

HW2

MPRI 2.11.1

Molecular Programming

23.09.2020 - Due on Wed. 30/09 before 16:15



You are asked to complete the exercise marked with a [★] and to send me your solutions to:

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as a PDF file named **HW2-Lastname.pdf** on Wed. 30/09 before 16:15.

■ **Exercise 1 (Exponential random variables & kTAM implementation).** Recall that an *exponential random variable* X with parameter $\lambda > 0$ is defined by: $(\forall x \geq 0) \Pr\{X \geq x\} = e^{-\lambda x}$.

► **Question 1.1)** Compute $\mathbb{E}[X]$.

▷ **Hint.** Recall that if X is a non-negative random variable, then $\mathbb{E}[X] = \int_0^\infty \Pr\{X \geq x\} dx$.

► **Question 1.2)** Show that the exponential distribution is memoryless, i.e. if X is exponentially distributed with parameter λ , then $(\forall t, u \geq 0) \Pr\{X \geq t + u \mid X \geq t\} = \Pr\{X \geq u\}$.

Let X and Y be two independent exponentially distributed random variables with respective parameters λ and μ .

► **Question 1.3)** Show that $\min(X, Y)$ is also exponentially distributed. What is its parameter?

► **Question 1.4)** What is the probability that $\min(X, Y) = X$?

► **Question 1.5)** Same questions as the two above for n independent exponentially distributed variables X_1, \dots, X_n with parameters $\lambda_1, \dots, \lambda_n$.

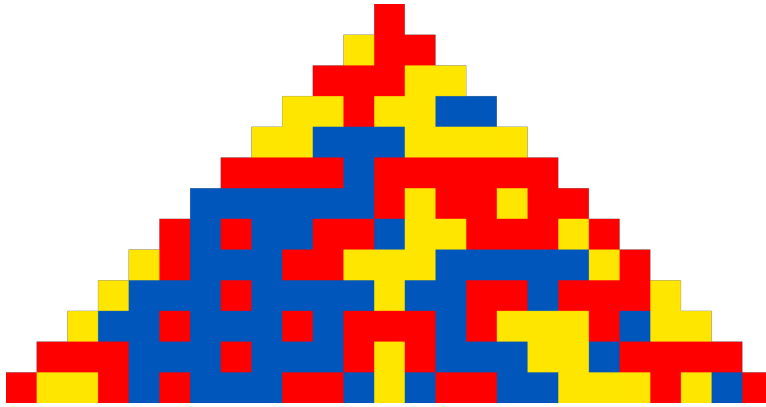
► **Question 1.6)** Assume that a non-negative random variable X is given by its tail distribution $F(x) = \Pr\{X \geq x\}$. Show that X is identically distributed as $F^{-1}(U)$ where U is a uniform random variable in $[0, 1]$.

Describe how to sample an exponential random variable of rate λ .

► **Question 1.7)** Propose an algorithm together with a data structure to implement the kTAM model with attachment rate $r_f = k_f[\text{Strand}] = k_f e^{-G_{mc}}$ and detachment rate $r_{s,b} = k_d e^{-b \cdot G_{se}}$ where b is the number of bonds made by the strand with the current aggregate.

Use parameters $k_f = 10^6/M/\text{sec}$, $G_{mc} = 12.9$ and $G_{se} = 6.5$ for the algorithmic phase.

[★] **Exercise 2 (Tilset for simulating cellular automata).** A cellular automaton consists of a finite set of *states* Q , a function $f : Q^3 \rightarrow Q$, called the *rule*, and an initial configuration $c^0 \in Q^*$. The configuration at time $t + 1$ is obtained from the configuration at time t as follows: $c_i^{t+1} = f(c_i^t, c_{i+1}^t, c_{i+2}^t)$ for $0 \leq i < |c^t| - 2$. The calculation stops at the first time T such that $|c^T| < 3$ and the result of the computation is c_0^T . A classic visualization of the computation of a cellular automaton consists of a pyramid where the bottom line is the initial configuration and time goes upwards. Here is an example:

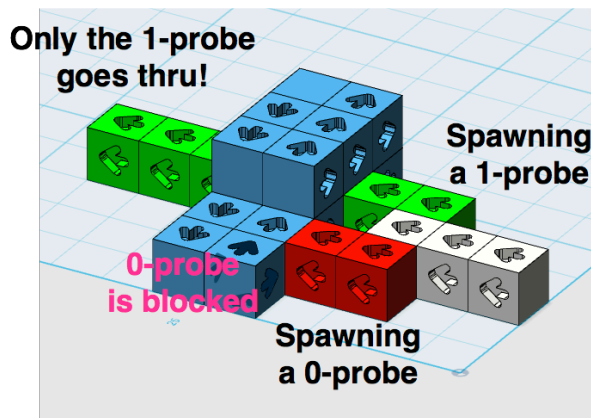


$$\text{for the rule } f(x, y, z) = \begin{cases} \text{yellow} & \text{if } \{x, y, z\} = \{\text{red}\} \text{ or } \{\text{yellow}, \text{blue}\} \\ \text{blue} & \text{if } \{x, y, z\} = \{\text{yellow}\} \text{ or } \{\text{blue}, \text{red}\} \\ \text{red} & \text{if } \{x, y, z\} = \{\text{blue}\} \text{ or } \{\text{yellow}, \text{red}\} \text{ or } \{\text{yellow}, \text{blue}, \text{red}\} \end{cases}$$

► **Question 2.1)** Propose a finite tileset whose self-assembly simulates the computation of any Q -state cellular automata from any initial configuration and whose size is independent of the initial configuration length. Give a generic example of the execution of your assembly for generic computation steps. Give the number of variants of each tile type as a function of $|Q|$. Provide the procedure which selects the tiles used to simulate a given Q -state cellular automaton.

▷ Hint. Do you need upscaling? Consider reshaping the pyramid to simplify your design.

■ **Exercise 3 (Probabilistic simulation Turing Machine at $T^\circ = 1$ in 2D).** Recall that in 3D, for any single-tape binary-alphabet Turing machine M , there is a tile set which simulates M using a clever trick to encode 0s and 1s. These are encoded with bridges and read using two probes where only one go through the bridge:



► **Question 3.1)** By adjusting the concentrations (and thus the rate at which the different tiles attached), describe a tile set together with concentrations for each tile type, that simulates a given single-tape binary-alphabet Turing machine M with an arbitrary small error ε for each symbol read in 2D at temperature $T^\circ = 1$.