HW 3: circuits, randomness.

(due Tuesday, April 6th)

Problem 1 (Circuits)**.** Show that any boolean function $f : \{0,1\}^n \to \{0,1\}$ can be computed by a circuit of size $O(2^n)$.

Bonus: Improve this bound to $O(\frac{2^n}{n})$.

Problem 2 (Sparse languages and $\mathbf{P/poly}$)**.** Recall that a language $A \subseteq \{0,1\}^*$ is said to be sparse if there is a polynomial $p(n)$ such that $|L \cap \{0,1\}^n| \leq p(n)$ for all positive integers n.

Show that $\mathbf{P/poly} = \cup_{A \text{ sparse}} \mathbf{P}^A$.

Problem 3. We define the class $\mathbf{BPP'}$ as the set of languages L such that there is a polynomial-time Turing machine M and a polynomial q such that

$$x \in L \implies \underset{r \in \{0,1\}^{q(n)}}{\mathbf{P}} \{M(x,r) = 1\} \geq \frac{1}{10} + \frac{1}{n}$$
$$x \notin L \implies \underset{r \in \{0,1\}^{q(n)}}{\mathbf{P}} \{M(x,r) = 1\} < \frac{1}{10},$$

where $n = |x|$ is the size of x. Prove that $\mathbf{BPP'} = \mathbf{BPP}$.