HW 4: Error-correcting codes

(due December 4th, before tutorial)

1. Let $A_q(n, d)$ be the largest k such that a code over alphabet $\{1, \ldots, q\}$ of block length n, dimension k and minimum distance d exists (recall that this corresponds to the notation $(n, k, d)_q$). Determine $A_2(3, d)$ for all integers $d \geq 1$.

2. Suppose C is a $(n, k, d)_2$-code with d odd. Construct using C a code C' that is a $(n + 1, k, d + 1)_2$-code.

3. By constructing the columns of a parity check matrix in a greedy fashion, show that there exists a binary linear code $[n, k, d]_2$ provided that

$$2^{n-k} > 1 + \binom{n-1}{1} + \cdots + \binom{n-1}{d-2} \, . \tag{1}$$

   This is a small improvement compared to the general Gilbert-Varshamov bound. In particular, it is tight for the $[7, 4, 3]_2$ Hamming code.