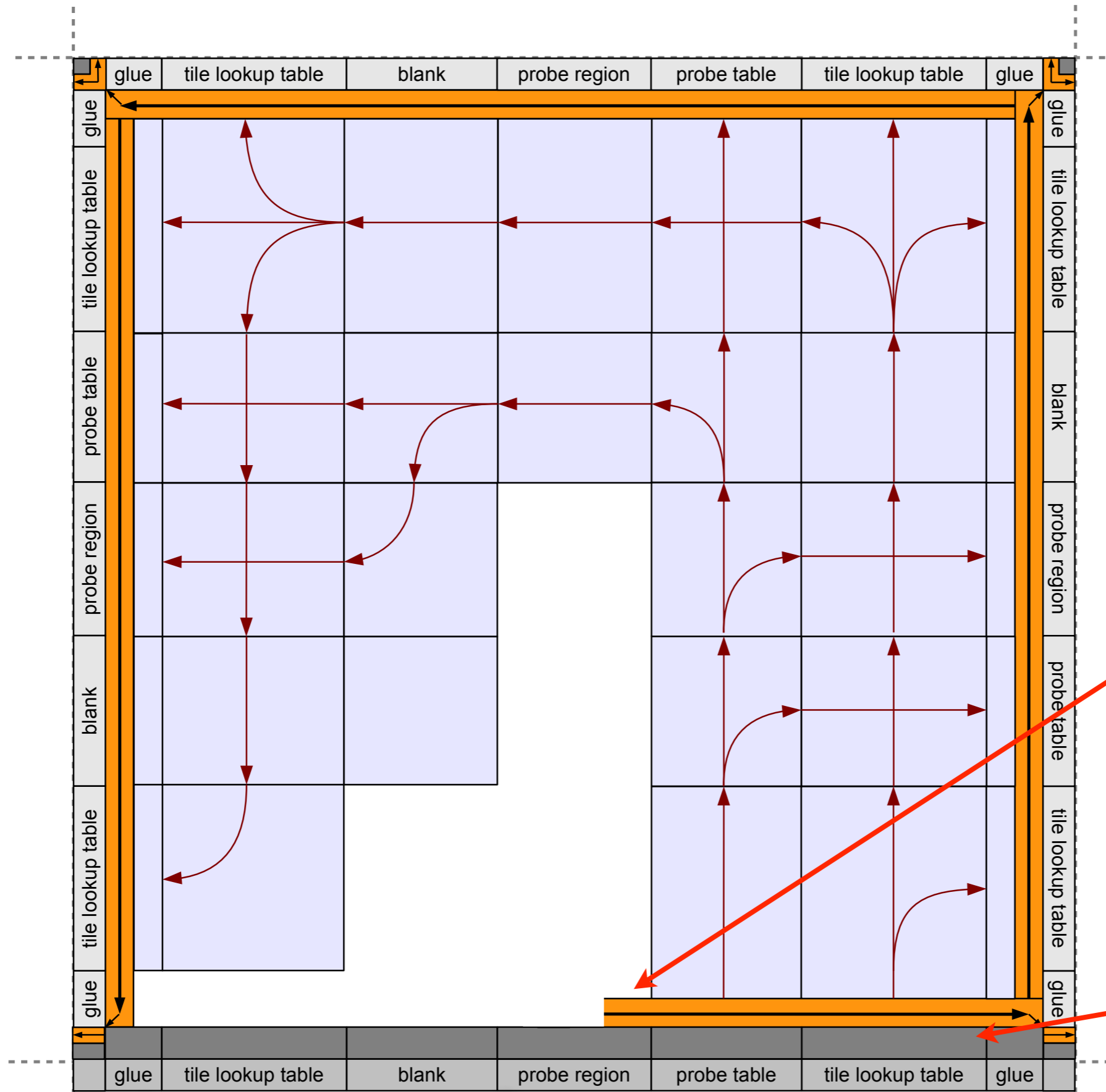
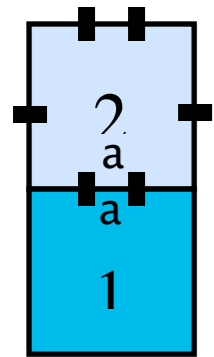
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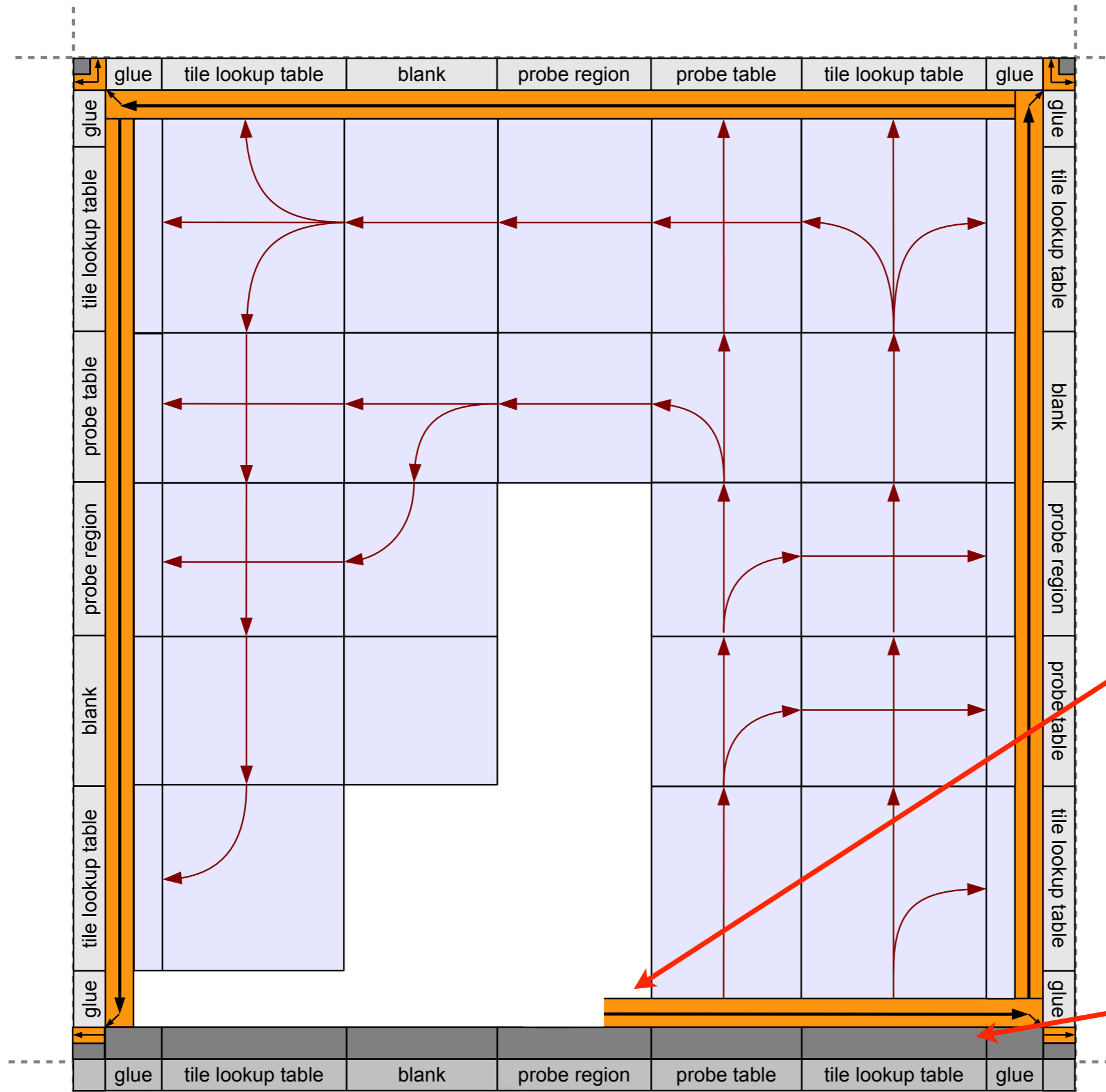
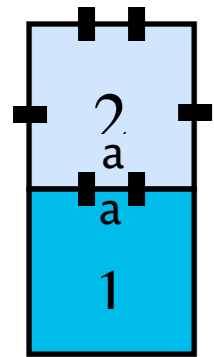
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One-sided binding with a single strength- τ south superside



Goal: place a description of T and "tile 2" around the 4 super-edges

Nondeterminism
Rotations

crawler encodes
glue of south
superside

"Genome" is
copied

"Genome" is
read

Glue "a" is
encoded here

Supertile 1

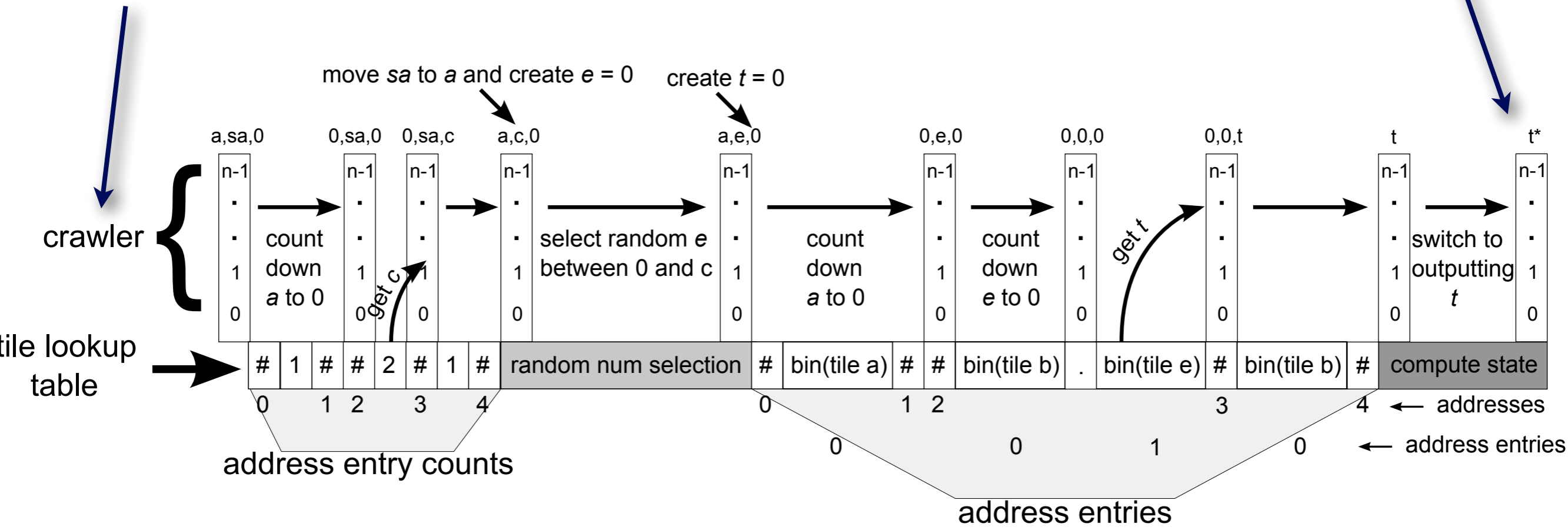
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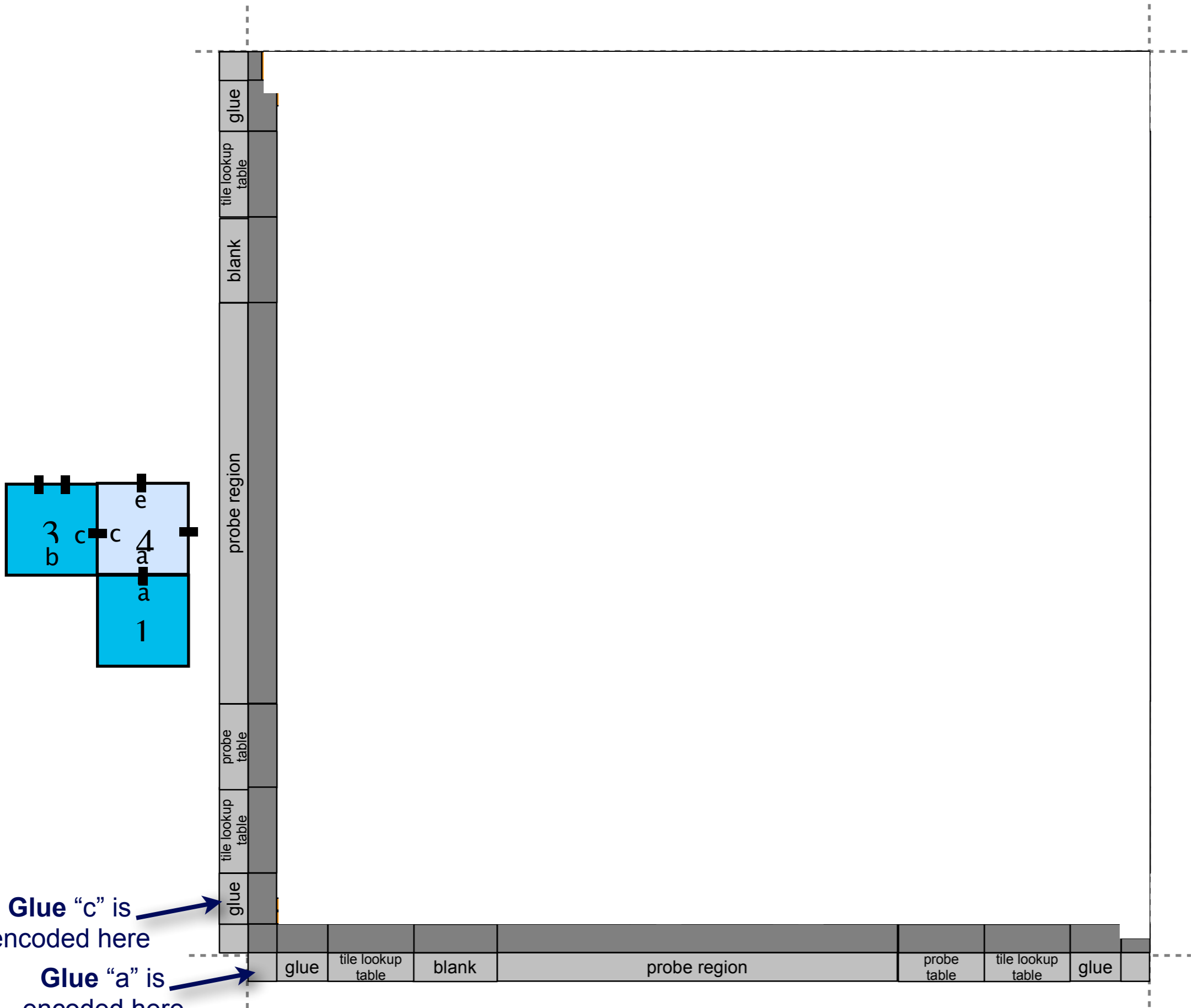
Crawler doing a tile lookup

crawler encodes "input" glues

crawler encodes "output" tile type



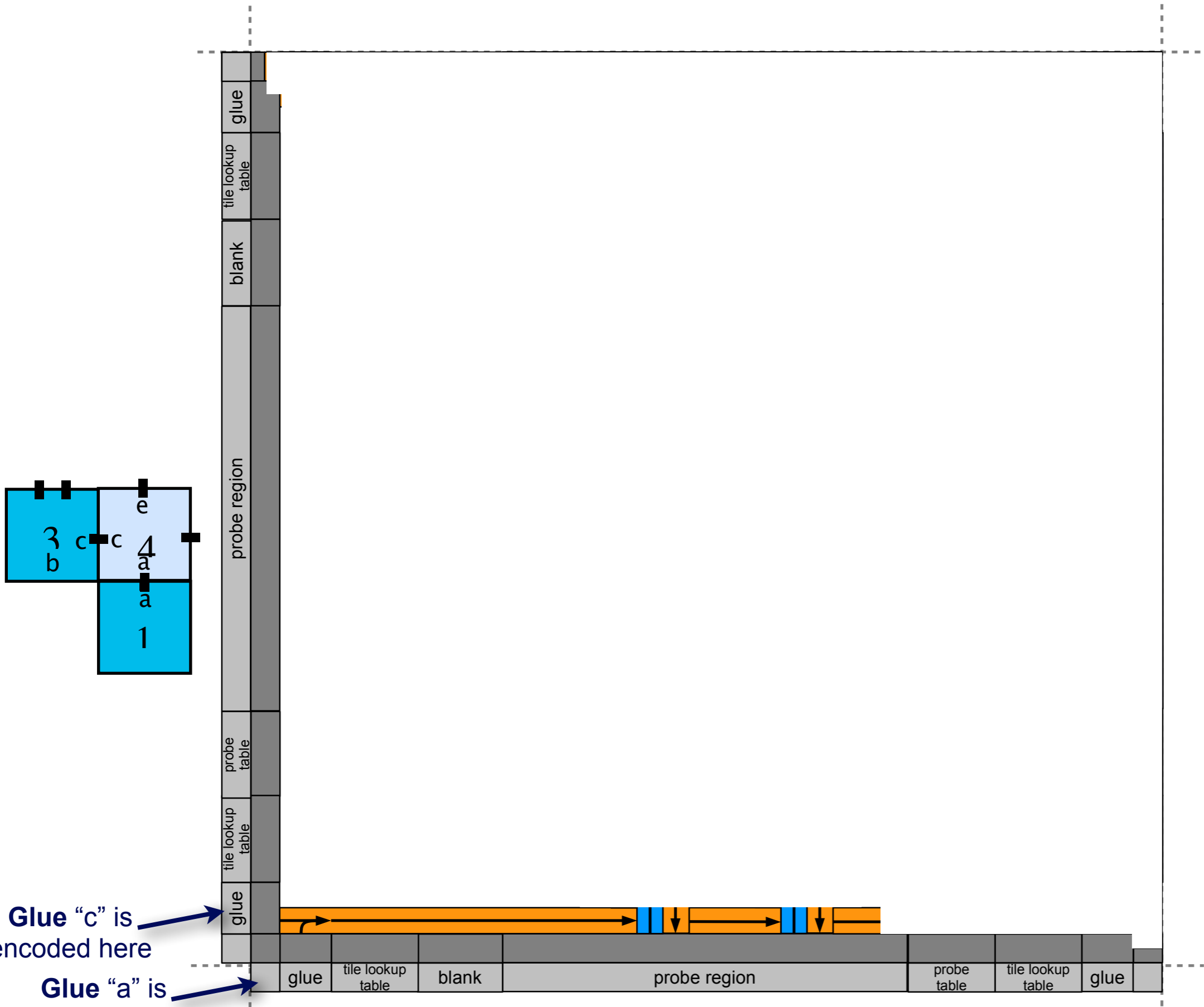
Two-sided binding with adjacent cooperating supersides



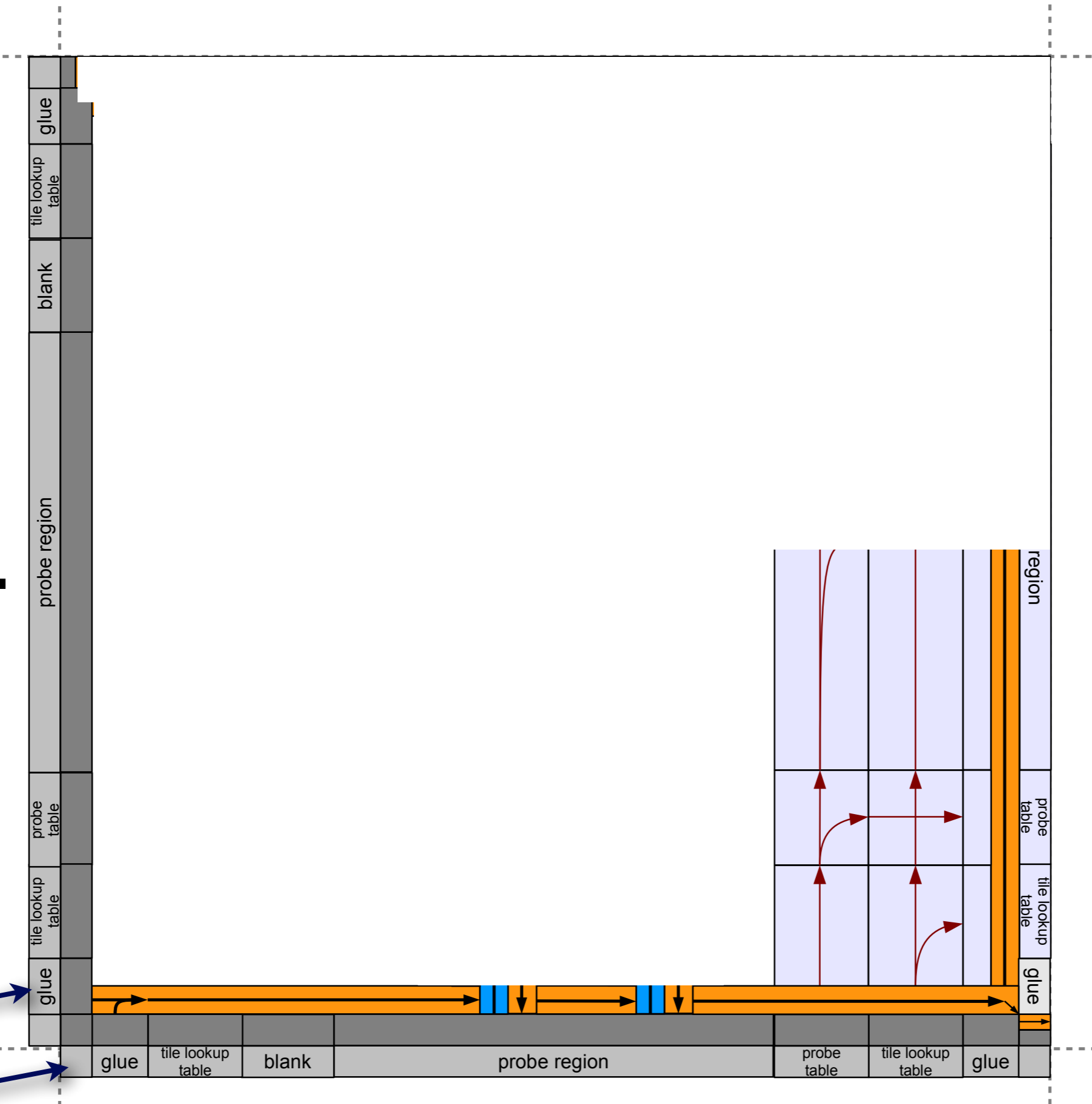
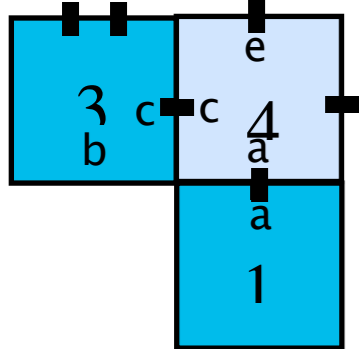
Glue "c" is encoded here

Glue "a" is encoded here

Two-sided binding with adjacent cooperating supersides



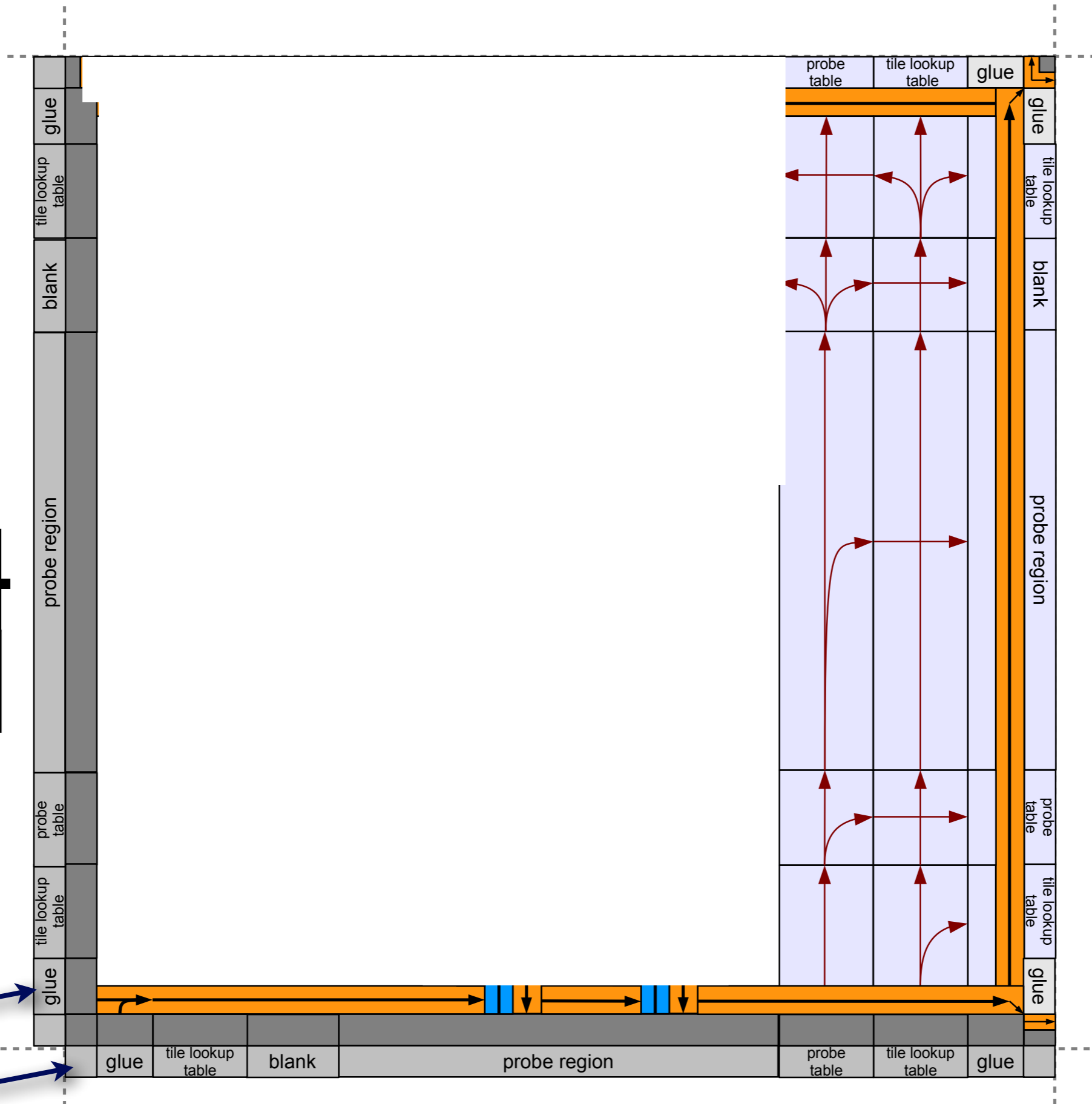
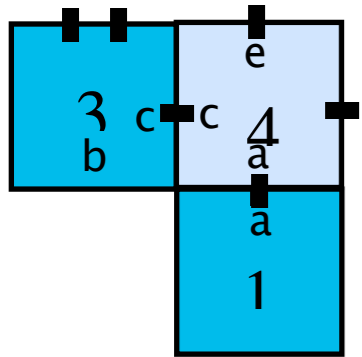
Two-sided binding with adjacent cooperating supersides



Glue "c" is encoded here

Glue "a" is encoded here

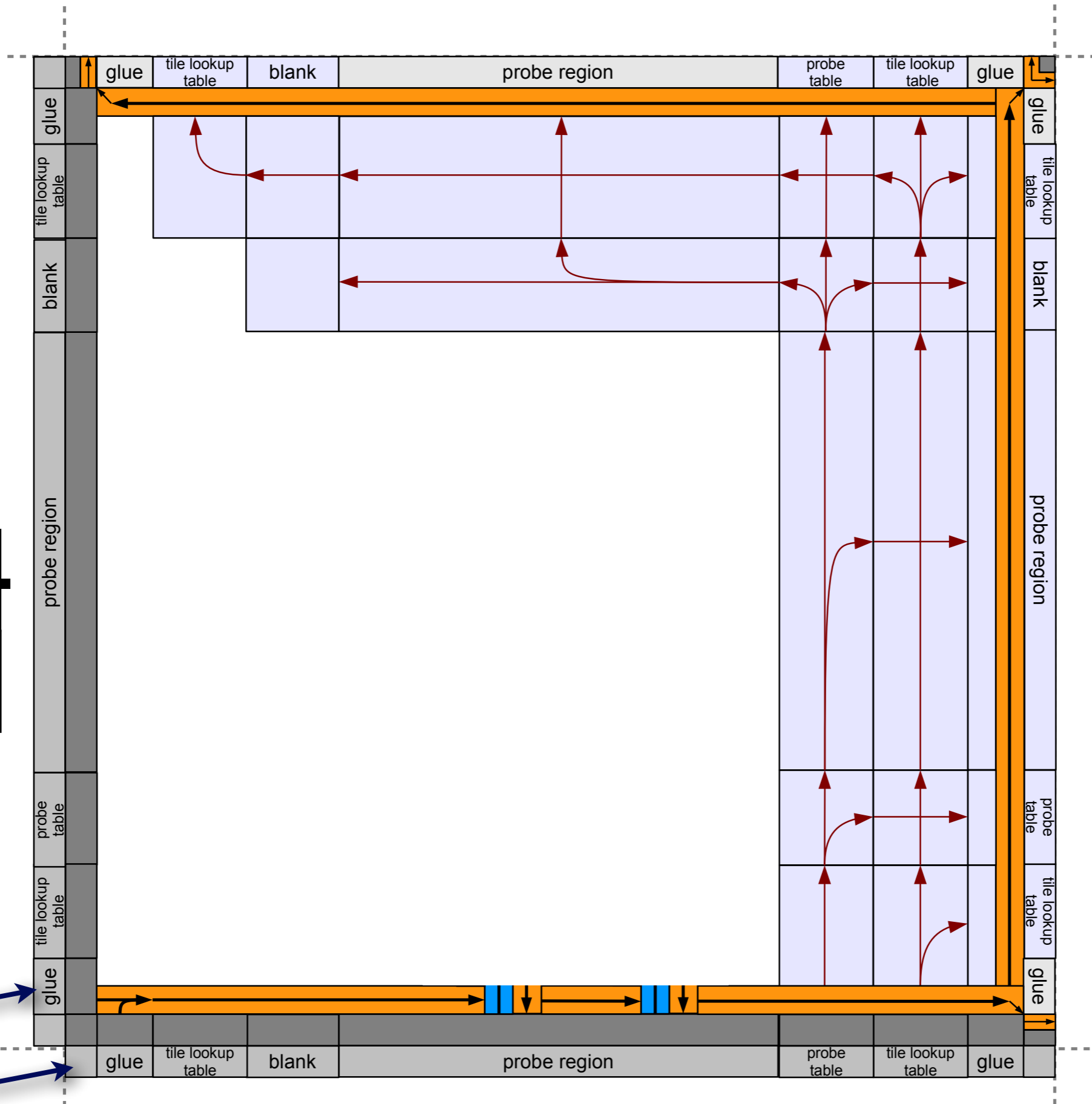
Two-sided binding with adjacent cooperating supersides



Glue "c" is encoded here

Glue "a" is encoded here

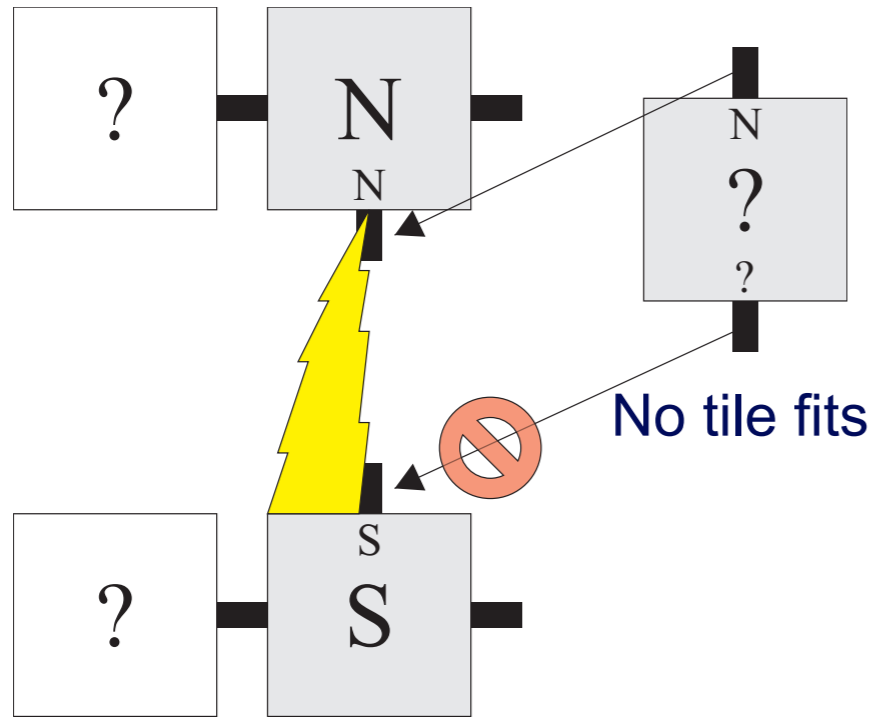
Two-sided binding with adjacent cooperating supersides



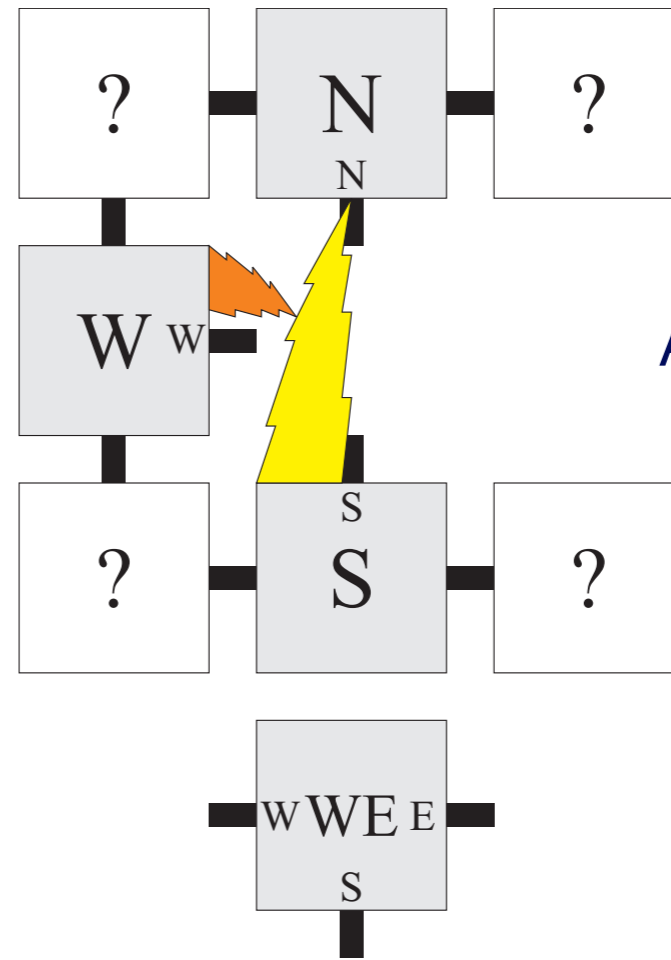
Glue "c" is encoded here

Glue "a" is encoded here

A key problem

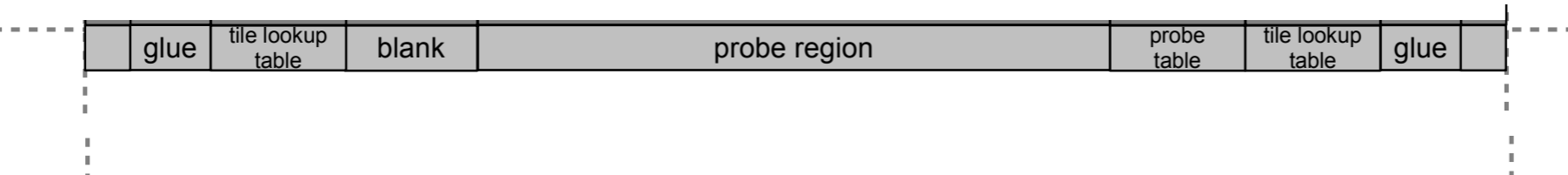
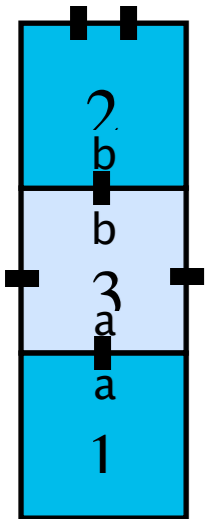
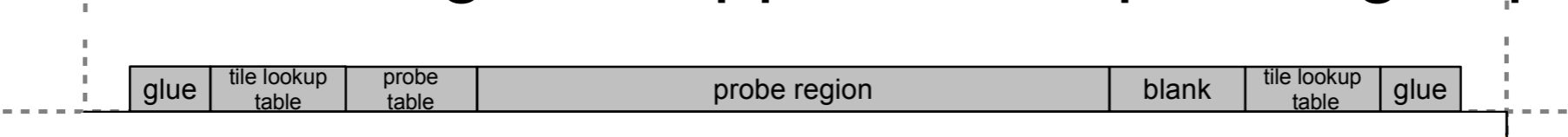


Better luck next time!

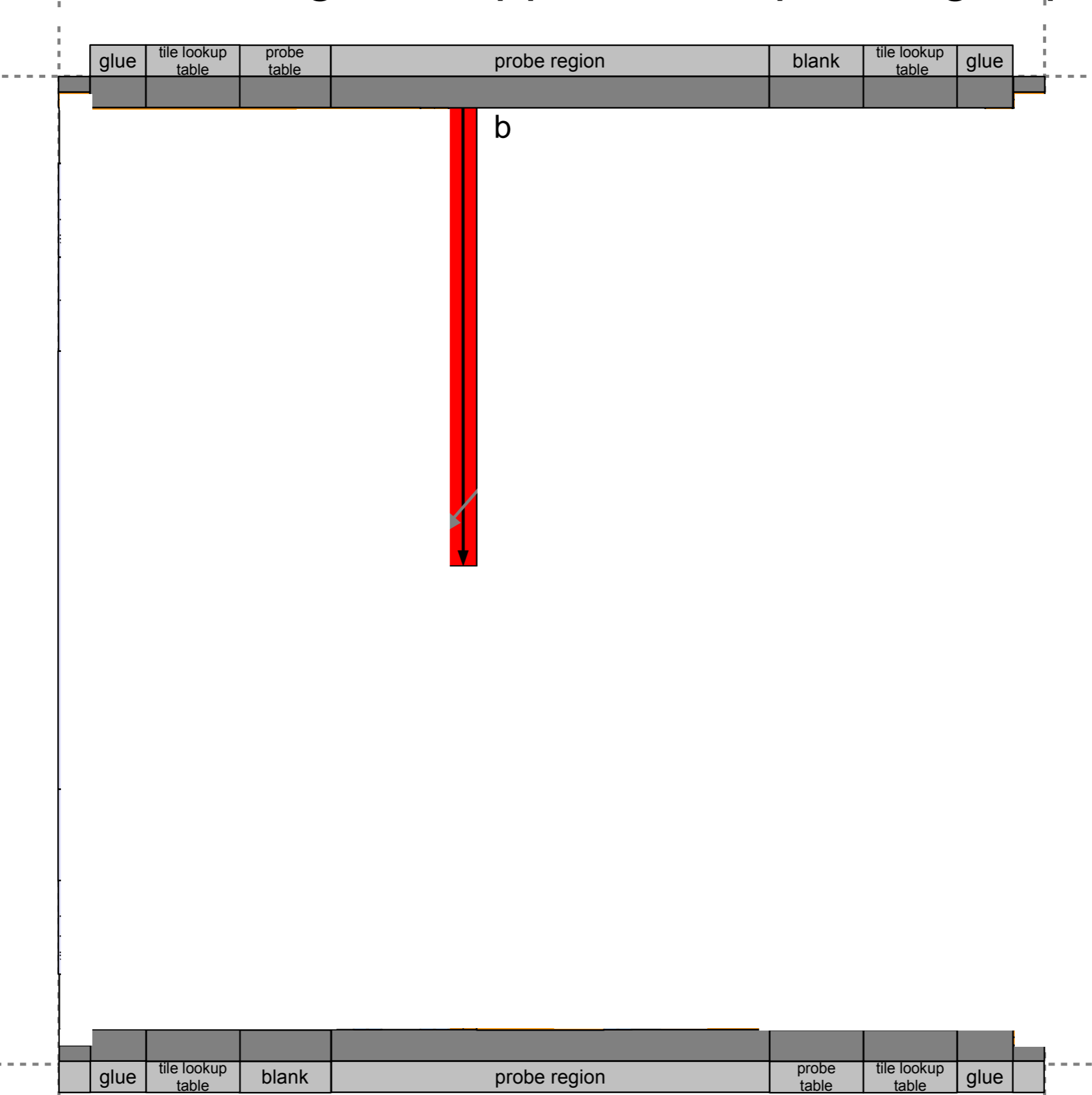
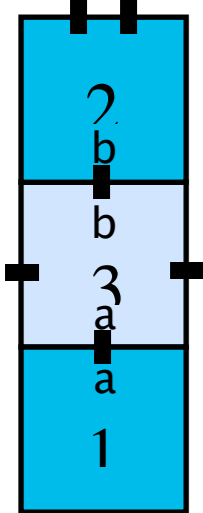


Uh oh!

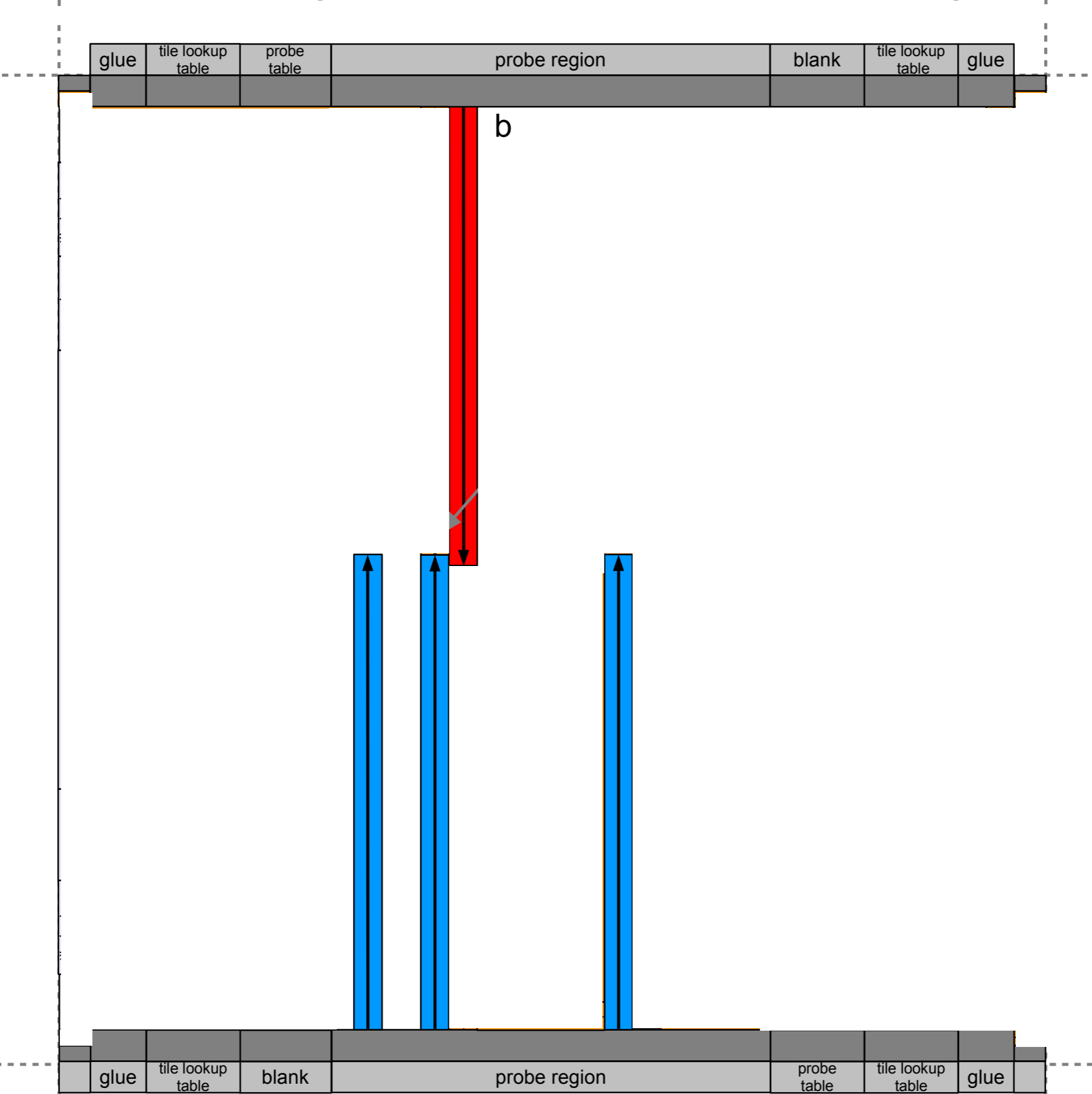
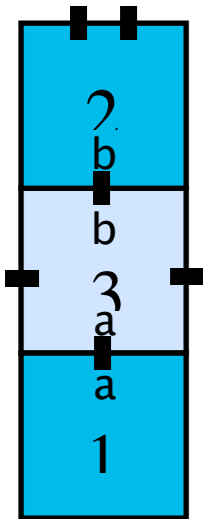
Two-sided binding with opposite cooperating supersides



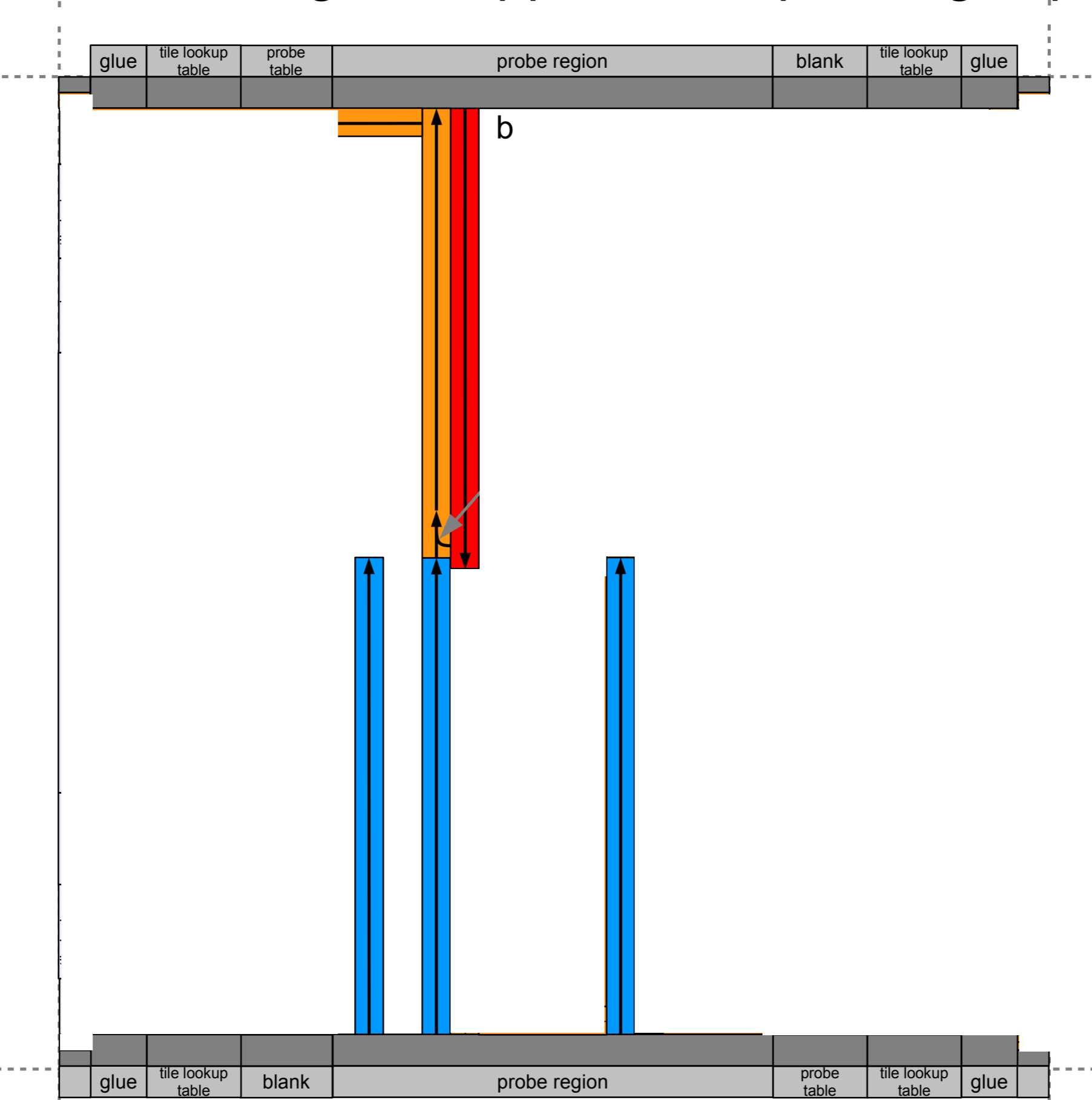
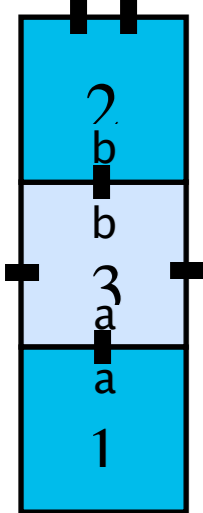
Two-sided binding with opposite cooperating supersides



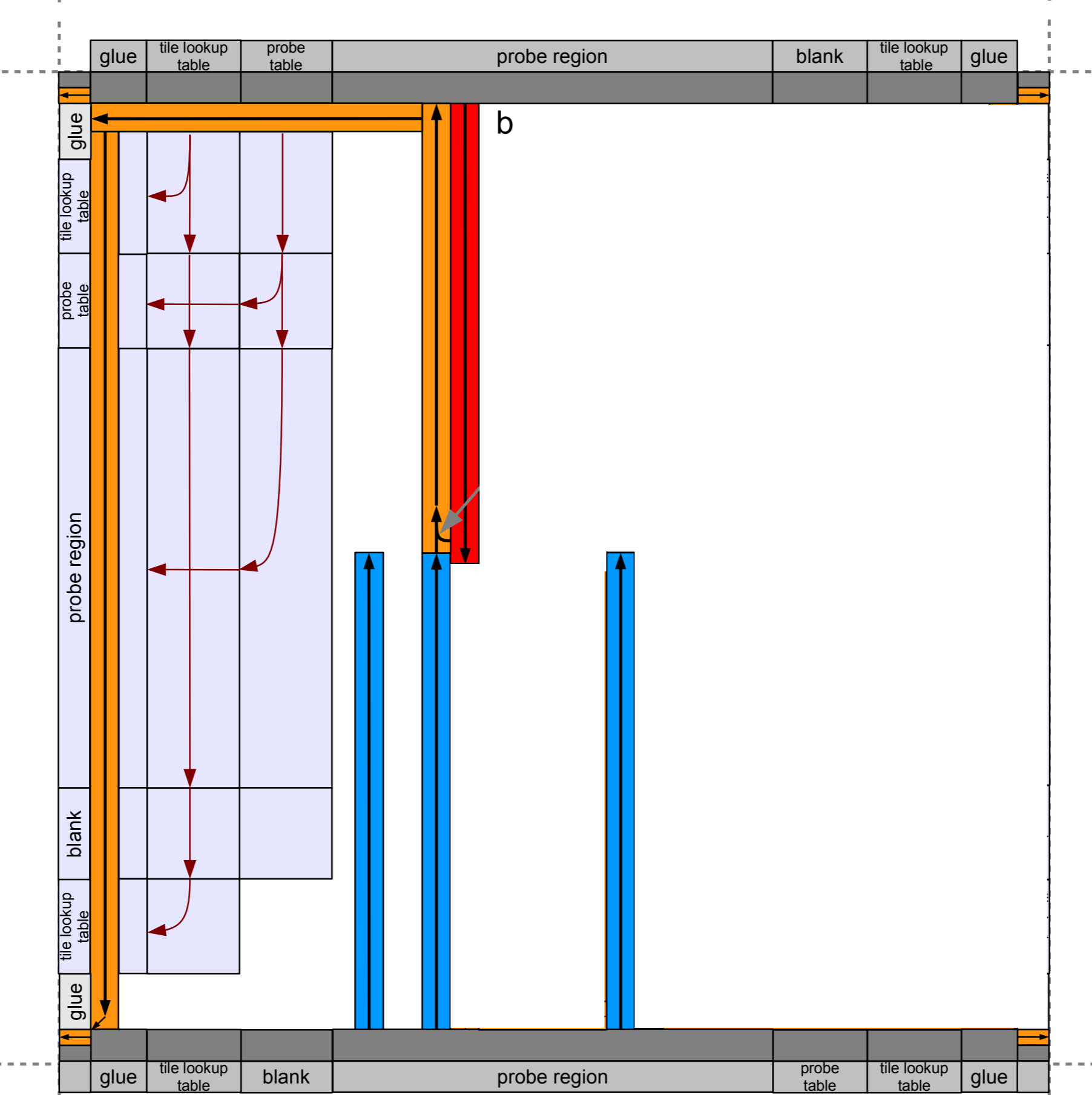
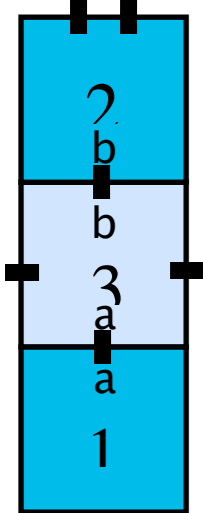
Two-sided binding with opposite cooperating supersides



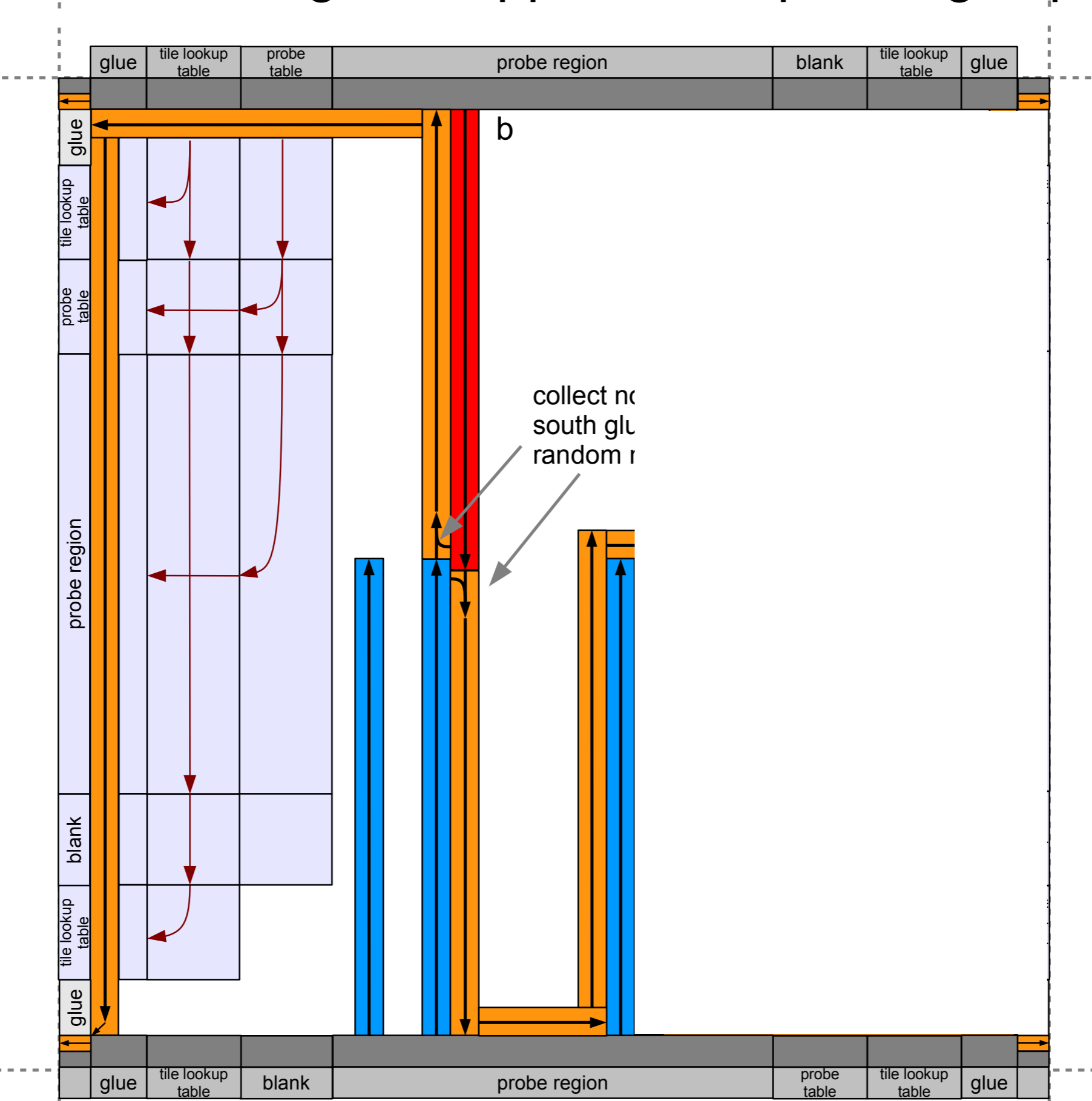
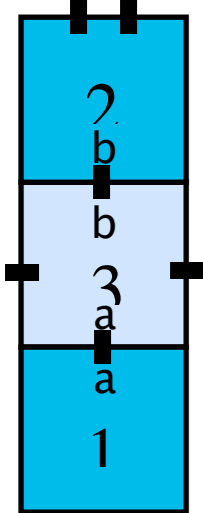
Two-sided binding with opposite cooperating supersides



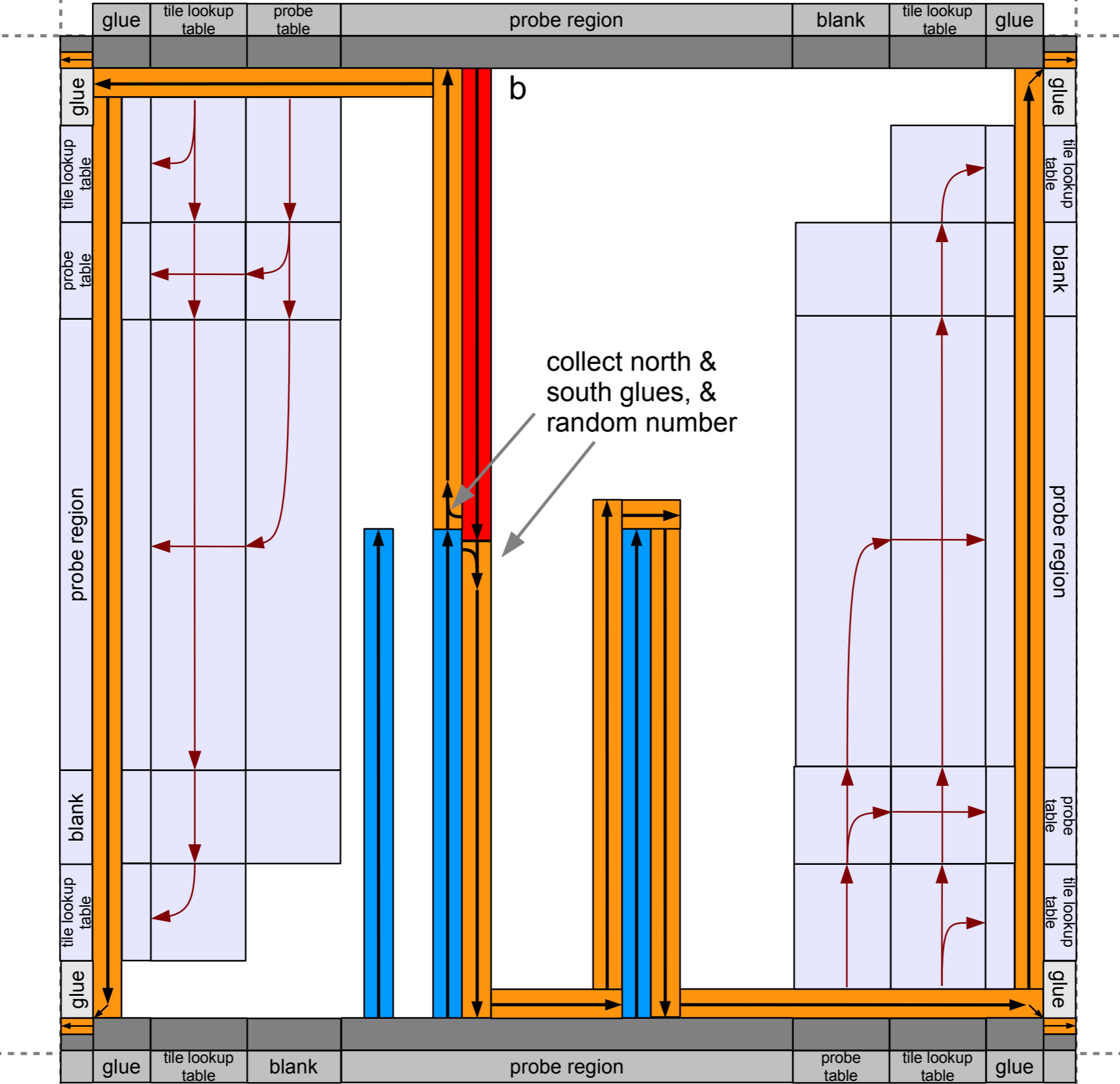
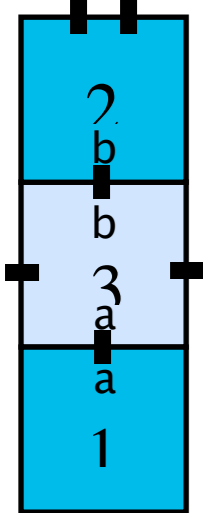
Two-sided binding with opposite cooperating supersides



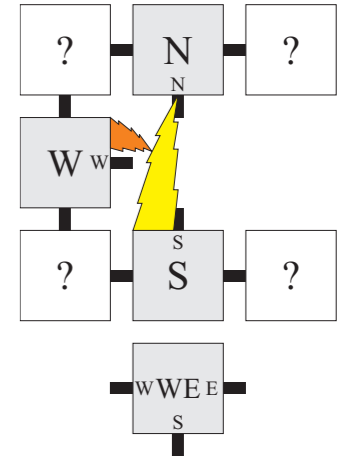
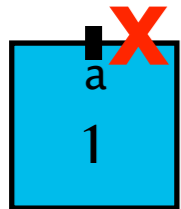
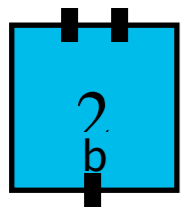
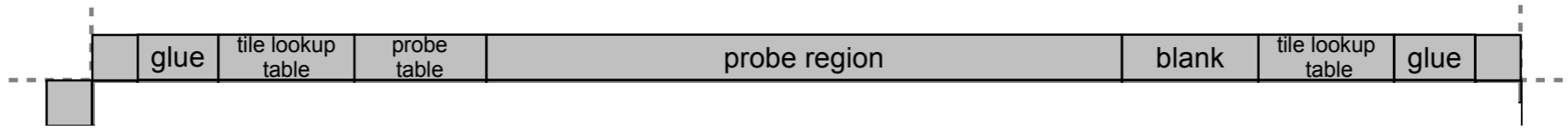
Two-sided binding with opposite cooperating supersides



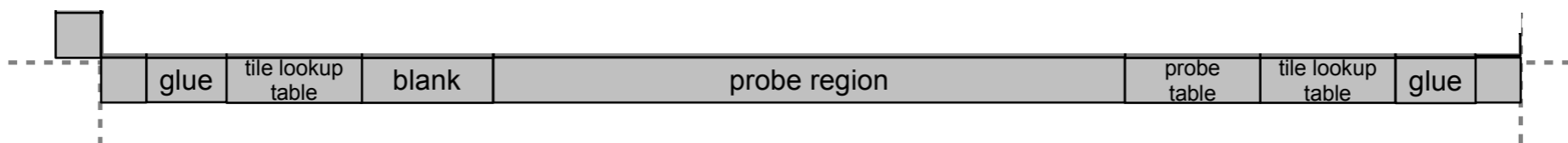
Two-sided binding with opposite cooperating supersides



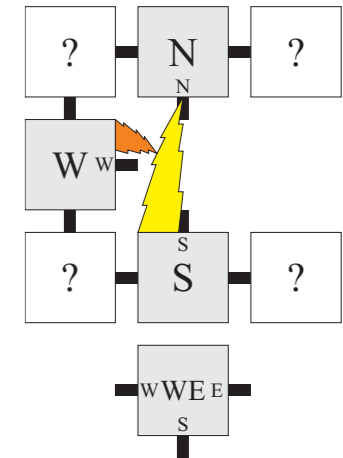
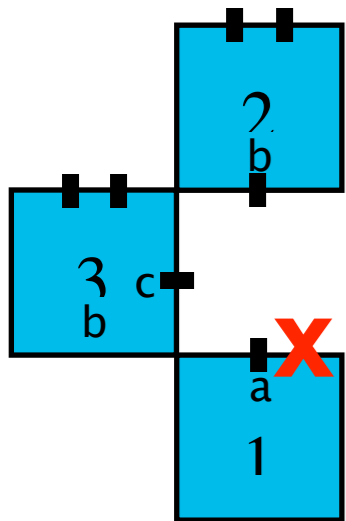
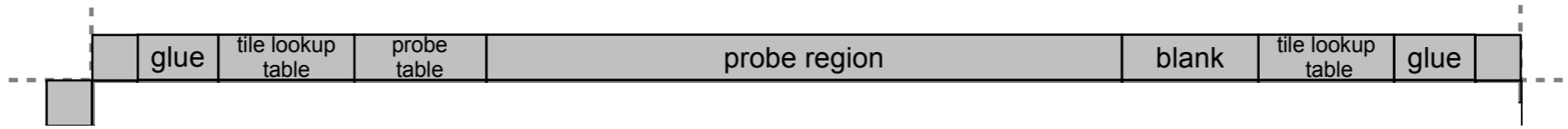
3-sided "uh-oh" example: probes miss each other



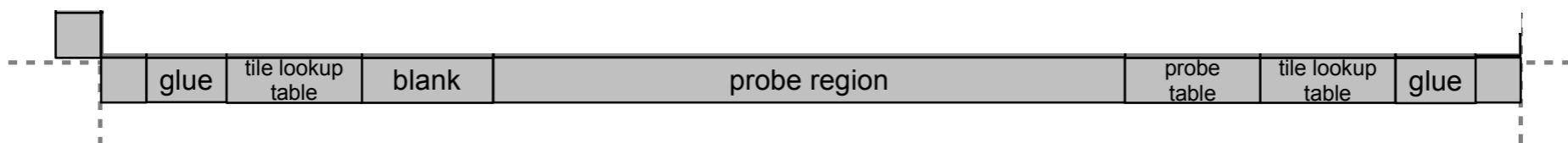
uh oh!



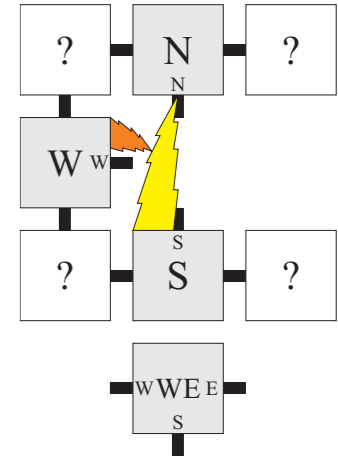
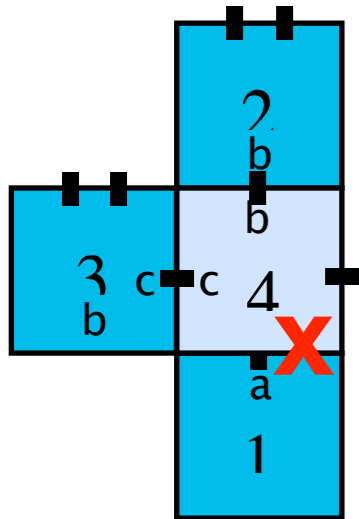
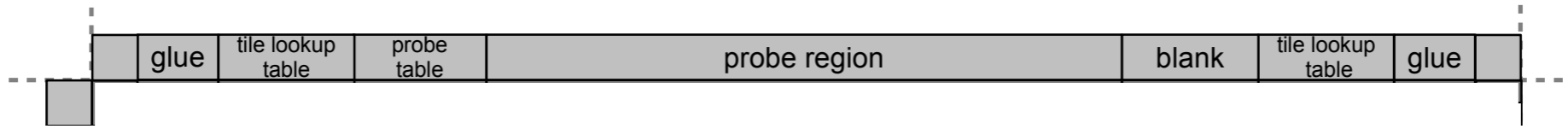
3-sided "uh-oh" example: probes miss each other



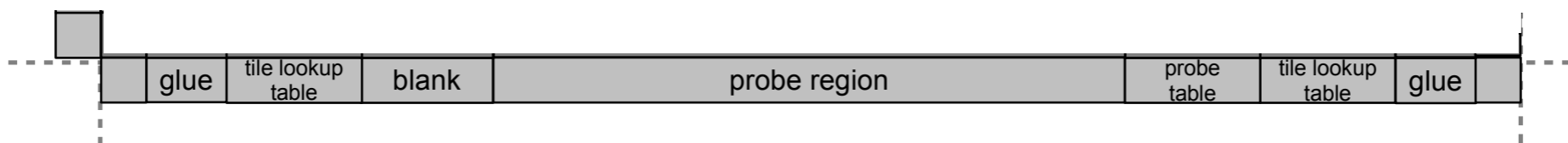
uh oh!



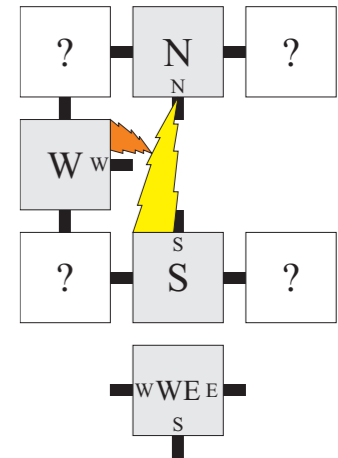
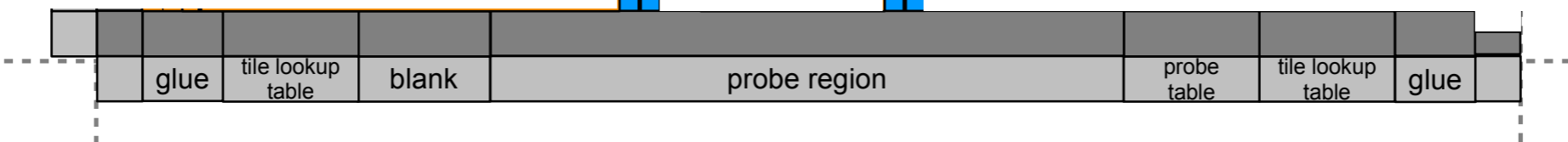
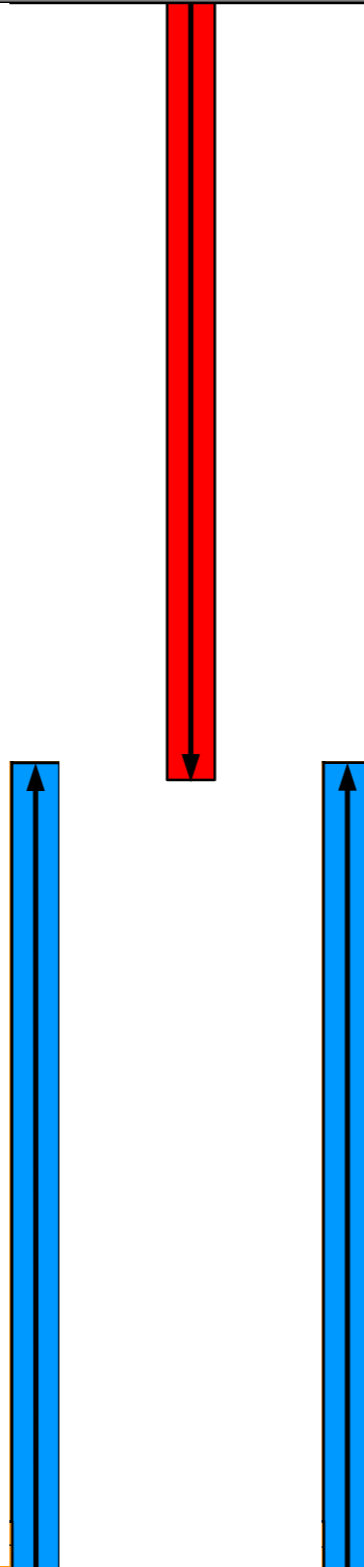
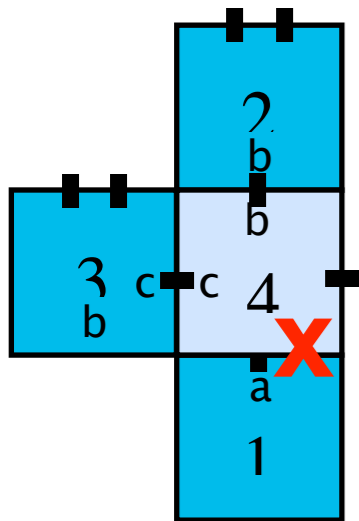
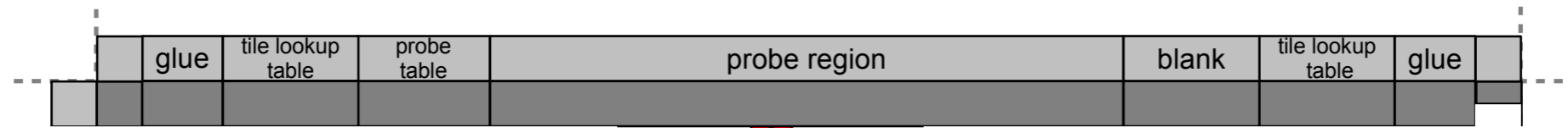
3-sided "uh-oh" example: probes miss each other



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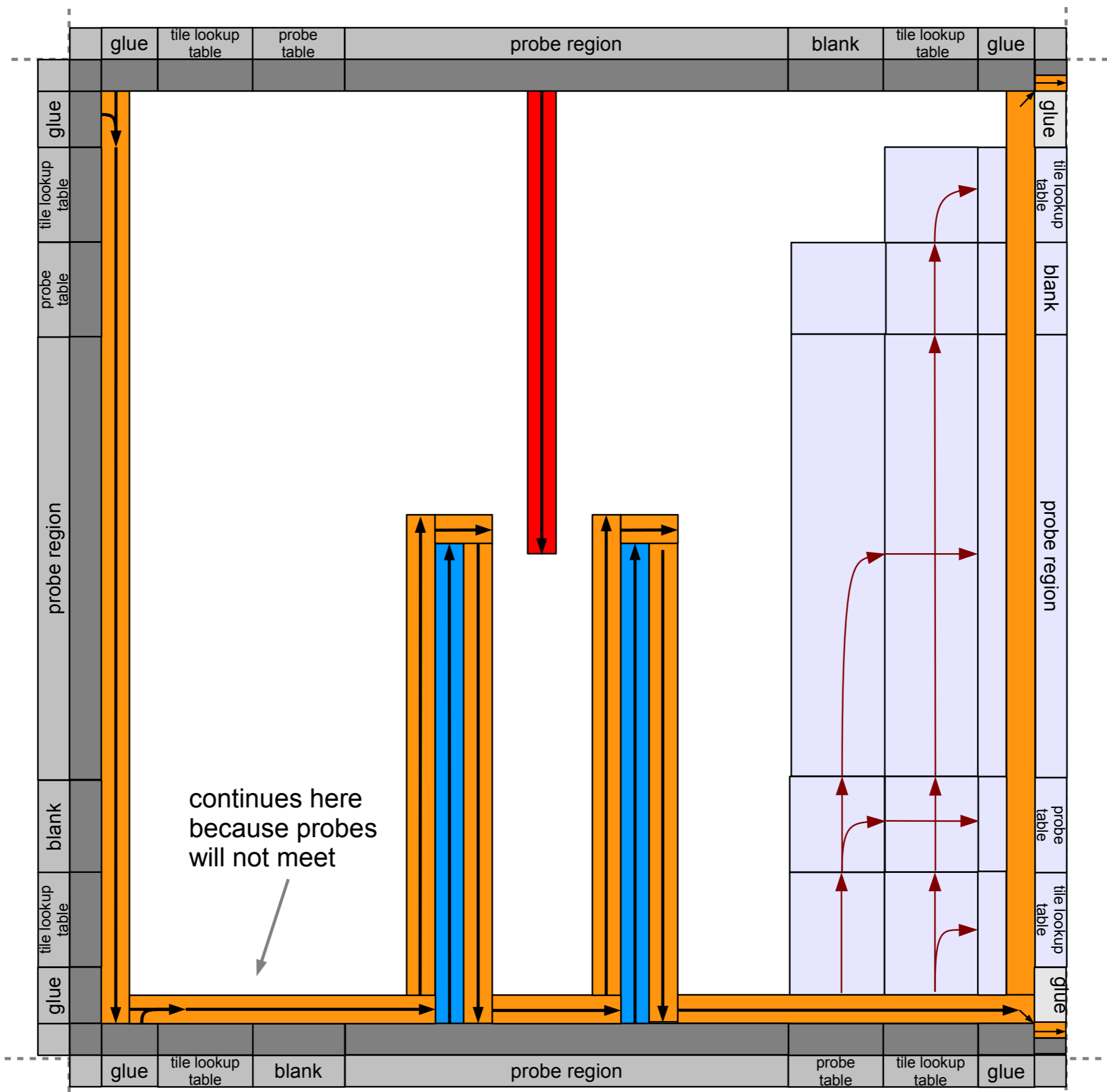
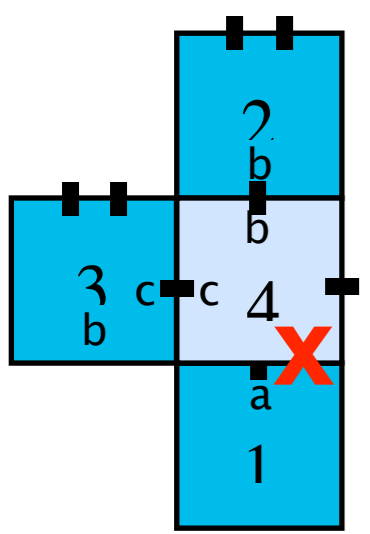


3-sided "uh-oh" example: probes miss each other

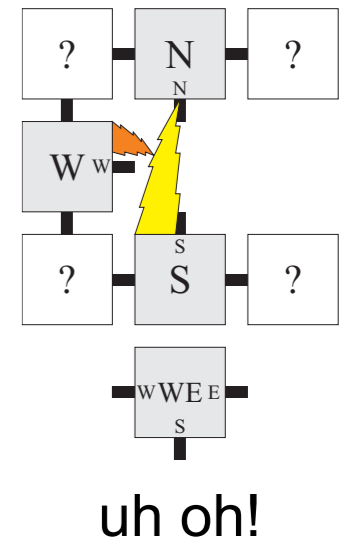


uh oh!

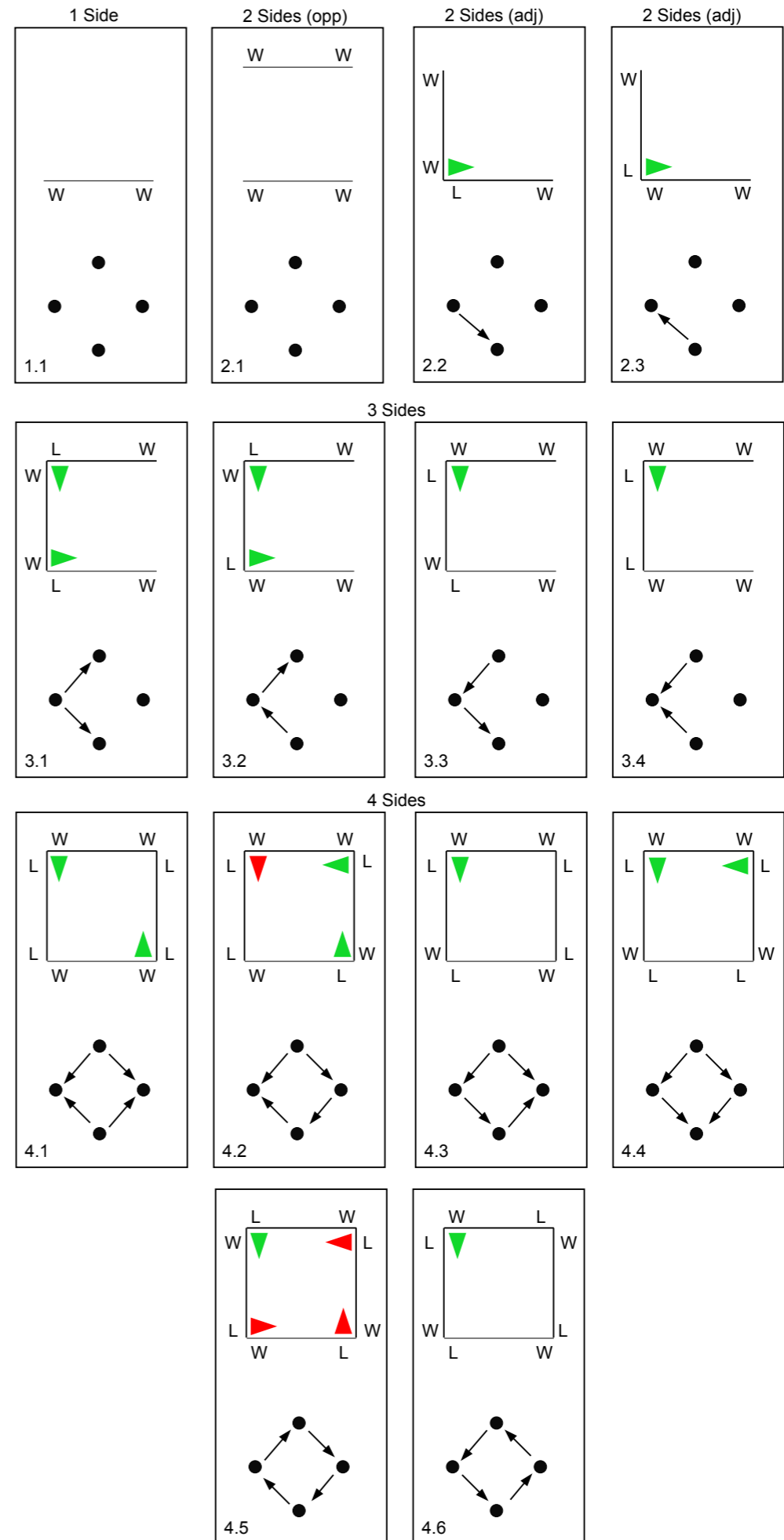
3-sided "uh-oh" example: probes miss each other



continues here because probes will not meet

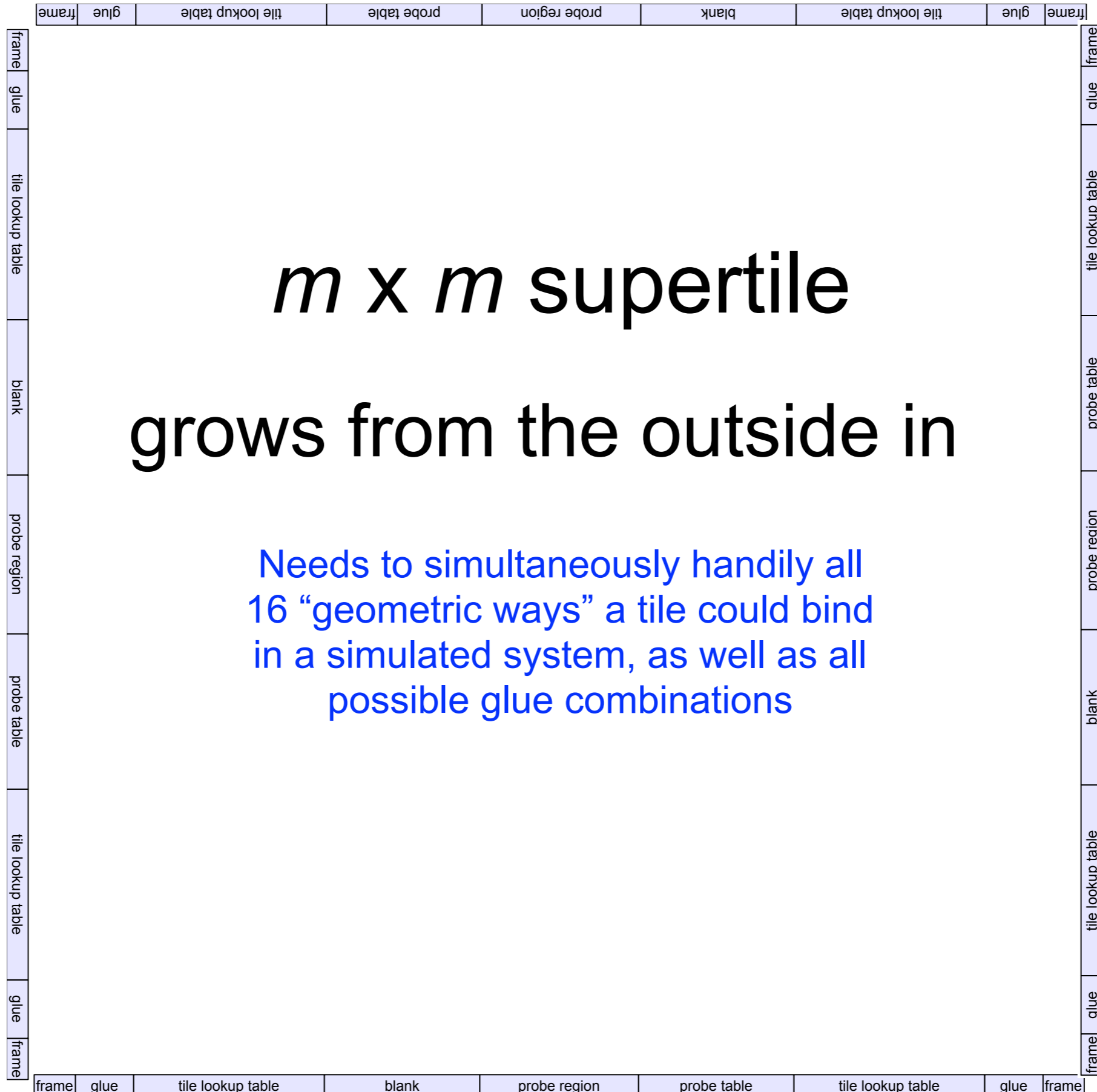


- Variety of cases for different orders of superside arrival
- Superside win/lose configurations and crawler initiation locations (green)
- Proof analogy:
 - Distributed game
 - Computation & geometry
 - Key challenge: make all the tricks work together

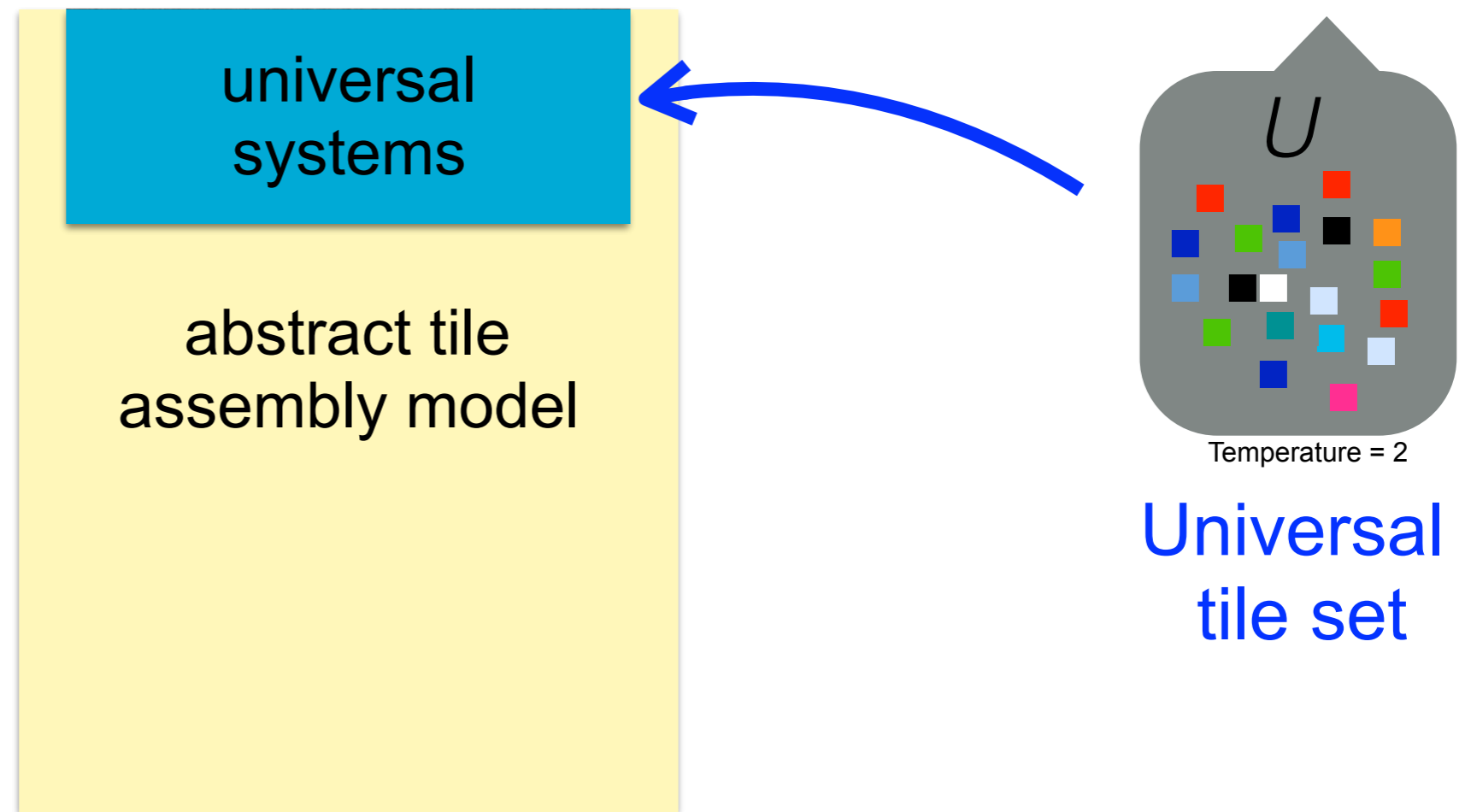


$m \times m$ supertile grows from the outside in

Needs to simultaneously handle all 16 “geometric ways” a tile could bind in a simulated system, as well as all possible glue combinations



Is the abstract tile assembly model intrinsically universal? **Yes!**

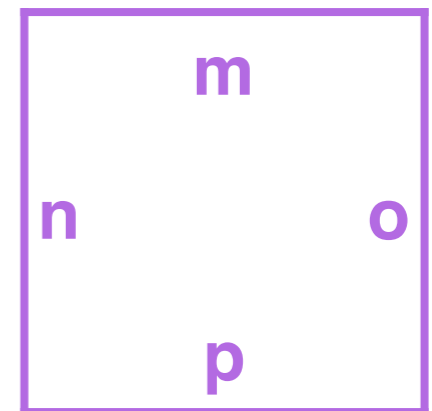
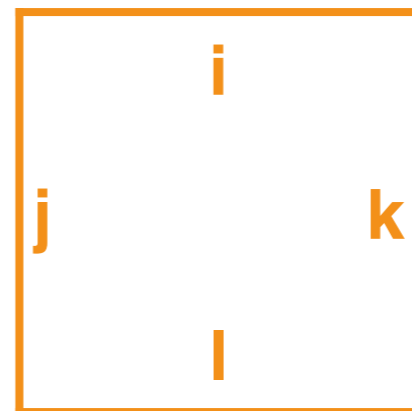
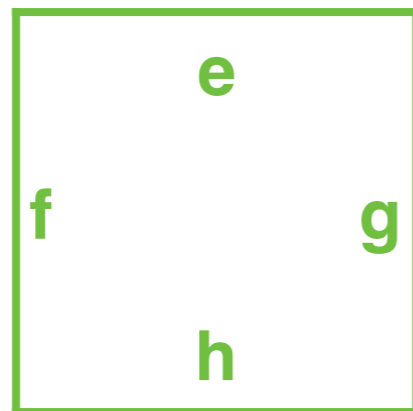
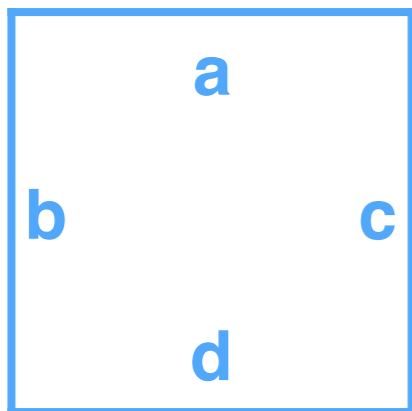


Theorem: There is a single intrinsically universal tile set U that simulates *any* tile assembly system

Doty, Lutz, Patitz, Schweller, Summers, Woods. FOCS 2012

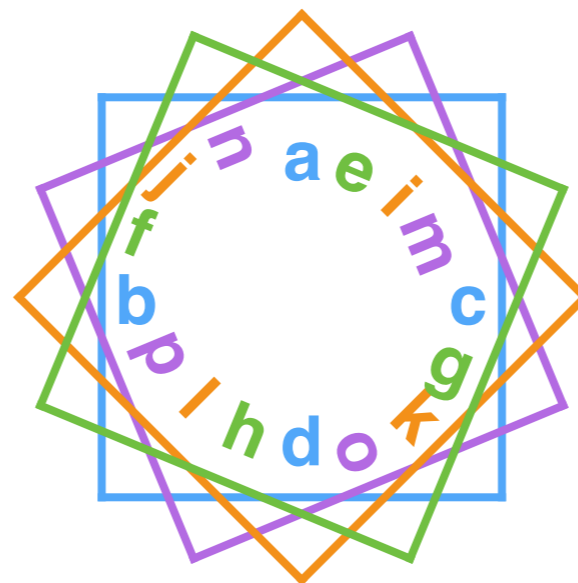
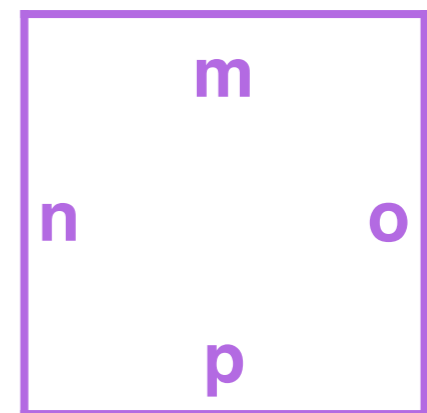
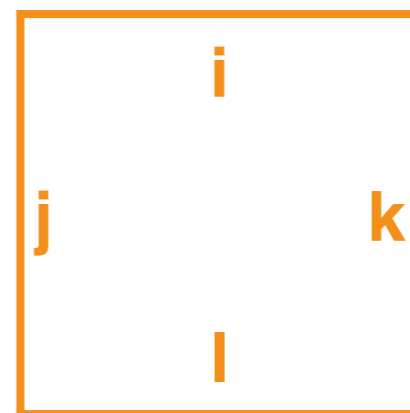
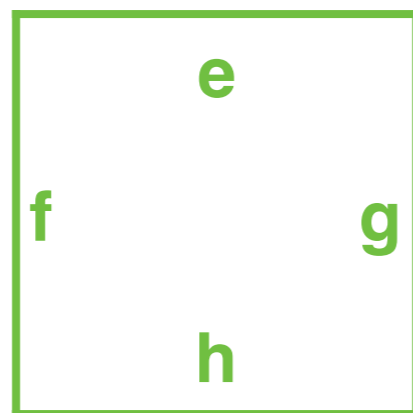
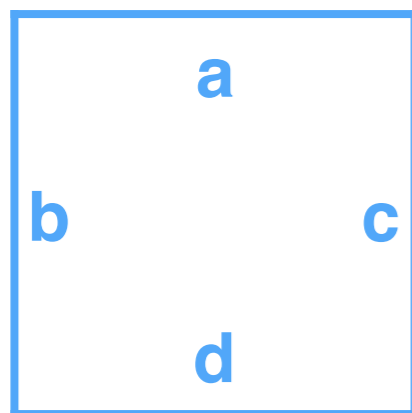
How many tiles do we need ?

- Just ONE with rotation!... What?!?... But a *polygonal* one



How many tiles do we need ?

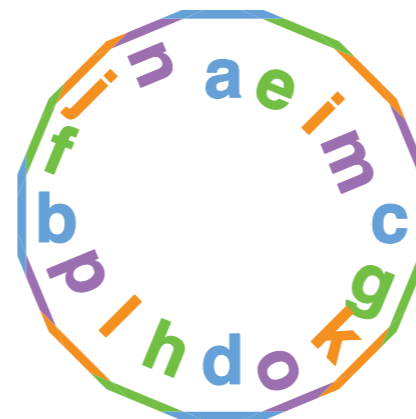
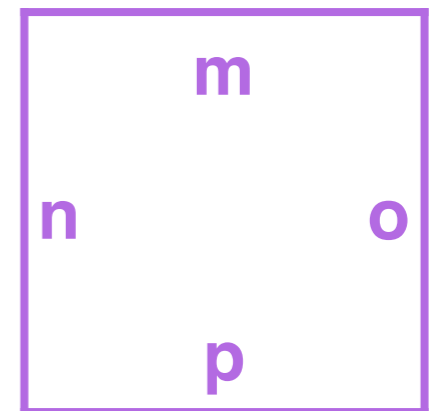
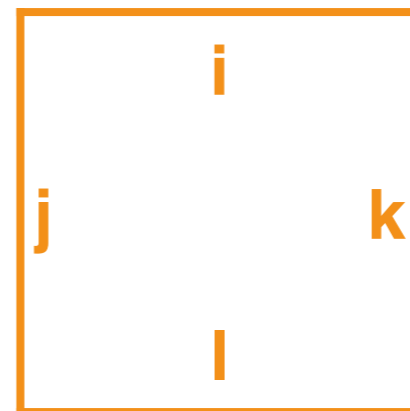
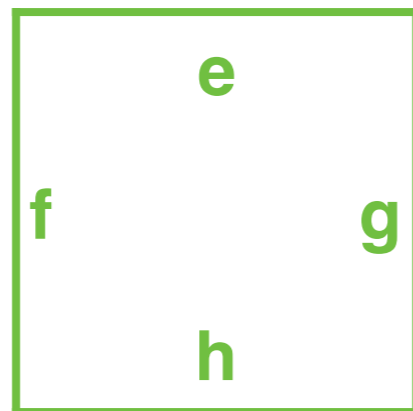
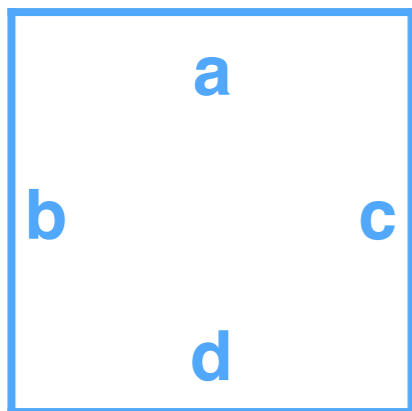
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Demaine Demaine Fekete Patitz Schweller Winslow Woods 2012

How many tiles do we need ?

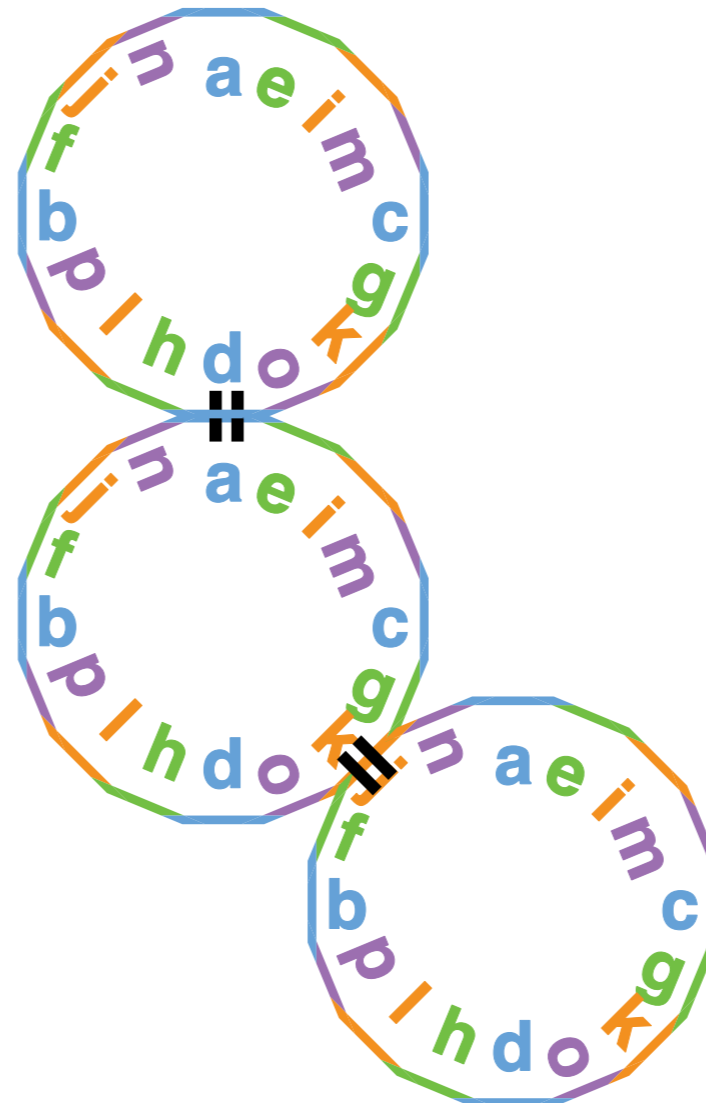
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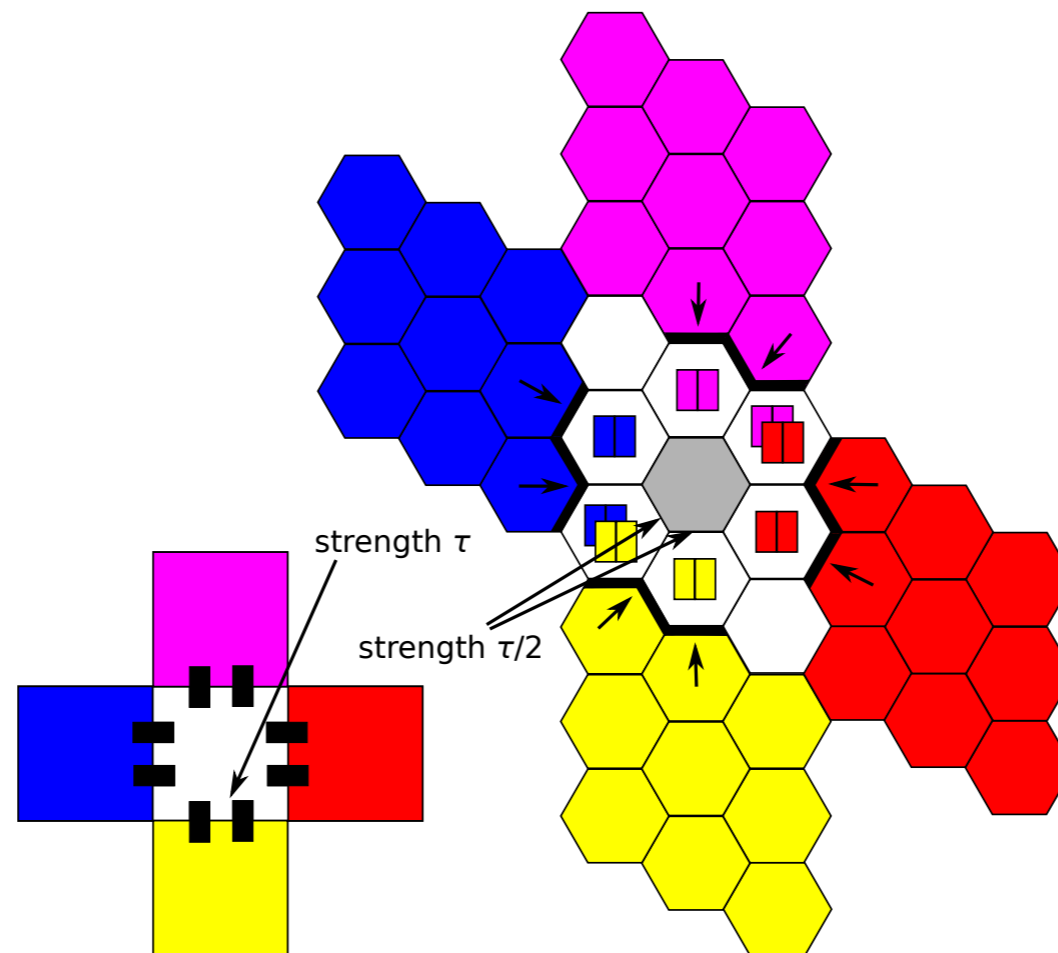
- Just ONE with rotation!... What?!?... But a *polygonal* one

**Problem with glue
of strength 2 !!!**

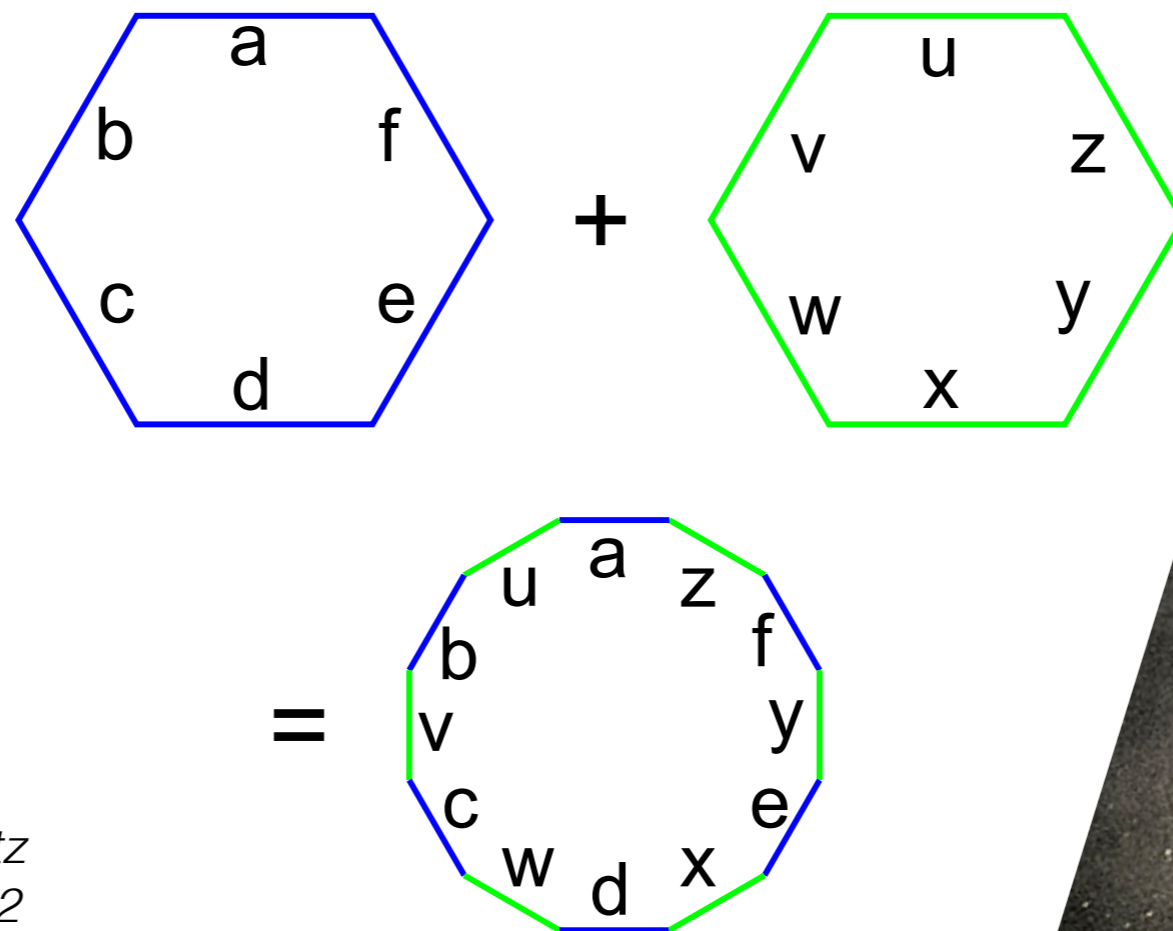


Demaine Demaine Fekete Patitz Schweller Winslow Woods 2012

Encoding strength 2 glues into strength 1 glue in hexagonal tiles



A **single** (polygonal) tile is enough !



*Demaine Demaine Fekete Patitz
Schweller Winslow Woods 2012*

The magic powder can
assemble anything!



One molecule is
enough !

Co-transcriptional folding

Joint work with Cody Geary Pierre-Étienne Meunier and
Shinnosuke Seki