HW 4: Data compression and channel coding
(due October 23rd, before tutorial)

1. This problem is to illustrate that some stream codes can use much less than one bit per symbol of the source. Assume the source is composed of symbols in $\mathcal{X}$ and $0 \in \mathcal{X}$ is one of the symbols. To simplify the calculations, you may assume $\mathcal{X} = \{0, 1\}$. Determine the length of the encoding of the bitstring $0^n$ (i.e., $n$ times the symbol 0) using the arithmetic coding method described in class. You can give your result in the form $O(f(n))$ (of course you should try to obtain the smallest possible $f$ that you can).

2. Consider the channel $W$ with input alphabet $\mathcal{X} = \{a, b, c\}$ and output alphabet $\{0, 1\}$, with $W(0|a) = 1, W(0|b) = \frac{1}{2}, W(1|b) = \frac{1}{2}$ and $W(1|c) = 1$. Then, let $W^n$ be $n$ independent copies of $W$.

   (a) For any $M$, determine the optimal (i.e., smallest possible) error probability for an $M$-code for $W^n$, as a function of $M$ and $n$.

   (b) Compute $C(W)$.

3. (Bonus) Consider the arithmetic coding method described in class. Suppose I use this compressor for an iid source $X^n = X_1 \ldots X_n$. Show that the expected encoding length is of order $nH(X_1) + o(n)$. 