

Natural Deduction and Sequent Calculus

LFPL Homework 1

September 27, 2021

Formulas

We consider formulas built from propositional variables by using the connectives \neg and \vee :

$$A ::= X \mid \neg A \mid A \vee A$$

Additive Natural Deduction NK

$$\frac{}{\Gamma_1, A, \Gamma_2 \vdash \Delta_1, A, \Delta_2} ax^a$$
$$\frac{\Gamma, A \vdash \Delta_1, \Delta_2}{\Gamma \vdash \Delta_1, \neg A, \Delta_2} \neg I \quad \frac{\Gamma \vdash \neg A, \Delta \quad \Gamma \vdash A, \Delta}{\Gamma \vdash \Delta} \neg E$$
$$\frac{\Gamma \vdash \Delta_1, A, \Delta_2}{\Gamma \vdash \Delta_1, A \vee B, \Delta_2} \vee_1 I \quad \frac{\Gamma \vdash \Delta_1, B, \Delta_2}{\Gamma \vdash \Delta_1, A \vee B, \Delta_2} \vee_2 I \quad \frac{\Gamma \vdash A \vee B, \Delta \quad \Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma \vdash \Delta} \vee E$$

Multiplicative Sequent Calculus LK

$$\frac{}{A \vdash A} ax^m \quad \frac{\Gamma \vdash A, \Delta \quad \Gamma', A \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'} cut^m \quad \frac{\Gamma \vdash \Delta}{\sigma(\Gamma) \vdash \Delta} exL \quad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \sigma(\Delta)} exR$$
$$\frac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta} wkL \quad \frac{\Gamma \vdash \Delta}{\Gamma \vdash A, \Delta} wkR \quad \frac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta} coL \quad \frac{\Gamma \vdash A, A, \Delta}{\Gamma \vdash A, \Delta} coR$$
$$\frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta} \neg R \quad \frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta} \neg L$$
$$\frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta} \vee R \quad \frac{\Gamma, A \vdash \Delta \quad \Gamma', B \vdash \Delta'}{\Gamma, \Gamma', A \vee B \vdash \Delta, \Delta'} \vee L$$

1. Prove the additive cut rule $\frac{\Gamma \vdash A, \Delta \quad \Gamma, A \vdash \Delta}{\Gamma \vdash \Delta} cut^a$ is derivable in NK.
2. Prove the derivability of the contraction rules in NK.
3. Prove the admissibility of the exchange rules in NK.
4. Prove the admissibility of the weakening rules in NK.
5. Prove the admissibility of all the LK rules in NK.
6. Prove that all the rules of NK are derivable in LK.