HW 4: Phase estimation  (due Oct 24th, before class)

The objective of this problem is to show that the phase estimation algorithm can still return a good approximation if the angle $\theta$ is not of the form $\theta = \frac{j}{2^m}$.

1. Show that after applying the phase estimation circuit described in class, we obtain (in the first register) the state

$$\frac{1}{2^m} \sum_{j=0}^{2^m-1} \left( \sum_{k=0}^{2^m-1} e^{2\pi ik(\theta-j2^{-m})} \right) |j\rangle .$$

2. We now measure this state in the standard basis. Compute the probability $p_j$ of obtaining outcome $j$.

3. Suppose we are aiming for a precision of $t < m$ bits, i.e., we would like the outcome to be some $j$ such that $|\theta - j2^{-m}| \leq 2^{-t-1}$. Compute a lower bound on the probability of obtaining such an outcome. The bound should be such that if $t$ is fixed and $m$ grows, the probability of success should go to 1. For this you may use the following inequality without proof: for $\gamma \in [-\pi, \pi]$, $|1 - e^{i\gamma}| \geq \frac{2}{\pi} |\gamma|$. 