Subject 4: Hyland-Ong game semantics

**Question 1**. Prove that well-bracketed strategies are preserved by composition. (Hint: adapt the proof that P-visible strategies are preserved by composition.)

**Question 2**. Calculate the P-view function of
\[ \lambda f \, (f \, tt)(f \, ff \, tt) : (bool^2 \to bool) \to bool \]

**Question 3**. Apply definability to find the normal form that denotes the following P-view function:

\[
\begin{array}{c}
B \Rightarrow B \Rightarrow B \\
\downarrow \quad \quad \downarrow \\
\quad q \\
\quad \downarrow \\
\quad b \\
\quad \downarrow \\
\quad ff
\end{array}
\]

**Question 4**. Calculate the composite strategy defining the substitution of the term of Question 3 for the variable \( f \) in the term of Question 2.

**Question 5**. Find an Idealized Algol context that distinguishes between \( \lambda x \, (if \, x \, then \, tt \, else \, ff) \) and \( \lambda x \, (if \, x \, then \, (if \, x \, then \, tt \, else \, ff) \, else \, ff) \) of type \( bool \to bool \).