1. Let $X_1, X_2$ be linear operators on $\mathbb{C}^{d_1}, \mathbb{C}^{d_2}$, respectively. Show that $X_1 \otimes I_{d_2}$ commutes with $I_{d_1} \otimes X_2$, where $I_d$ is the identity operator on $\mathbb{C}^d$. Show that if $X_1, X_2$ are unitary, then so is $X_1 \otimes X_2$.

2. As any state, the state $H^\otimes_n |x\rangle$ for $x \in \{0, 1\}^n$ can be written as $\sum_{y \in \{0, 1\}^n} \alpha_y(x) |y\rangle$. Compute $\alpha_y(x)$.

3. Define the 2-qubit state $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle_{AB} - |10\rangle_{AB})$ shared between Alice and Bob. Let $U$ be a 1-qubit unitary. Compute $U \otimes U |\psi\rangle_{AB}$. Conclude that any unitary operation done by Alice on her system $A$ can be undone by Bob only by acting on his system $B$.

4. Alice has a 2-qubit unknown state $|\psi\rangle_{A_1A_2}$. Show how Alice can transmit these two qubits to Bob by sending 4 classical bits and 2 shared EPR pairs.

   Assume now that the state is guaranteed to be of the form $|\psi\rangle_{A_1A_2} = \alpha |00\rangle_{A_1A_2} + \beta |11\rangle_{A_1A_2}$, with $\alpha, \beta$ arbitrary. Show that this task can be done using only 2 classical bits of communication and one shared EPR pair.