Article Report: No extension of quantum theory can have improved predictive power by R. Colbeck and R. Renner

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Table of contents

Abstract

In [EPR35] authors postulated that uncertainty in quantum mechanics originates from our incomplete understanding of the laws of nature. This idea clashed with the Copenhagen interpretation of quantum mechanics stating that physical systems do not have definite properties prior to being measured and thus uncertainty is indispensable. Starting with [Bel64] various so called *no-go theorems* proved that under certain assumptions no extension of quantum mechanics that would agree with its predictions and give more information about the outcomes of measurements is possible. An article we are presenting includes a novel approach for *no-go theorems*.

1 Overview

While preparing this report I have faced three major difficulties all of them related to the fact that this is an article of theoretical physics and not of computer science or quantum information theory. First of all I have had to understand the motivations and historical context behind this work that include more than 80 years of discussions and experiments testing the very foundations of quantum theory and our believes about the laws of nature. Then the treatment of the quantum states in this article differs from what we have seen during the course and this coupled with the theoretical physics style heuristic arguments abundant in the text make the article difficult to fully understand. And finally the results provided are far from being unquestionable as can be seen from the *Controversies* section based on the critique presented in [Lan15], [GR13a] and [GR13b]. Thus the goal of this report is not to give the formal proof of the main theorem but rather to present the historical context, motivations of this work and the controversies it has provoked.

2 Historical context

2.1 EPR

Einstein, Podolsky and Rosen in their article [EPR35] concluded that Quantum mechanics is an incomplete theory and there should exist some *hidden variables* that would explain the uncertainty predicted by quantum theory. These might be the microscopic properties of fundamental particles that we are not yet able to observe due to the technological limitations. However their discussion depended on somehow vague definitions of an element of reality (*if, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical* *quantity*) and complete theory (*every element of the physical reality must have counterpart in the physical theory*) and was objected by the proponents of the Copenhagen interpretation, notably Niels Bohr in [Boh35].

In [EPR35] authors presented a thought experiment that became known as EPR paradox. They have demonstrated how quantum theory allows quantum entanglement of two particles P and Q such that the measurement on P determines an outcome of the measurement on Q even if the particles are separated in space. Thus EPR paradox defies *the principle of locality* stating that physical processes occurring at one place should have no immediate effect on the elements of reality at another location and authors argued that *no reasonable definition of reality could be expected to permit this.* This point of view became known as *local realism.*

2.2 Bell's theorem

John Bell in [Bel64] formulated mathematically the ideas of *local realism* and showed that under these conditions the correlations between outcomes of different measurements performed on separated entangled physical systems must satisfy certain constraints known as Bell inequalities. Author then showed that these constraints differ from the ones predicted by quantum mechanics and concluded that *in a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously.*

Great number of Bell experiments have been conducted in order to confirm the predictions given by quantum mechanics. Up until 2015 these experiments relied on various unverifiable assumptions known as loopholes, however the first loophole-free Bell test supporting quantum non-locality was reported in [HBD$^+$15].

In [Bel64] author makes an implicit assumption that measurement settings can be chosen freely. Thus his theorem does not rule out the possibility of *super determinism* which is a class of theories in which experimenter's choice to carry one set of measurements rather that another is predetermined.

2.3 Free Will Theorem

In [CK08] authors present three axioms

1. SPIN - measurements of the squared components of the spin of a spin 1 particle in three orthogonal directions always give the answers 1,0,1 in some order.
2. TWIN - for twinned (*entangled*) spin 1 particles, suppose experimenter A performs a triple experiment of measuring the squared spin component of particle a in three orthogonal directions x, y, z, while experimenter B measures the twinned particle b in one direction, w. Then if w happens to be in the same direction as one of x, y, z, B's measurement will necessarily yield the same answer as the corresponding measurement by A.
3. MIN - assume that the experiments performed by A and B are spacelike separated (*authors treat this experiment in a conventional way which considers a finite number of directions [Per91]*). Then experimenter B can freely choose any one of the 33 particular directions w, and a's response is independent of this choice. Similarly and independently, A can freely choose any one of the 40 triples x, y, z, and b's response is independent of that choice.

Authors proceed by proving *The Strong Free Will Theorem*

Theorem 1. *The axioms SPIN, TWIN and MIN imply that the response of a spin 1 particle to a triple experiment is free - that is to say, is not a function of properties of that part of the universe that is earlier than this response with respect to any given inertial frame.*

Authors then conclude that there is no relativistic deterministic theory of nature that could reproduce the results predicted by quantum mechanics.

3 What's new?

3.1 Additional information

In the previously considered *no-go* theorems authors were interested if quantum mechanics or at least certain experiments could be made deterministic by observing some *deterministic hidden variables.* In the article we present [CR11] authors keep the description of *hidden variables* (that are called *additional information* in the article) way more general. It is not assumed to be encoded in a classical system, but instead characterized by how it behaves when observed. The only restriction on this additional information is that it can be accessed at any any time and that it is *static*, that is its behavior does not depend on where and when it is observed. Thus authors allow hidden variables to be a quantum system.

3.2 Improvement of predictions

In [CR11] authors do not ask if additional information can complete quantum mechanics into deterministic theory which was an initial idea of Einstein, Podolsky and Rosen. The question is more modest one: can it provide any improvement on the predictions of this theory?

3.3 Free Will

Authors try to formalize mathematically the free will hypothesis stating that the experimenters can choose what to measure independently from each other's choice. As we will see in the section *Controversies* this formulation stimulated lots of discussions which at the end might help to clarify the notion of the free will itself.

3.4 Motivation

Results obtained are of significance for the foundations of quantum mechanics as they to the certain extent confirms the inherent randomness in the quantum theory. This is not only of theoretical importance, but also relevant to the tasks that exploit this randomness, such as quantum cryptography.

4 Presentation

In this section we present informally the settings and assumptions used in the article and give an outline of the proof.

4.1 Settings

Authors model an experiment as a triplet of a system being measured, measurement device which has a number of possible settings and some additional information that is chosen to be observed. If A is a setting of a device, X is an outcome of the measurement and Ξ is an additional information then within quantum mechanics we can compute the distribution $P_{X|A}$ and the goal is to show that a Markov chain condition

$$X \leftrightarrow A \leftrightarrow \Xi$$

holds, meaning that the distribution of X given A and Ξ is the same as the distribution of X given only A.

4.2 Assumptions

We consider author's two assumptions on an informal level. The first one (called QM) is that the present quantum theory is correct. This is a natural assumption as the main question is whether current quantum theory can be extended. And the second (called FR) is that measurement settings can be chosen freely. It is noted that assumption FR is common in physics, for example it is a crucial ingredient of Bell's theorem. In this paper FR is interpreted as an equality

$$P_{A|BCYZ} = P_A$$

Stating that the choice of the parameters A is independent from the choice of the parameters for other devices and measurements of those devices.

4.3 Proof

On an informal level the proof consists of the two main parts. In the first part authors consider a system of two entangled particles that are being measured in two space like separated devices. They show that FR forces certain equalities of distributions that they call *non-signaling constraints*. These mean that the settings of measurements of the two devices and a choice of what additional information to observe does not interfere with each other. Then they show that these *non-signaling constraints* guarantee that no additional information Ξ can help to predict the outcome of the experiment. This part is somehow technical and draws on ideas from *non-signaling cryptography*. In the second part of the proof authors use an assumption QM to argue that the previous conclusion applies to the arbitrary states by appending an additional measurement constructed in such a manner that the state before the measurement can be considered entangled, so that they can use the result obtained in the first part of the proof. For any further details considering the proof a reader should check [CR11] as we will move on to the discussion of the controversies caused by this theorem.

5 Controversies

In [CR11] quantum states are treated as random variables which differs from the formulations we have seen during the course. This and a somehow vague use of such terms as *correlation* makes the article hard to fully understand. In addition to that some intermediate results seemed questionable to me, notably the *non-signaling constraint* presented in the page 4:

$$P_{XY|ABC} = P_{XY|AB}$$

This constraint is obtained from an assumption FR using simple manipulations of conditional probabilities. Here X, Y are the outcomes of the measurements on the entangled particles, A, B are the settings of the two devices and C is the choice of what additional information should be observed. Informally this means that choosing what to observe as hidden variables does not give any additional information on the outcomes of the experiment. However if hidden variables are classical this *non-signaling constraint* seems to trivially exclude all the deterministic completions of the quantum theory without using any assumptions QM on quantum theory which seems to be too strong as a result, because Bell's theorem uses QM extensively.

This idea encouraged me to research the papers citing [CR11] and discover [GR13a] which stresses exactly the same point. Authors argue that the arguments given in [CR11] *are basically flawed by an inappropriate use of the assumption of free will FR.* They show that even if a [CR11] is formally correct, the conditions by which an assumption FR is formalized *are, by no means, physically necessary and appropriate.* Authors indeed note that the case of deterministic completions of quantum mechanic are trivially excluded by an assumption FR. In yet another paper [GR13b] the same authors provide a different formalization of the free choice assumption from which it follows that there exist extensions of quantum theory satisfying assumption QM and the modified assumption FR that are predictively inequivalent to quantum theory in a possibly experimentally testable way.

In yet another critical paper [Lan15] authors argue that treating the quantum states as random variables is *not conventional* and they rewrite the proof of [CR11] replacing the *theoretical physics style heuristic arguments* by rigorous mathematics. Authors show *that additional assumptions are necessary to make the proof work, so that the theorem is weaker than it may appear to be at first sight: it does not show that quantum mechanics is complete, but that (informative) extensions are subject to (possibly undesirable) constraints.* Some of these additional assumptions are purely technical and invented for the proof to work, which seems to indicate that the starting hypotheses are not well chosen.

6 Conclusions

This article presents a *no-go* theorem while further relaxing the notions of *hidden variables* and *completions* of the quantum theory to be excluded. The novel approach to formulate the free will hypothesis and the treatment of mathematics in the paper are questionable ([GR13a], [Lan15]), however the ideas presented here certainly stimulated further discussion. Transforming the proofs of the main theorem given in [CR11] and [Lan15] into the framework of quantum information would certainly be helpful and might provide further insights on how the free will hypothesis should be formulated.

References

[Bel64] John S. Bell. On the Einstein-Podolsky-Rosen paradox. *Physics*, 1:195–200, 1964.

[Boh35] N. Bohr. Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.*, 48:696–702, Oct 1935.

[CK08] John Conway and Simon Kochen. The Strong Free Will Theorem. July 2008.

[CR11] R. Colbeck and R. Renner. No extension of quantum theory can have improved predictive power. *Nature Communications*, 2011.

[EPR35] A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.*, 47:777–780, May 1935.

[GR13a] GianCarlo Ghirardi and Raffaele Romano. About possible extensions of quantum theory. *Foundations of Physics*, 43(7):881–894, 2013.

[GR13b] GianCarlo Ghirardi and Raffaele Romano. Ontological models predictively inequivalent to quantum theory. *Phys. Rev. Lett.*, 110:170404, Apr 2013.

[HBD$^+$15] B. Hensen, H. Bernien, A. E. Dréau, A. Reiserer, N. Kalb, M. S. Blok, J. Ruitenberg, R. F. L. Vermeulen, R. N. Schouten, C. Abellán, W. Amaya, V. Pruneri, M. W. Mitchell, M. Markham, D. J. Twitchen, D. Elkouss, S. Wehner, T. H. Taminiau, and R. Hanson. Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres. *Nature*, 526(7575):682–686, October 2015.

[Lan15] Klaas Landsman. On the colbeck-renner theorem. *Journal of Mathematical Physics*, 56(12), 2015.

[Per91] A. Peres. Two simple proofs of the kochen-specker theorem. *Journal of Physics A: Mathematical and General*, 24(4):L175, 1991.