HW1 Molecular Programming
MPRI 2.11.1

16.12.2021 - Due on Thu. 06/01 before 08:45

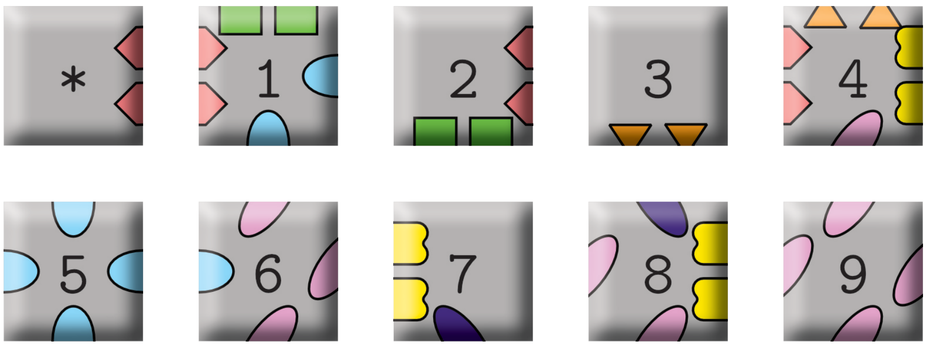
You are asked to complete the exercise marked with a [★] and to send me your solutions to:

nicolas.schabanel@ens-lyon.fr

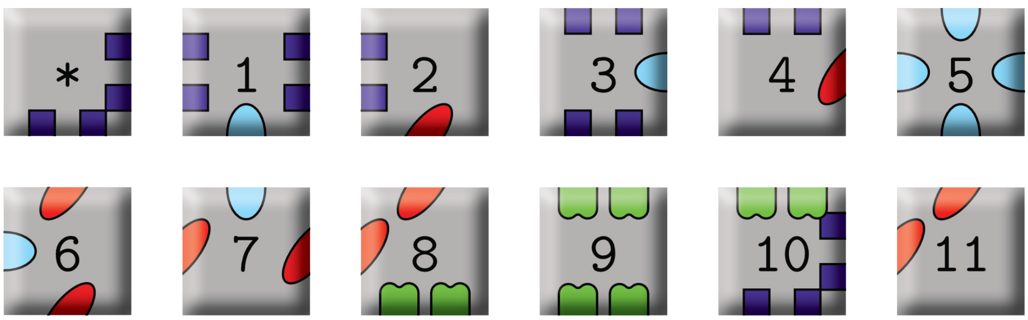
as a PDF file named **HW1-Lastname.pdf** on **Thu. 06/01 before 08:45**.

■ **Exercise 1 (Algorithmic Self-Assembly).** Recall that the self-assembly process consists in, given a finite tileset (with infinitely many tiles of each type), starting from the seed tile (marked with a ★), glueing tiles with matching colors to the current aggregate so that each new tile is attached by at least *two* links to the aggregate (either on the same border or on two borders). Recall that a shape is *final* if no tiles can be attached to it anymore.

► **Question 1.1)** *What is the exact family of final shapes self-assembled by the following tileset? (No proof nor justification is asked.) Indicate the local order of assembly by drawing arrows over the tiles of a generic final shape. Which are the two competing tiles that decide the size of the resulting final shape?*

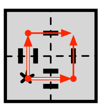


[★] Exercise 2. What is the family of shapes built by this tileset at temperature $T^\circ = 2$? *(no justification asked)*



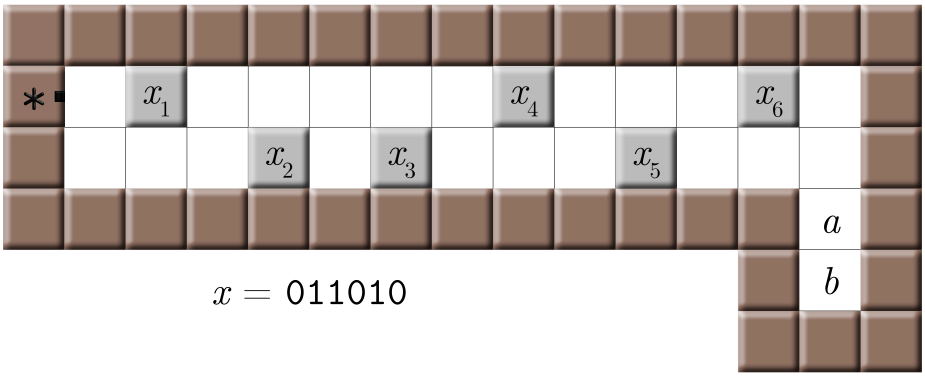
Indicate the assembly order with arrows[1] on a generic production. Is this a well-ordered tileset?

---

[1] Draw an arrow $(i,j) \to (i',j')$ iff tile (i,j) is attachedbefore tile (i',j'): for instance, 

■ **Exercise 3 (Bit-cave).** We want to design a tileset that computes at temperature $T^\circ = 1$ the parity of a number x written in binary $x_1 \dots x_n$: parity$(x) = (x_1 + \dots + x_n) \mod 2$. Remember that *glue mismatch* are allowed: a tile can attach to the agregate as soon as there is at least on side of the tile that agrees with a side of the current agregate (even if the other sides do not agree).

Consider the "cave" seed (in brown below) where the i-th bit of x is encoded in the $(2i+1)$-th column by a (grey) tile placed either on the top row if $x_i = 0$ or in the bottom row if $x_i = 1$.



► **Question 3.1)** *Give a (absolute constant size) tileset that at temperature $T^\circ = 1$, in presence of the seed above:*

- *places a tile at position a and none at position b if and only if* parity$(x) = 0$*; and*
- *places a tile at position b if and only if* parity$(x) = 1$.

Give a generic assembly (run your tileset on the example above) indicating the order of assembly. How many tile types do you have? No mathematical justification asked however a simple explanation of the role of each tile type is necessary.

■ **Exercise 4.** Assume a random Poisson model where the random time X between two consecutive appearances of a tile of a given type τ at a given empty location follows an exponential law: $p(x) = c \cdot e^{-cx}$ where $c > 1$ is the concentration of the tiles of type τ. We want to prove the following theorem:

Theorem 1 (Adleman et al, 2001). *Consider an ordered tile system $\mathcal{T}$ that assembles deterministically a single shape S. Let $\prec$ be the partial order of the assembly, i.e. such that $(i,j) \prec (k,l)$ if the tile at position (i,j) is attached before the tile at (k,l) by $\mathcal{T}$. With very high probably, the assembly time of a shape S by $\mathcal{T}$ is:*

$$O(\gamma \times \text{rank}(S))$$

where γ only depends on the concentrations and rank(S) *is the highest rank in the shape S (i.e. the length of the longest path in $\prec$).*

► **Question 4.1)** *Let X be an exponential random variable such that $p(X = x) = ce^{-cx}$ for all real $x \geqslant 0$, for some $c > 0$. Show that X is memoryless, i.e. for all $u, t \geqslant 0$,*

$$p(X = t + u | X \geqslant u) = p(X = t)$$

Let T be the assembly time of the shape S, i.e. the time at which the last tile of shape S is attached. Let $X_{i,j}$ be the independant exponential random variable for the time between two consecutive appearances of the tile to be attached at position (i,j) in S. We denote by $w(P)$ the random variable for the weight of a $\prec$-path P, defined as: $w(P) = \sum_{(i,j)\in P} X_{i,j}$.

▶ **Question 4.2)** *Show that:*

$$T = \max_{\prec\text{-path } P} w(P)$$

▷ *Hint. Proceed by recurrence on the rank of the tiles and show that for all tile (i,j), its assembly time is the random variable $T_{ij} = \max_{\prec\text{-path } P \text{ from } (0,0) \text{ to } (i,j)} w(P)$.*

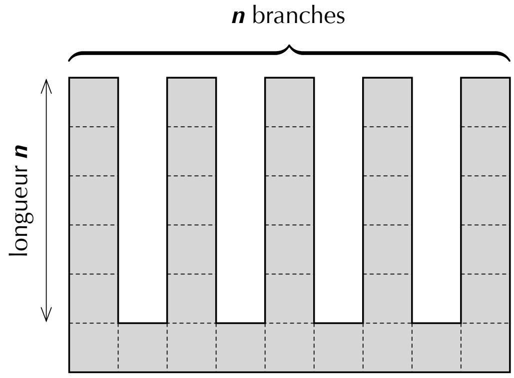
▶ **Question 4.3)** *Let $X_1, \ldots, X_\ell$ be ℓ independent exponential variables s.t. $p(X_i = x) = c_i e^{-c_i x}$ with $c_i > 1$. Show that there is γ which depends only of $\min_i c_i$ such that: for all $n \geqslant \ell$,*

$$\Pr\{X_1 + \cdots + \ell \geqslant \gamma \cdot n\} \leqslant 1/4^\ell \cdot e^{-\gamma(n-\ell)}$$

▷ *Hint. Note that $\mathbb{E}[e^{X_i}] < \infty$ and apply Markov inequality to $Z = e^{X_1+\cdots+X_\ell}$.*

▶ **Question 4.4)** *Conclude.*

■ **Exercise 5.** Propose a staged assembly scheme at temperature $T^\circ = 1$ of the shape family E of candelabrums with n branches of length n.



Describe the tiles, glues, their number, the number of stages and the number of different bechers needed. Give an illustration of the stages to build a generic production.