CR 17)

Rappels

connecteur $*$

- introduction / élimination

$$\frac{}{*} \qquad \frac{*}{}$$

- gauche / droite

$$* \vdash \qquad \vdash *$$

- multiplicatif / additif
  - binaire

$$\frac{\Gamma \qquad \Delta}{\Gamma, \Delta} \qquad \frac{\Gamma \qquad \Gamma}{\Gamma}$$

  - unaire

$$\frac{A \quad B}{A * B} \quad \Bigg| \quad \frac{A}{A * B} \qquad \frac{B}{A * B}$$

  - 0-aire

$$\frac{}{\vdash} \qquad \frac{}{\Gamma \vdash \Delta}$$

- multiplicatif $\xleftrightarrow[\text{structurelles}]{\text{règles}}$ additive

---

elim G $\to$:

$$\xcancel{\frac{\Gamma, A \to B \vdash \Delta}{\Gamma, B \vdash \Delta}} \qquad \xcancel{\frac{\Gamma, A \to B \vdash \Delta}{\Gamma \vdash A, \Delta}}$$

correct

COUPURE

$$\frac{\Gamma \vdash A, \Delta \qquad \Gamma', A \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\,\text{Cut}$$

$$\dfrac{\dfrac{\Gamma, A \to B \vdash \Delta}{\Gamma, \neg A \vdash \Delta}}{\Gamma \vdash A, \Delta}$$

- introduction G $\xleftrightarrow[\text{axiome}]{\text{coupure}}$ élimination D

$$\frac{\Gamma, A \vdash \Delta}{\Gamma, A \land B \vdash \Delta} \qquad\qquad \frac{\Gamma \vdash A \land B, \Delta}{\Gamma \vdash A, \Delta}$$

$$\dfrac{\Gamma, A \vdash \Delta \qquad \dfrac{\dfrac{}{A \land B \vdash A \land B}\,\text{ax}}{A \land B \vdash A}}{\Gamma, A \land B \vdash \Delta}\,\text{Cut} \qquad\qquad \dfrac{\Gamma \vdash A \land B, \Delta \qquad \dfrac{\dfrac{}{A \vdash A}\,\text{ax}}{A \land B \vdash A}}{\Gamma \vdash A, \Delta}$$

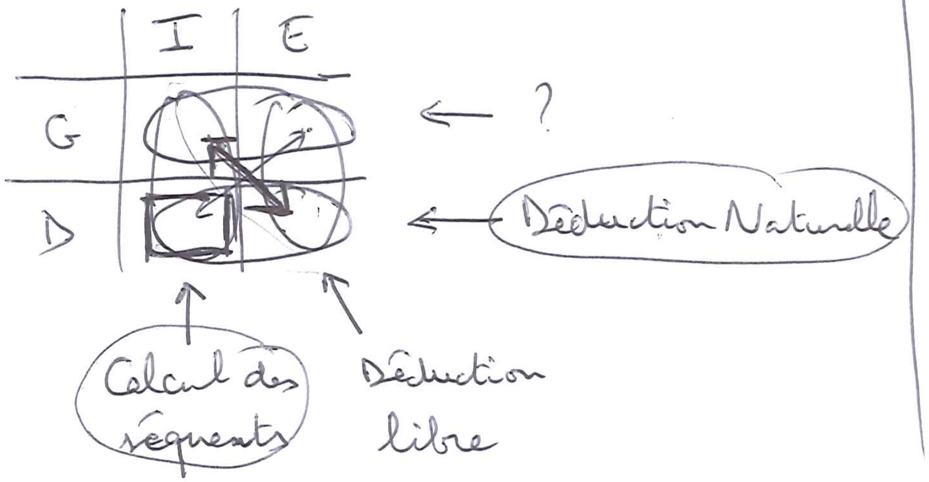
- élimination G $\underset{\text{axiome}}{\overset{\text{coupure}}{\longleftrightarrow}}$ introduction D

$$\dfrac{\Gamma, A\land B \vdash \Delta}{\Gamma, A, B \vdash \Delta} \qquad \dfrac{\Gamma \vdash A, \Delta \quad \Gamma' \vdash B, \Delta'}{\Gamma, \Gamma' \vdash A\land B, \Delta, \Delta'}$$

$$\dfrac{\Gamma, A\land B \vdash \Delta \qquad \dfrac{\dfrac{}{A \vdash A} \quad \dfrac{}{B \vdash B}}{A, B \vdash A\land B}}{\Gamma, A, B \vdash \Delta}$$

$$\dfrac{\Gamma \vdash A, \Delta \qquad \dfrac{\Gamma' \vdash B, \Delta' \quad \dfrac{\dfrac{}{A\land B \vdash A\land B}\,\text{ax}}{A, B \vdash A\land B}}{\Gamma', A \vdash \Delta', A\land B}}{\Gamma, \Gamma' \vdash A\land B, \Delta, \Delta'}$$

~~$A\land B \vdash A\land B$~~



Déduction Naturelle (additive) NK

groupe ~~logique~~ identité

$$\dfrac{}{\Gamma, A \vdash A, \Delta}\,\text{ax}$$

groupe structurelle

$$\dfrac{\Gamma \vdash \Delta}{\sigma(\Gamma) \vdash \Delta}\,\text{ex G} \qquad \dfrac{\Gamma \vdash \Delta}{\Gamma \vdash \sigma(\Delta)}\,\text{ex D}$$

$$\dfrac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta}\,\text{wk G} \qquad \dfrac{\Gamma \vdash \Delta}{\Gamma \vdash A, \Delta}\,\text{wk D}$$

$$\dfrac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta}\,\text{ctr G} \qquad \dfrac{\Gamma \vdash A, A, \Delta}{\Gamma \vdash A, \Delta}\,\text{ctr D}$$

groupe logique

$\boxed{\land}$

$$\dfrac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A\land B, \Delta}\,\text{ID} \qquad \dfrac{\Gamma \vdash A\land B, \Delta}{\Gamma \vdash A, \Delta}\,\text{ED}_1 \qquad \dfrac{\Gamma \vdash A\land B, \Delta}{\Gamma \vdash B, \Delta}\,\text{ED}_2$$

$\boxed{\lor}$

$$\dfrac{\Gamma \vdash A, \Delta}{\Gamma \vdash A\lor B, \Delta}\,\text{ID}_1 \qquad \dfrac{\Gamma \vdash B, \Delta}{\Gamma \vdash A\lor B, \Delta}\,\text{ID}_2$$

~~[illegible]~~

$$\dfrac{\Gamma \vdash A\lor B, \Delta \quad \Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma \vdash \Delta}\,\text{ED}$$

$\boxed{\top}$ $$\dfrac{}{\Gamma \vdash \top, \Delta}\,\text{ID}$$

$\boxed{\bot}$ ~~$\Gamma \vdash \bot, \Delta$~~ $$\dfrac{\Gamma \vdash \bot, \Delta}{\Gamma \vdash \Delta}\,\text{ED}$$

$\boxed{\neg}$ $$\dfrac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\,\text{ID} \qquad \dfrac{\Gamma \vdash \neg A, \Delta \qquad \Gamma \vdash A, \Delta}{\Gamma \vdash \Delta}$$

$\boxed{\rightarrow}$ $$\dfrac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \to B, \Delta}\,\text{ID} \qquad \dfrac{\Gamma \vdash A \to B, \Delta \qquad \Gamma \vdash A, \Delta}{\Gamma \vdash B, \Delta}\,\text{ED}$$

Coupure

$$\dfrac{\Gamma \vdash A, \Delta, B \qquad \dfrac{\dfrac{\Gamma, A \vdash \Delta, B}{\Gamma, A \vdash \Delta, B, B}\,\text{wkD}}{\Gamma \vdash A \to B, \Delta, B}\,\text{ID}\to}{\dfrac{\Gamma \vdash B, \Delta, B}{\Gamma \vdash B, \Delta}\,\text{ctrD}}\,\text{ED}\to$$

$\Gamma \vdash A, \Delta \qquad \boxed{\Gamma, \cancel{A} \vdash \Delta}$

Classique / Intuitionniste

$\Gamma \vdash \Delta \qquad \Gamma \vdash A$

$A \to B := \neg A \vee B \qquad A \to B \not\equiv \neg A \vee B.$

Calcul des Séquents LK

groupe identité

$$\frac{}{A \vdash A}\,ax \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma', A \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\,cut$$

groupe structurel

idem

groupe logique

$\boxed{\wedge}$ additif

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta} \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \qquad \frac{\Gamma, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}$$

multiplicatif

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma' \vdash B, \Delta'}{\Gamma, \Gamma' \vdash A \wedge B, \Delta, \Delta'} \qquad \frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}$$

$\boxed{\vee}$ additif

$$\frac{\Gamma \vdash A, \Delta}{\Gamma \vdash A \vee B, \Delta} \qquad \frac{\Gamma \vdash B, \Delta}{\Gamma \vdash A \vee B, \Delta} \qquad \frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}$$

multiplicatif

$$\frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta} \qquad \frac{\Gamma, A \vdash \Delta \quad \Gamma', B \vdash \Delta'}{\Gamma, \Gamma', A \vee B \vdash \Delta, \Delta'}$$

$\boxed{\top}$ additif $\frac{}{\Gamma \vdash \top, \Delta}$

multiplicatif $\frac{}{\vdash \top}$ $\qquad \frac{\Gamma \vdash \Delta}{\Gamma, \top \vdash \Delta}$

$\boxed{\bot}$ additif $\frac{}{\Gamma, \bot \vdash \Delta}$

multiplicatif $\frac{\Gamma \vdash \Delta}{\Gamma \vdash \bot, \Delta} \qquad \frac{}{\bot \vdash}$

$\boxed{\neg}$

$$\frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta} \qquad \frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}$$

$\boxed{\rightarrow}$ additif

$$\frac{\Gamma, A \vdash \Delta}{\Gamma \vdash A \rightarrow B, \Delta} \qquad \frac{\Gamma \vdash B, \Delta}{\Gamma \vdash A \rightarrow B, \Delta} \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \rightarrow B \vdash \Delta}$$

multiplicatif

$$\frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \rightarrow B, \Delta} \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma', B \vdash \Delta'}{\Gamma, \Gamma', A \rightarrow B \vdash \Delta, \Delta'}$$

$$\neg(A\wedge B) \overset{\overrightarrow{\vdash}}{\underset{\dashv}{\equiv}} \neg A \vee \neg B \qquad \neg\top \overset{\longrightarrow}{\equiv} \bot$$

$$\neg\neg A \underset{\longrightarrow}{\equiv} A$$

$$A\wedge B \equiv \neg(\neg A \vee \neg B)$$

Systèmes monolatères

- Formules

$A ::= X \mid A\wedge A \mid A\vee A \mid \top \mid \bot \mid \neg A$

$A ::= X \mid A\wedge A \mid A\vee A \mid \top \mid \bot \mid \neg X$

$\neg A$ n'est pas une formule

$\neg$: formule → formule

$\neg(A\wedge B) := \neg A \vee \neg B$

$\neg(A\vee B) := \neg A \wedge \neg B$

$\neg\top := \bot$

$\neg\bot := \top$

$\neg X := \neg X$

$\neg\neg X := X$

lemme:

$\forall A,$

$\neg\neg A = A$

$$\neg(\underbrace{X\wedge(\bot\vee\neg Y)}) := \underbrace{\neg X \vee (\top\wedge Y)}$$

$$\Gamma\vdash\Delta \rightsquigarrow \vdash\neg\Gamma, \Delta$$

- Séquents $\vdash\Gamma$

$$\frac{}{A\vdash A} \rightsquigarrow \boxed{\frac{}{\vdash\neg A, A}}$$

$$\frac{\Gamma\vdash A,\Delta \quad \Gamma', A\vdash\Delta'}{\Gamma,\Gamma'\vdash\Delta,\Delta'} \rightsquigarrow \frac{\vdash\neg\Gamma, A, \Delta \quad \vdash\neg\Gamma', \neg A, \Delta'}{\vdash\neg\Gamma, \neg\Gamma', \Delta, \Delta'}$$

$$\boxed{\frac{\vdash\Gamma, A \quad \vdash\Delta, \neg A}{\vdash\Gamma, \Delta}}$$

LK $\underbrace{\text{correct}}$ et $\underbrace{\text{complet}}$ pour la logique classique?

prouvable ⇓ vrai

vrai ⇓ prouvable

vrai ⟹ prouvable dans $\boxed{\text{Hilbert}}$ $\boxed{\swarrow ?}$

$A\to B\to A$

[illegible]

$(A\to B\to C)\to(A\to B)\to A\to C$

$((A\to B)\to A)\to A$

$$\frac{A\to B \quad A}{B}$$