HW 3: Channel coding

(due November 6th, before tutorial)

1. Consider a source given by $X^n = X_1 \dots X_n$ with X_i independent and identically distributed bits with $\mathbf{P}\{X_i = 1\} = \frac{1}{4}$. Describe the distribution of the random variable $h_{X^n}(X^n) = -\log_2 P_{X^n}(X^n)$. How many values does it take? What is the probability for each different value? What is the expectation?

2. Consider the channel W with input alphabet $\mathcal{X} = \{a, b, c\}$ and output alphabet $\{0, 1\}$, with $W(0|a) = 1, W(0|b) = \frac{1}{2}, W(1|b) = \frac{1}{2}$ and $W(1|c) = 1$. Then, let $W^{\times n}$ be n independent copies of W.

   (a) For any M, determine the optimal (i.e., smallest possible) error probability for an M-code for $W^{\times n}$, as a function of M and n.

   (b) Compute $\mathrm{C}(W)$.

3. Let $a \in \{1, 2\}$. Consider the additive noise channel with input alphabet $\mathcal{X} = \{0, 1\}$ and output alphabet $\mathcal{Y} = \{0, 1, 2, 3\}$, where the output Y is given by $x + Z$ when x is the input symbol and Z is a random variable with distribution $\mathbf{P}\{Z = 0\} = \mathbf{P}\{Z = a\} = \frac{1}{2}$. Compute the information capacity of this channel.