**HW 2: circuits, randomness.**
(due Thursday, April 11th, before tutorial)

**Problem 1** (Circuits). Show that any boolean function \( f : \{0, 1\}^n \to \{0, 1\} \) can be computed by a circuit of size \( O(2^n) \).

**Bonus:** Improve this bound to \( O(2^n n^2) \).

**Problem 2.** The class \( \text{PP} \) is defined as the set of languages \( L \) such that there exists a polynomial \( q(n) \) and a polynomial time machine \( M \) such that the following holds.

\[
x \in L \implies \mathbb{P}_{r \in \{0,1\}^{q(n)}} \{ M(x, r) = 1 \} > \frac{1}{2}
\]

\[
x \notin L \implies \mathbb{P}_{r \in \{0,1\}^{q(n)}} \{ M(x, r) = 1 \} \leq \frac{1}{2},
\]

where \( n = |x| \).

1. Show that \( \text{NP} \subseteq \text{PP} \).

2. Show that the problem \( \#\text{SAT} = \{ (\varphi, k) : \text{the formula } \varphi \text{ has } > k \text{ satisfying assignments} \} \) is \( \text{PP} \)-complete for Karp reductions.