HW VIII: Convergence and Markov chains

(due before April 25th at 8:00)

1. Convergence

   (a) Let $\{X_n\}$ be a sequence of random variables with $\mathbf{E}\{X_n\} = 5$ and $\mathbf{Var}\{X_n\} = \frac{1}{\sqrt{n}}$ for all n. Is it true that X_n must converge in probability to 5?

   (b) Let $\{X_n\}$ be independent and identically distributed random variables with $\mathbf{E}\{X_n\} = 4$ and $\mathbf{Var}\{X_n\} = 9$ for all n. Find $C(n,x)$ such that

   $$\lim_{n\to\infty} \mathbf{P}\{X_1 + \cdots + X_n \leq C(n,x)\} = \Phi(x) ,$$

   where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt$.

   (c) Give an example of sequence $\{Y_n\}$ such that Y_n converges in probability to 0, $\frac{Y_n}{n}$ converges almost surely to 0, but Y_n does not converge almost surely to 0.

2. Markov chains

   (a) For a Markov chain with state space S of size d and transition matrix $\{P_{i,j}\}$ what is the largest value of N such that $P^N_{i,j} > 0$ but $P^n_{i,j} = 0$ for all $1 \leq n < N$.