HW II: Random variables and expectation (due Feb 14th, before tutorial)

1. (From GS) Let $X_1, \ldots, X_n$ be independent discrete random variables that are symmetric about 0, i.e., X_i and $-X_i$ have the same distribution. Let $S_n = \sum_{i=1}^{n} X_i$. Show that for all t, $\mathbf{P}\{S_n \geq t\} = \mathbf{P}\{S_n \leq -t\}$. What if we drop the independence assumption? Justify your answer.

2. (a) Let $X_1, \ldots, X_n$ be independent and uniform $\{0, 1\}$ random variables. A streak is a consecutive sequence of 0's or 1's. For a given integer $p \in \{1, \ldots, n\}$. For example, the sequence 00011001 has 4 streaks of length 2. What is the expected number of streaks of length p? For which value p^* is this expectation 1 (or closest to 1 if not integer)?

   (b) Show that for $p \leq \alpha p^*$ with $\alpha < 1$, the probability that there is a streak of length p goes to 1 as $n \to \infty$.