Homework I (due March 7st, before tutorial)

Problem 1 (NP-hardness). Prove that the following problems are NP-hard

- The halting problem \( \text{HALT} = \{ (\alpha, x) : M_\alpha \text{ halts on input } x \} \).
- The problem Integer Linear Programming

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\text{ILP} = \{ (A, b) : A \text{ integer } n \times m \text{ matrix and } b \text{ column vector of } m \text{ integers s.t. there exists a vector } x \text{ of } n \text{ integers such that } Ax \geq b \}\]

We use the notation \( v \geq w \) for vectors \( v \) and \( w \) when \( v_i \geq w_i \) for all coordinates \( i \).

Are they NP-complete? For this, you may choose one of three options with justification: not in NP, in NP, seems to be in NP but there is a difficulty and say what the difficulty is.

Problem 2 (Reductions). We saw that a language \( L \) is polynomial-time Karp reducible to \( L' \) if there is a polynomial-time computable functions \( f \) such that \( x \in L \) iff \( f(x) \in L' \). Give an example of a class that is closed under polynomial-time Karp reductions (i.e., if \( L \) is reducible to \( L' \) and \( L' \in C \), then \( L \in C \)) and one that is not.

Problem 3 (Difference of NP problems). Let \( \text{DP} = \{ L_1 \setminus L_2 : L_1 \in \text{NP} \text{ and } L_2 \in \text{NP} \} \). Show that the problem \( \text{EXACTINDSET} = \{ (G, k) : \text{ the largest independent set of } G \text{ has size exactly } k \} \) is DP-complete (for the usual polynomial-time Karp reductions).