Ranking Transport Projects by their Socioeconomic Value or Financial Internal Rate of Return ?

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Abstract

This paper discusses the choice by the public authority of the most efficient programme of infrastructure investments. More specifically, it studies the optimal ranking of the projects implementations when these projects are partially self-financed by their own revenues such as tolled highways. In this case, the optimal investment programme must be defined under a constraint of annual subsidies. This paper demonstrates that the optimal ranking is not necessarily the ranking of decreasing socio-economic IRR.

This counter-intuitive result can be demonstrated by a general approach. Analytical calculations are not useful in this discrete problems because each programme is an ordered subset of projects. Therefore, there is no continuous variation linking the various programmes and the usual tools of optimisation are useless, such as differential calculus. Thus, we adopt here a discrete optimisation analysis based on standard techniques in the physics area, such as Monte Carlo sampling.

Introduction

For most of the last century, the roles of private and public actors in the transport sector was clear. For instance, public authorities were generally in charge of financing and building new infrastructures. Nevertheless, an inflexion was observed during the nineties with a significant development of public-private partnership (PPP). This is paradoxical since the most profitable lines were already service and the subsequent projects show a financial IRR below the self-financing level. The paradox of the appearance (or reappearance) of the PPP in these circumstances has already been explained by Bonnafous, (2001, 2002) who deals with the formalization of the need of subsidies we will propose in paragraph 3.

This paper deals with the new issues raised by the PPP system or, more generally, by any system in which the new infrastructure is partially financed by its users. Is there, in this case, a new economic rationality of public authorities? Particularly, is there an optimal way to rank projects ?

2. The encounter of public and private investors

Before answering to these questions, we have to recall the logic of investment for both the private operator and the public sector in order to explore the financial logic of a PPP. From the classical point of view of microeconomics, it is assumed that a private operator implements a project if the expected IRR covers:

- the market interest rate,
- plus a risk premium which takes account of the uncertainties that necessarily affect assessments of, for example, costs and future traffic and revenue[1],
- plus a profit margin.

Thus, with a market interest rate of 4 %, a risk premium of 4 % and a profit margin of 4 % too, the minimum targeted IRR will be 12 %. If the IRR of the project is any lower than this the operator will require a subsidy in order to reach 12 %.

The public authority is likewise using the IRR of the project, namely the discount rate which cancels out its Net Present Value (NPV). Nevertheless the valuation of this NPV takes into account not only the future accounting of the operation but also those of concurrent operators and, more generally earnings and losses of all the concerned agents, including external effects such as the users surplus, consequences on safety or environmental effects. For this socio-economic assessment, we will use the notation IRRse for the internal rate of return.

In the tradition of public evaluation, a project is considered to be implemented when its IRRse is higher than a standard level[2]. This border-line can be interpreted as a collective profitability condition: for any project having a lower IRRse than this standard level it is assumed that the destroyed wealth would be higher than the created wealth[3].

In the case of infrastructures exclusively financed by public subsidies (excluding any user contribution), if we consider not only one project but a program of scheduled projects, the objective function is the NPVse provided by the program. Thus, the question of the optimal ranking is solved by the decreasing order of the IRRse's and the rhythm of their implementation depends of the available budget. In the case of a PPP, and more generally when the projects are partially financed by the users, the objective function of the public authority still being the total NPVse of the program, it is not obvious that the decreasing order of the IRRse's provides the optimal ranking.

In this case, which is typically the case of railways infrastructures, the socio-economic efficiency of each unit of subsidy result not only of the IRRse of each project but also of its need of subsidy, which is itself depending of its financial IRR. Thus, we have to use the relationship between the need for subsidies and the level of the IRR (Bonnafous, 1999).

3. A fundamental relationship

In order to formalize this relationship we shall consider a standard project for which an investment C will be made for a duration d, which is the number of years over which it has been assumed that expenditure will be evenly spread. The net profit made by the operation of the project once it has come into service is denoted by a, and it has been assumed that this will increase by an annual amount b.

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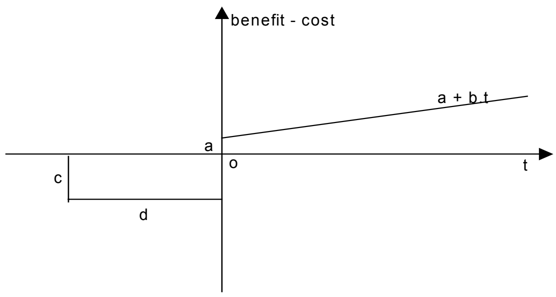
1 The risk premium may also include an additional amount to cover uncertainties about the stability of the country in question. This country spread can be very important in some developing countries.
2 In the French case, this standard level (« taux d'actualisation du Plan »), is 8 % from the early eighties.

This corresponds to the stylized, but nevertheless quite familiar, account of costs and benefits that is set out in Figure 1. If it is assumed that the project comes into service at the date t = 0, annual expenditure between the dates –d and 0 will be given by c = C/d. The profit made once the project comes into service is assumed to take the form (a+b.t).

Figure 1: Cost-benefit function for a stylized project as a function of time.



The Internal Rate of Return (IRR) of the project, namely the discount rate which cancels out its Net Present Value (NPV), is therefore a function of the four parameters c, d, a and b. We must compare this IRR to the Rate of Return that an operator can reasonably expect.

In what follows we shall use the following notation:

α is the discount rate used to calculate the Net Present Value (NPV),

α_0 is the discount rate which cancels out the NPV of the project, which is therefore its IRR,

δ is the amount by which the subsidy increases the IRR,

τ is the rate of subsidy, i.e. the percentage of c which is financed by subsidies.

For a discount rate α, and the present value of cost and benefits from date –d to date T, the Net Present Value is given by the following expression:

[3] That means that the NPVse of the project, evaluated with the standard level of interest rate, would be negative.

$$\mathrm{NPV} = \int_{-d}^{0} -\mathrm{c.e}^{-\alpha t}.\mathrm{dt} + \int_{0}^{T} (\mathrm{a} + \mathrm{b.t}).\mathrm{e}^{-\alpha t}.\mathrm{dt} \qquad (1)$$

In order to simplify the calculations that follow, we shall assume that the present value calculation has been extended to infinity, which will have little effect on the results because of the small influence of the distant future (and the known convergence of these integral functions). Equation (1) therefore becomes:

$$\mathrm{NPV} = \left[\frac{\mathrm{c}}{\alpha}\mathrm{e}^{-\alpha t}\right]_{-d}^{0} + \left[-\frac{\mathrm{a}}{\alpha}\mathrm{e}^{-\alpha t}\right]_{0}^{+\infty} + \left[-\frac{-\mathrm{b.t}}{\alpha}\mathrm{e}^{-\alpha t}\right]_{0}^{+\infty} + \left[-\frac{\mathrm{b}}{\alpha^{2}}\mathrm{e}^{-\alpha t}\right]_{0}^{+\infty} \qquad (2)$$

or alternatively:

$$\mathrm{NPV} = \frac{1}{\alpha}\left[\mathrm{c}(1-\mathrm{e}^{\alpha d}) + \mathrm{a} + \frac{\mathrm{b}}{\alpha}\right] \qquad (3)$$

The IRR of the project, α_0 is therefore given by :

$$\mathrm{c}(1-\mathrm{e}^{\alpha_0 d}) + \mathrm{a} + \frac{\mathrm{b}}{\alpha_0} = 0 \qquad (4)$$

A rate of subsidy τ lowers the annual cost of construction c to c(1-τ) and raises the IRR α_0 to ($\alpha_0+\delta$) such that equation (4) becomes (4’)

$$(1-\tau)c(1-e^{(\alpha_0+\delta)d})+a+\frac{b}{\alpha_0+\delta}=0 \qquad (4')$$

Which allows us to express the required rate of subsidy :

$$\tau = 1-\frac{a(\alpha_0+\delta)+b}{c(\alpha_0+\delta)(e^{(\alpha_0+\delta)d}-1)} \qquad (5)$$

Rather to analyze the properties of this function, we will use its graphic representation which shows clearly some economic consequences of these properties.

4. Concavity and tyranny of the financial profitability

What is of prime importance to us in this function is clearly the relationship between τ and δ. However, equation (5) also shows that this relationship obviously depends on the values of the parameters c, d, a, b and, of course, α_0, which characterize the economics of the project and which are moreover linked together by equation (4) which established the IRR of the project α_0. If we wish to represent equation 5 we therefore need to keep some of these 5 parameters constant and vary just those whose role we wish to demonstrate. This is the well-known nomogram technique.

In this paper we shall only reproduce one of these nomograms, which will be sufficient to illustrate the point we wish to make (Figure 2). The values of c have been kept constant (assumed to be 100); as have those of d (5 years) and b (taken as 1). We have plotted α_0 against a. The IRR of the project α_0 thus takes on a series of values between 2% and 14 % with a step equal to 0.4%. The function (5) which links the rate of subsidy to δ is then shown for each of these values of α_0.

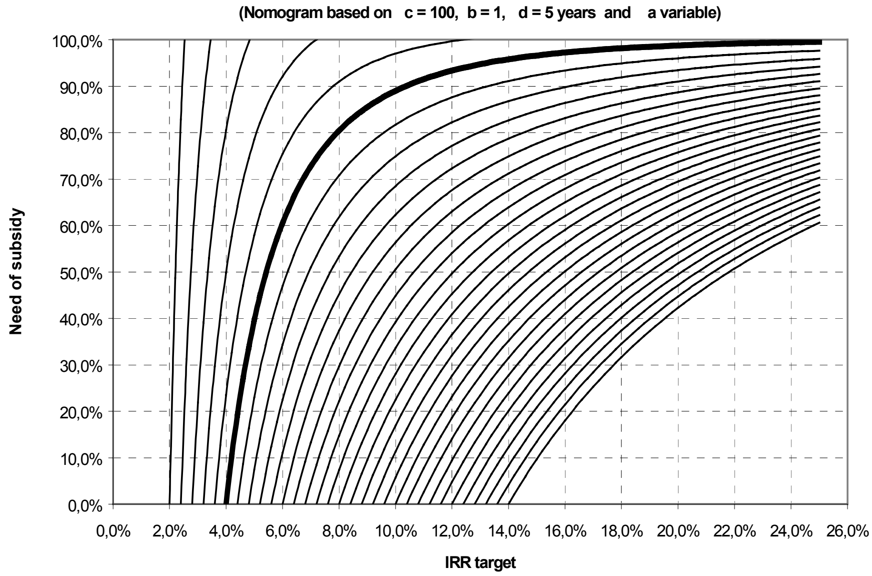


Figure 2 : Nomogram showing the need of subsidy as a function of the target IRR, for different project's IRR (from equation (5)).

Of course, these functions must in no way be considered as completely general because of the hypotheses which were made in order to develop them and which, in particular, relate to the time series of the costs and benefits of the project in question. For instance, it is possible (and easy) to modify equation [5] by hypothesizing of an exponential instead of a linear increase in demand. Nevertheless, none of our hypotheses are extraordinary, and the conclusions suggested by the shape of these curves can be accepted.

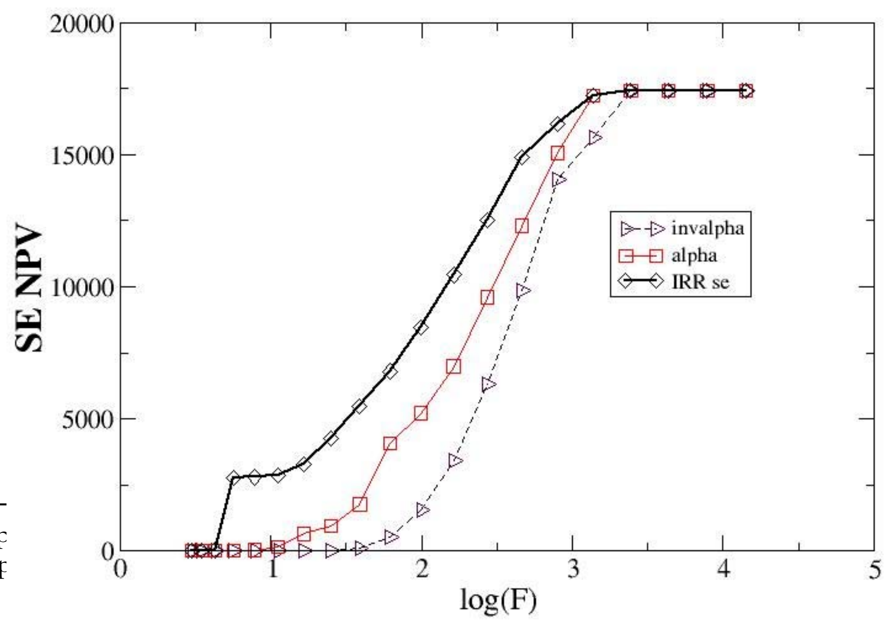
It is quite natural for the need for subsidy to be an increasing function of the additional IRR which the operator must receive. However, the gradient of the curve decreases in a marked manner. This concavity is a counter-intuitive result: it means that the first differences between the targeted IRR and the IRR of the operation are extremely costly, particularly in the case of projects with a low IRR. For example, in the case of a project for which $\alpha_0 = 4$ % the plot of the subsidy rate against δ (shown in bold on Figure 2) shows that a subsidy rate in excess of 80 % is required to raise the IRR for the operator to only 8%.

This first observation suggests that *the leverage effect of public finance on the rhythm of investment will be more powerful when preference is given to projects, if* *any exist, whose IRR is very close to the target IRR required by the operator*. It suggests more generally that under a public budget constraint, the role of the financial IRR could be primordial in the determination of an optimized PPP program.

5. How to find the optimal order for a set of PPP projects?

To confirm this point, we will take the set of 17 toll highway projects for which homogeneous economical data can be obtained[4]. Subsidizing rates have been calculated from equations (4) and (5), taking 8% as the target IRR. We assume that there exists a budget constraint the first year (F, in MEuros), this constraint increasing by 2,5 % yearly. To make clear the role of the budget constraint, we have changed its value between 1 and 1000 MEuros. For a given order of the projects, the value of F determines the rhythm of completion of projects, since each project needs to draw on this budget an amount $\tau .c$ of public subsidies each year.

We first examine the interest of using the IRRse as a ranking criterion. For this, we calculate the NPVse returned by three different programs obtained by ranking the projects by alphabetical or inverse alphabetical order (simulating a random combination of the projects) or by decreasing IRRse. Figure 3 shows that the IRRse is clearly more efficient as a ranking criterion than randomness, which is not surprising.



[4] These 17 French p... all contribution from p... ne European reform).

Figure 3 : Comparison of the NPVse returned by "random" (alphabetical or inverse alphabetical) ranking of projects or by ranking with IRRse. Clearly, the IRRse order leads to a higher NVPse.

However, our previous discussion suggests that the IRRse is probably not the best ranking criterion, mainly if budget constraints are important. Therefore, we can calculate the NPVse outputs obtained by ranking the projects with two other criteria: the financial IRR and the « output », defined as O(i) = NPVse(i)/sub(i), where sub(i) represents the amount of public subsidies required by this project to obtain the targeted IRR (8%, as assumed above). In figure 4, we compare the total NPVse returned by the 17 highway projects for different rankings, as a function of the mean subsidy required by all the projects (<F>= 472). More precisely, we plot the % of gain obtained by choosing the specified ranking criterion compared to the total NPVse returned by using the IRRse as the ranking criterion. Note that the curve labeled "Optimum" (triangles) will be discussed below. The results presented in Figure 4 confirm our intuition : the pure financial IRR is a better ranking criterion than the IRRse, and this is the truer the tighter the budget constraint. When this constraint is lower than some value (close to one third of the average subsidy needed for a project), the NPVse output of the program obtained with the IRRse ranking is frankly disastrous when compared with the pure financial IRR order.

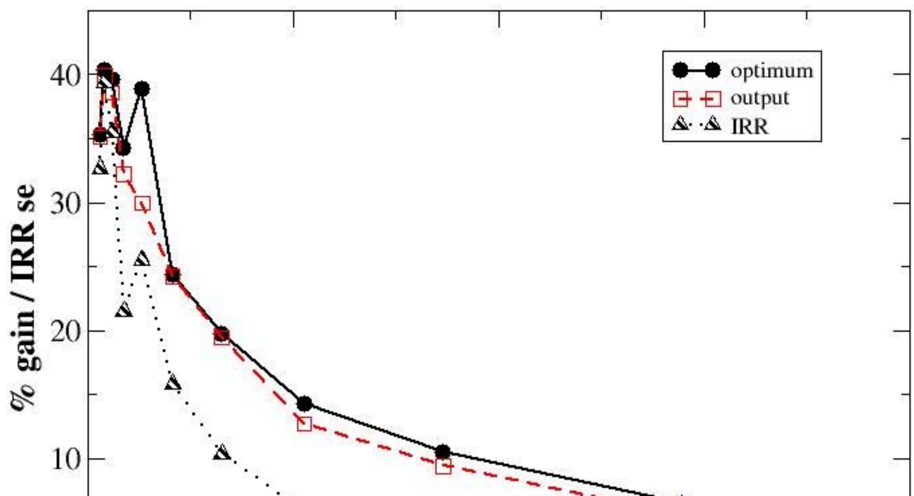


Figure 4 : Comparison of the total NPVse returned by the 17 highway projects for different rankings. We plot the % increase of the program NVPse compared to that obtained by the IRRse ranking.

Figure 4 also shows that our “output” criterion is even better than the IRR, for all the values of the public budget constraint. The point now is : can we find a better criterion ? Can we be sure to have found the best possible ranking? The problem is that, since a program is a *discrete* combination of all the single projects, we cannot use the standard analytical optimization techniques such as Lagrangian multipliers or functional derivatives to find rigorously the best criterion. Therefore, we are left with exploring the different combinations of projects to find the one that yields the highest NPVse. However, the number of possible orders ($17! = 3\ 10^{14}$) forbids an exhaustive examination of all the possibilities. Fortunately, several tools to explore efficiently the “landscape” of different combinations have been developed by several disciplines.

6. Simulation approach to optimization

Let’s present briefly the three most widely used methods. First, the “simulated annealing” (Kirkpatrick *et al.*, 1983, van Laarhoven and Aarts, 1992) method mimics the way physical systems approach thermal equilibrium. The system is taken to a high “temperature” (introducing disorder, which allows to explore a large number of configurations), then “cooled” slowly, hoping that the internal dynamics of the system will steer it towards the optimum configuration, as happens with crystallization of materials. Second, "genetic" algorithms (Mitchell, 1996) draw inspiration from natural selection, where the convergence to the optimum is obtained by random generation of individuals (each configuration), their reproduction (for example, a linear interpolation) and their selection following the desired criteria. Finally, several algorithms were recently proposed mimicking the behavior of ant colonies (Bonabeau *et al.*, 1999) which are able to find shortest paths from their nest to the food sites. For our problem, which is relatively simple, we have chosen a reliable algorithm, inspired from the Monte Carlo method.

The algorithm can be summarized as follows. We start from an arbitrary initial state (random or given by the IRR order, this has no consequence on the final result, as shown in figure 5), we iterate the following process as many times as needed to converge.

(1) Two randomly chosen projects, i and j, are permutated.

(2) The total NVPse for this new ranking is calculated.

(3) The permutation is systematically accepted the new total NVPse is larger than the previous one. In this case, the "temperature" (explained later) is slightly decreased and we start over (step 1).

(4) However, to avoid the jamming of the system in a given configuration, we allow for a (small) probability of accepting the permutation even if it leads to a lower total NVPse, as in the simulated annealing algorithm. For this, we calculate p=exp((NVPse_new – NVPse_old)/T), where T is a "temperature".

(5) We throw a random number q ($0 < q < 1$) and compare it with p. If it is lower, we authorize the permutation, otherwise, we cancel it. If the permutation is accepted, the temperature is slightly decreased. In any case, we start over, step (1).

We have tested the robustness of the algorithm by changing the initial value of the "temperature" (see figure 5). This temperature, as in the simulated annealing algorithm, is an arbitrary variable that allows more or less large fluctuations around the maximum value at any iteration of the search. If a higher value is fixed, the fluctuations are larger, because rankings with lower total NVPse can be accepted, but the algorithm always converges to the same ranking. Figure 5 also shows that convergence to the same final, optimal ranking of projects is ensured for any initial state (IRR or random ranking). Our algorithm can therefore be trusted to find the unique optimal ranking of projects.

Figure 5 : Convergence of the program NVPse to the optimum configuration as the simulation iterates.

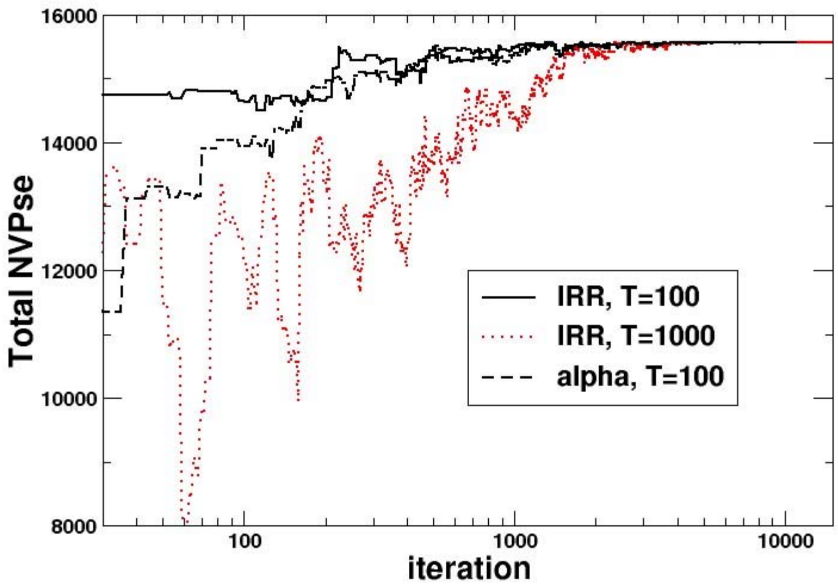


Figure 4 shows the results obtained by the optimization algorithm, labelled "Optimum" (triangles). It confirms that the R ranking is the optimum, since it always corresponds to the value found by our algorithm. Even if this does not constitute a rigorous proof of the output criterion as the best ranking criterion, we can trust the generality of our optimization criterion.

This kind of algorithms will certainly show its full power to study programs of interdependent projects, namely programs where TRI of one project depends on its rank (or even the date) of implementation.

7. Conclusion

Let's summarize here our three main results, which may seem paradoxical but are a direct consequence of the present financial constraints.

1) When financial constraints are tight, the social return of a program of investments is higher when projects are ranked according to their pure financial characteristics (such as financial IRR), instead of their pure socioeconomic characteristics.

2) We have also confirmed the *hybrid* optimum ranking criterion : the "output", defined as the ratio of the socioeconomic NPV to the amount of subsidy it needs.

3) A simulation approach, for example using our optimization algorith above, will be particularly useful in complex situations. For example, when the characteristics of the projects depend on the precise date of completion (because of external factors, such as international development) or on the precise order of completion (a project IRR can depend on the fact that another project has been completed previously). In those complex situations, analytical treatments are certainly difficult to imagine, and simulation tools such as those developed in this paper may turn out to be a powerful tool.

In the case of infrastructure financed exclusively by public subsidies, the public objective function has traditionally been the NPVse provided by the program of scheduled projects, the question of their optimal ranking being solved by the decreasing order of their IRRse's and the rhythm of their implementation depending on the available budget. We have shown that in the case of a PPP, and more generally when the projects are partially financed by the users, the objective function of the public authority still being the total NPVse of the program, the decreasing order of the IRRse's does not provide the optimal ranking: the pure financial IRR is a better ranking criterion, and this is the truer the tighter the budget constraint.

The ratio of the socioeconomic NPV to the amount of subsidy required is a still better criterion – in fact, the best. We can conclude, therefore, that both the tyranny of financial profitability and the error of ranking by the IRRse become issues as soon as the user becomes involved in the financing of transport infrastructure.

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