Homework II  (due Tuesday, March 23rd)

Problem 1 (Difference of NP problems). Let $\text{DP} = \{L_1 \setminus L_2 : L_1 \in \text{NP} \text{ and } L_2 \in \text{NP}\}$. Show that the problem $\text{EXACTINDSET} = \{(G,k) : \text{the largest independent set of } G \text{ has size exactly } k\}$ is DP-complete (for the usual polynomial-time Karp reductions).

Problem 2 (Graph isomorphism). Two graphs $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ on the same vertex set are said to be isomorphic if there exists a permutation $\pi$ of $V$ such that $(u,v) \in E_1 \iff (\pi(u), \pi(v)) \in E_2$. Suppose that there is a polynomial-time algorithm $A$ that decides whether two given graphs $G_1$ and $G_2$ are isomorphic. Using $A$ as a subroutine, describe a polynomial-time algorithm that finds the permutation $\pi$ when the graphs are isomorphic.

Problem 3 (Oracles and NP). Show that $\text{NP}^A = \text{NP}$ if and only if $A \in \text{NP} \cap \text{coNP}$. 