

HW1 Molecular Programming

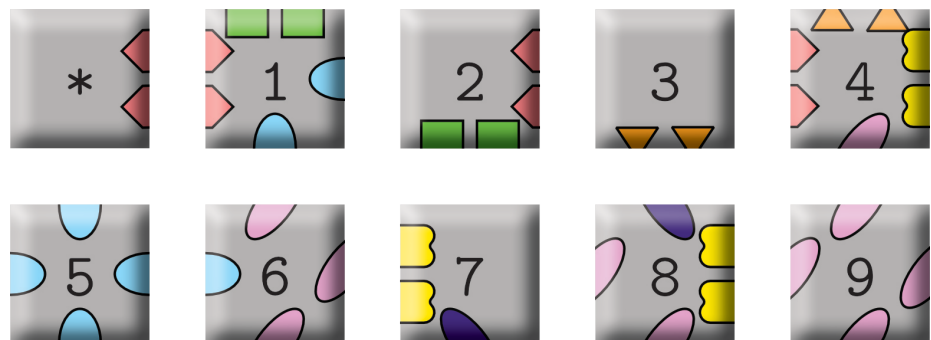
MPRI 2.11.1 16.09.2020 - Due on Wed. 23/09 before 16:15



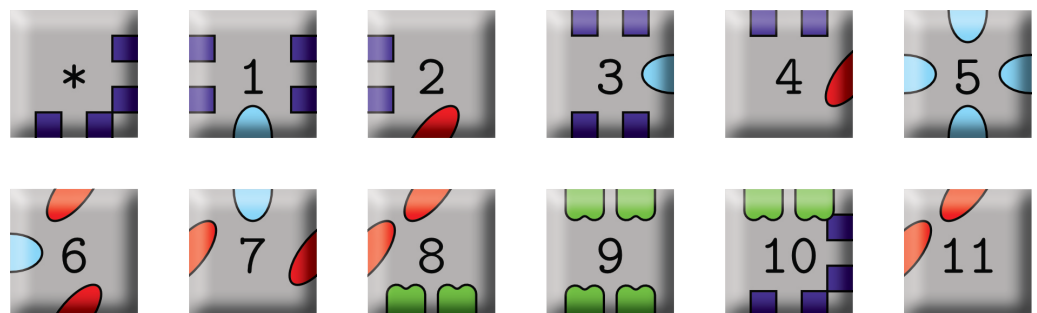
You are asked to complete the exercise marked with a [★] and to send me your solutions to:
 nicolas.schabanel@ens-lyon.fr
 as a PDF file named HW1-Lastname.pdf on Wed. 23/09 before 16:15.

■ **Exercise 1 (Algorithmic Self-Assembly).** Recall that the self-assembly process consists in, given a finite tileset (with infinitely many tiles of each type), starting from the seed tile (marked with a ★), glueing tiles with matching colors to the current aggregate so that each new tile is attached by at least two links to the aggregate (either on the same border or on two borders). Recall that a shape is *final* if no tiles can be attached to it anymore.

► **Question 1.1)** What is the exact family of final shapes self-assembled by the following tileset? (No proof nor justification is asked.) Indicate the local order of assembly by drawing arrows over the tiles of a generic final shape. Which are the two competing tiles that decide the size of the resulting final shape?

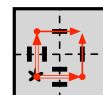


[★] **Exercise 2.** What is the family of shapes built by this tileset at temperature $T^0 = 2$? (no justification asked)



Indicate the assembly order with arrows⁽¹⁾ on a generic production. Is this a well-ordered tileset?

¹Draw an arrow $(i, j) \rightarrow (i', j')$ iff tile (i, j) is attached before tile (i', j') : for instance,



■ **Exercise 3.** Assume a random Poisson model where the random time X between two consecutive appearances of a tile of a given type τ at a given empty location follows an exponential law: $p(x) = c \cdot e^{-cx}$ where $c > 1$ is the concentration of the tiles of type τ . We want to prove the following theorem:

Theorem 1 (Adleman et al, 2001). Consider an ordered tile system \mathcal{T} that assembles deterministically a single shape S . Let \prec be the partial order of the assembly, i.e. such that $(i, j) \prec (k, l)$ if the tile at position (i, j) is attached before the tile at (k, l) by \mathcal{T} . With very high probability, the assembly time of a shape S by \mathcal{T} is:

$$O(\gamma \times \text{rank}(S))$$

where γ only depends on the concentrations and $\text{rank}(S)$ is the highest rank in the shape S (i.e. the length of the longest path in \prec).

► **Question 3.1)** Let X be an exponential random variable such that $p(X = x) = ce^{-cx}$ for all real $x \geq 0$, for some $c > 0$. Show that X is memoryless, i.e. for all $u, t \geq 0$,

$$p(X = t + u | X \geq u) = p(X = t)$$

Let T be the assembly time of the shape S , i.e. the time at which the last tile of shape S is attached. Let $X_{i,j}$ be the independent exponential random variable for the time between two consecutive appearances of the tile to be attached at position (i, j) in S . We denote by $w(P)$ the random variable for the weight of a \prec -path P , defined as: $w(P) = \sum_{(i,j) \in P} X_{i,j}$.

► **Question 3.2)** Show that:

$$T = \max_{\prec\text{-path } P} w(P)$$

▷ Hint. Proceed by recurrence on the rank of the tiles and show that for all tile (i, j) , its assembly time is the random variable $T_{ij} = \max_{\prec\text{-path } P \text{ from } (0,0) \text{ to } (i,j)} w(P)$.

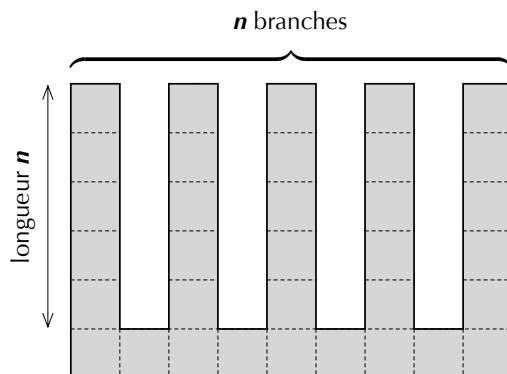
► **Question 3.3)** Let X_1, \dots, X_ℓ be ℓ independent exponential variables s.t. $p(X_i = x) = c_i e^{-c_i x}$ with $c_i > 1$. Show that there is γ which depends only of $\min_i c_i$ such that: for all $n \geq \ell$,

$$\Pr\{X_1 + \dots + X_\ell \geq \gamma \cdot n\} \leq 1/4^\ell \cdot e^{-\gamma(n-\ell)}$$

▷ Hint. Note that $\mathbb{E}[e^{X_i}] < \infty$ and apply Markov inequality to $Z = e^{X_1 + \dots + X_\ell}$.

► **Question 3.4)** Conclude.

■ **Exercise 4.** Propose a staged assembly scheme at temperature $T^\circ = 1$ of the shape family E of candelabrams with n branches of length n .



Describe the tiles, glues, their number, the number of stages and the number of different bechers needed. Give an illustration of the stages to build a generic production.

■ **Exercise 5 (Turns).**

► **Question 5.1)** Provide a fixed tile set which at temperature $T^\circ = 2$ assembles from a top line of $n + 2$ seed tiles encoding n bits surrounded by \star , into a $(n + 2) \times (n + 2)$ -square where these n bits are copied on its right, left and bottom sides as shown on figure 1. How many tiles do you have?

No justification asked, just provide a large enough typical assembly together with its assembly order.

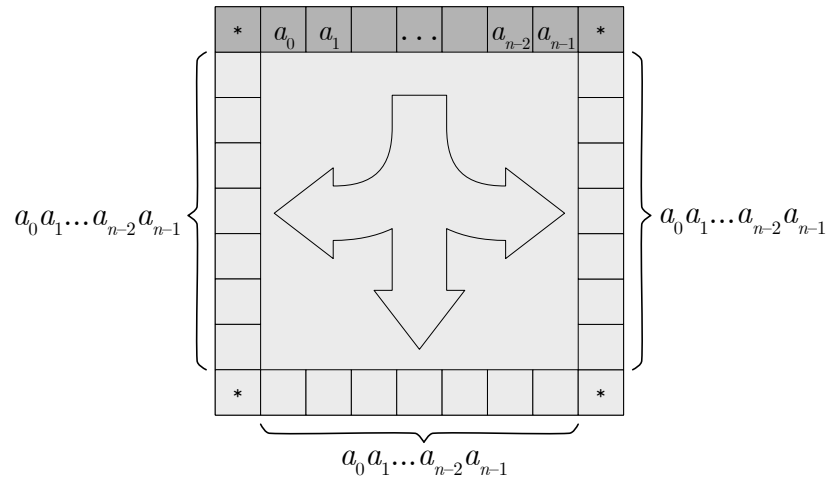


Figure 1: Question 5.1: All turns