Bonus HW

(due January 8th, before exam)

1. In this question, we relate the different ways of measuring the error probability of a code. Recall that for an M-code for the channel W, we defined P_{err} as the average (over the messages) error probability and $P_{err,max}$ as the maximum (over the messages) error probability. Show that if there is an M-code for the channel W with $P_{err} \leq \delta$, then there exists an $\lfloor M/2 \rfloor$-code for the channel W with $P_{err,max} \leq 2\delta$.

   Now answer this question from the midterm: "We showed in class that for any channel W, any $R < \mathrm{C}(W)$, $\delta > 0$ and large enough n, there exists a 2^{nR}-code for $W^{\times n}$ with error probability at most δ. Show that the same statement holds if we replace the usual error probability (which is an average over the messages) with a *maximum* error probability (which is a maximum over the messages)."

2. Let C be a linear code with block length n, dimension k over $\mathbb{F}_q$. Let $i \in \{1, \ldots, n\}$ and $\alpha \in \mathbb{F}_q$ with $\alpha \neq 0$. Show that the number of codewords $c \in C$ such that $c_i = \alpha$ is either equal to zero or equal to q^{k-1}.