Workshop on χ-bounded classes

Lyon, March 15th – 21st 2012

The workshop takes place at ENS-Lyon, site Jacques Monod. All talks are given in room 116 (on the first floor) except on Friday morning where they are given at "Amphi B" (on the 3rd floor). All rooms have blackboards and beamers.

Schedule

Thursday

09:30 Welcome coffee.

10:00 S. Thomassé. Introduction.

10:30 J. Fox. A disproof of Scott's conjecture.

11:30 Lunch break.

14:00 P. Ossona de Mendez. Low tree-depth decompositions and homomorphism bounds.

15:00 Open problems session.

Friday

9:00 P. Seymour. Structure theory for tournaments.

10:00 J. Fox. Graph regularity lemmas.

11:00 A. Scott. T.B.A.

12:00 Lunch break.

14:00 V. Chepoi. Colouring hyperplanes of CAT(0) cube complexes.

15:00 Working session.

Saturday

9:30 K. Vušković. Local structural properties and decomposition.

10:30 N. Bousquet. Scott's conjecture holds for maximal triangle-free graphs.

11:00 Working session.

12:30 Lunch Break.

Afternoon. Working session.

Monday

9:30 M. Chudnovsky. Forcing large transitive subtournaments.

Rest of the day: working sessions.

Tuesday

9:30. L. Esperet. (List) coloring claw-free graphs with small clique

10:00 F. Maffray. On $\{P_5, K_4\}$-free graphs.

Rest of the day: working sessions.

Wednesday

Full day working session.

Titles and abstracts

Scott's induced subdivision conjecture for maximal triangle-free graphs, by **Nicolas Bousquet**, LIRMM (Montpellier).
Abstract: Scott conjectured in 1997 that the class of graphs with no induced subdivision of a given graph is χ-bounded. We verify his conjecture for maximal triangle-free graphs. Joint work with Stéphan Thomassé.

Colouring hyperplanes of CAT(0) cube complexes by **Victor Chepoi**, LIF (Marseille).
Abstract: In the talk, we will present positive and negative results on colouring contact and crossing graphs of hyperplanes of CAT(0) cube complexes X with uniformly bounded degrees. The crossing graph of X is the intersection graph of the hyperplanes of X and the contact graph of X is the intersection graph of carriers of the hyperplanes of X. The crossing graph is always a subgraph of the contact graph of X. The chromatic number of the crossing graph is the minimum number of tree factors so that X isometrically embeds into their Cartesian product. The chromatic number of a subgraph of the contact graph is the minimum number of labels in a nice labeling of the event structure whose domain is X.

We show that for any 2-dimensional CAT(0) cube complex of maximum degree d, the chromatic number of its contact graph (and therefore of its crossing graph) is at most $q(d) = Md^{15}$, for a fixed constant M. On the other hand, we present an example (based on Burling's construction) of a 4-dimensional CAT(0) cube complex with uniformly bounded degrees whose contact graph has infinite chromatic number. The first result is based on a join paper with Mark Hagen.

Forcing large transitive subtournaments by **Maria Chudnovsky**, Columbia University (New York).
Abstract: The Erdős Hajnal Conjecture states roughly that a graph with some induced subgraph excluded has a large clique or a large stable set. A similar statement can be formulated for tournaments (a tournament is an orientation of a complete graph), replacing cliques and stable sets by transitive subtournaments; and the two conjectures turn out to be equivalent. This talk will survey a number of recent results related to the latter conjecture. In particular, we will discuss a new infinite class of tournaments excluding which forces large transitive subtournaments; to the best of our knowledge this is the first such class not obtained by the so-called substitution operation.

(List) coloring claw-free graphs with small clique. number by **Louis Esperet** (CNRS, G-SCOP, Grenoble)
Abstract: We prove that every claw-free graph with no K_4 is 4-choosable, and every claw-free graph with no K_5 is 7-choosable. Both bounds are tight. Joint work with A. Gyárfás and F. Maffray.

Graph regularity lemmas by **Jacob Fox**, MIT (Boston).
Abstract: Survey.

A disproof of Scott's conjecture by **Jacob Fox**, MIT (Boston).
Abstract: Paul Erdős conjectured a long time ago that there is a constant C such that every triangle-free intersection graph of segments in the plane is C-colorable. As observed by János Pach and myself, this would follow from Scott's conjecture that for every graph H, the class of graphs that do not contain any subdivision of H as an induced subgraph is χ-bounded. A beautiful construction disproving the conjecture of Erdős was recently discovered by Jakub Kozik, Tomasz Krawczyk, Michał Lasoń, Piotr Micek, Arkadiusz Pawlik, Tom Trotter, and Bartosz Walczak. Hence, Scott's conjecture is false. In this talk, I will present their proof and discuss related problems in graph theory and combinatorial geometry.

On $\{P_5, K_4\}$-free graphs by **Frédéric Maffray**, CNRS, G-SCOP (Grenoble).
Abstract: Gyárfás conjectured that for any tree T every T-free graph has chromatic number at most $f_T(w)$, where w is the maximum clique size in G and f_T is a function that depends only on T. Gyárfás proved the conjecture for paths. The value of f_T in the case of paths was slightly improved later by Gravier, Hoàng and Maffray. In particular, every $\{P_5, K_4\}$-free graph is 9-colorable. We prove that every such graph is 5-colorable and that this is the best possible bound. This is joint work with Louis Esperet, Laetitia Lemoine and Grégory Morel.

Low tree-depth decompositions and homomorphism bounds by **Patrice Ossona de Mendez**, CNRS, EHESS (Paris)

T.B.A. by **Alex Scott**, Oxford University.

Structure theory for tournaments by **Paul Seymour**, Princeton.
Abstract. Here are two well-known results by Neil Robertson and the speaker, from the Graph Minors series of papers:

(1) For any graph H, the graphs not containing H as a minor all have bounded pathwidth if and only if H is a forest.

(2) For any graph H, the graphs not containing H as a minor all have bounded treewidth if and only if H is planar.

There are other, similar results about (undirected) graphs and minor containment, relating a property of the excluded minor to a property of the graphs that do not contain it.

Recently we have been looking for analogues to the above, using tournaments instead of graphs, and replacing minor containment with a more appropriate containment relation. In this talk we survey our results. This is joint work with Maria Chudnovsky, Alexandra Fradkin, and Ilhee Kim.

Local structural properties and decomposition by **Kristina Vušković**, Leeds University and Union University (Belgrad).
Abstract. In this talk we survey several hereditary classes of graphs for which χ-boundedness results are obtained using decomposition theory.

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