Remember that the category \( \text{Rel} \) has sets as objects and binary relations as morphisms: \( \text{Rel}(A, B) = \mathcal{P}(A \times B) \).

1. Prove that \( \text{Rel} \) is isomorphic to the Kleisli category of \( \text{Set} \) for the monad \( \mathcal{P} \) (remark page 3 of handout of lecture 10).

2. Prove that \( \text{Rel} \) has finite co-products (remember that it is enough for that to have binary co-products and an initial element).

3. Prove that \( \text{Rel} \) has finite products.

4. Given a category \( \mathcal{C} \) and a monad \( M \) in \( \mathcal{C} \), assuming that \( f : A \to B \) is an isomorphism in \( \mathcal{C} \), prove that \( \eta_B \circ f \) is an isomorphism in the Kleisli category \( \mathcal{C}_M \).

5. Monoidal structure on \( \text{Rel} \).
   
   (a) Prove that \( \times \) defines a bifunctor on \( \text{Rel} \).
   
   (b) Prove that \( (\text{Rel}, \times, \{\star\}) \) has a structure of monoidal category (MC).
   
   (c) Prove that \( (\text{Rel}, \times, \{\star\}) \) has a structure of symmetric monoidal category (SMC).
   
   (d) Prove that \( (\text{Rel}, \times, \{\star\}) \) has a structure of symmetric monoidal closed category (SMCC).

6. Explain why there is an isomorphism \( (A + B) \times C \cong (A \times C) + (B \times C) \) in \( \text{Rel} \) (see handout of lecture 10).

7. Prove that \( d_A = \{(a, (a, a)) \mid a \in A\} : A \to A \times A \) does not define a natural transformation from the identity functor \( \text{Id} \) to the diagonal functor \( \Delta \).

8. Compute the interpretation of the following \( \text{IMLL} \) proof in \( \text{Rel} \):

\[
\begin{align*}
A \vdash A & \quad \text{ax} \quad B \vdash B & \quad a \chi \quad \vdash 1 \quad R \\
A \to B, A \vdash B \& 1 & \quad -L \\
A, A \to B \vdash B \& 1 & \quad -ex \\
A \& (A \to B) \vdash B \& 1 & \quad -L \\
\vdash (A \& (A \to B)) \to (B \& 1) & \quad -R
\end{align*}
\]