HW3 Molecular Programming

MPRI 2.11.1

10.02.2022 - **Due on Thu. 24/02 before 08:45**

You are asked to complete the exercise marked with a [★] and to send me your solutions to:

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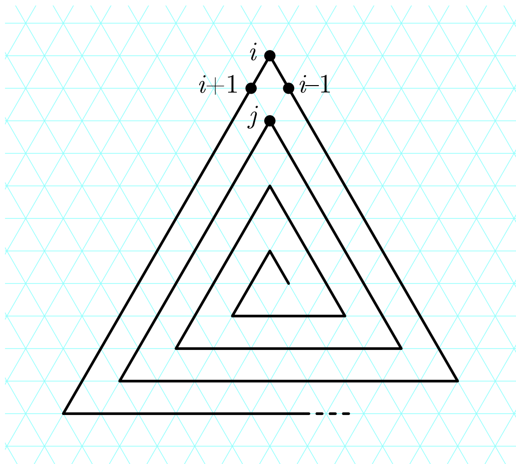
as a PDF file named **HW3-Lastname.pdf** on **Thu. 24/02 before 08:45**.

[★] Exercise 1 (Oritatami — Impossible triangle path). We want to prove that no deterministic oritatami system with delay $\delta \leqslant 2$ can fold according to the infinite triangular spiral below. Recall that the transcript t of an oritatami system (t is the sequence of bead types) is *ultimately periodic*, i.e. there is an i_0 and a period T such that for all $i \geqslant 0$, $t_{i_0+i} = t_{i_0+T+i}$.



► ***Question 1.1)*** *Prove than no deterministic delay-1 oritatami system can fold according to this spiral.*

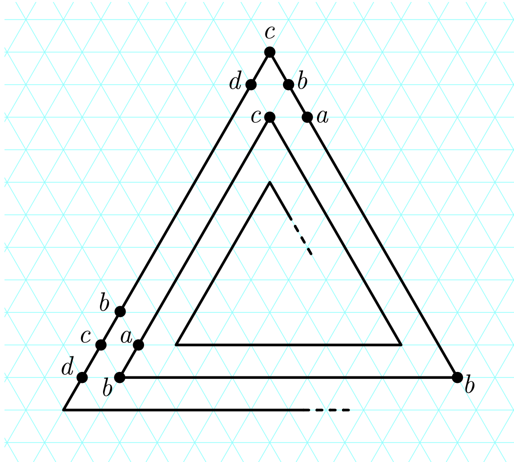
Answer. ▷ The bead at the corner cannot make any bond with anyone, it is thus impossible to place correctly at delay 1. ◁

Let us consider now a deterministic delay-2 oritatami system that would fold according to the infinite triangular spiral.

► ***Question 1.2)*** *Prove that 2 bonds are required to place the bead correctly at each corner.*

Answer. ▷ Using the notations at the top corner from the figure, let i be the index of the bead at a corner. At delay 2, only i and $i+1$ can make bonds. i cannot make bonds with anyone from its final position. If i is placed correctly, $i+1$ can only make bonds with j and $i-1$. In all case, if $i+1$ makes only one bond, the triangle $(i-1)i(i+1)$ can move around and i is not stabilized. Thus, 2 bonds are needed and 2 bonds are enough: $(i+1)$—$(i-1)$ and $(i+1)$—j. ◁

► ***Question 1.3)*** *Show that there are 4 consecutive bead types a, b, c, d in the transcript that get placed as follows:*



Answer. ▷ Consider i_0 be the index from which the transcript is periodic and let $T > i_0$ be a multiple of its period larger than i_0. Consider the side of length T in the spiral and let b be the identical bead type at its two extremities and let a, c and d be the preceding and the two following bead types respectively. They are located as illustrated above. ◁

▶ ***Question 1.4)*** *Show that in order to stabilize c in the lower left corner, c must bind with a.*
Answer. ▷ From question 1.2, in order to fold the top corner properly, d must bind with b. Thus, in the lower left corner, when folding c and d, d can make two bonds with the two bs if located at c's place. At least three bonds must then be made in order to place c at its correct location. Furthermore, only three bonds can be made if c is at its correct location: d—b, c—a and c—b. It follows that c must bind to a. ◁

▶ ***Question 1.5)*** *Conclude that c cannot be placed deterministically at the top corner.*
Answer. ▷ When c and d are folded at the top corner, only two bonds can be made if c is correctly located. But, if c and d are located to the east and to the north-east of b resp., c binds with a and d with b. c is thus unstable and cannot be placed correctly deterministically. ◁

▶ ***Question 1.6 (★★★))*** *What about deterministic oritatami systems with larger delays?*
Answer. ▷ We suspect it is impossible for all delay, but... no one never knows until it's proven... help wanted! ◁