

## On the emergence of an 'intention field' for socially cohesive agents

This content has been downloaded from IOPscience. Please scroll down to see the full text.

J. Stat. Mech. (2014) P03010

(<http://iopscience.iop.org/1742-5468/2014/3/P03010>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 140.77.67.190

This content was downloaded on 12/03/2014 at 09:21

Please note that [terms and conditions apply](#).

# On the emergence of an ‘intention field’ for socially cohesive agents

Jean-Philippe Bouchaud<sup>1,2</sup>, Christian Borghesi<sup>3</sup> and Pablo Jensen<sup>4,5</sup>

<sup>1</sup> CFM, 23 rue de l’Université, F-75007 Paris, France

<sup>2</sup> Ecole Polytechnique, F-91120 Palaiseau, France

<sup>3</sup> Laboratoire de Physique Théorique et Modélisation, UMR-8089 CNRS-Université Cergy Pontoise, France

<sup>4</sup> Laboratoire de Physique, UMR-5672 CNRS, ENS de Lyon, France

<sup>5</sup> Institut Rhônalpin des Systèmes Complexes, IXXI, 5 Rue du Vercors, F-69007 Lyon, France

E-mail: [jean-philippe.bouchaud@cfm.fr](mailto:jean-philippe.bouchaud@cfm.fr), [christian.borghesi@u-cergy.fr](mailto:christian.borghesi@u-cergy.fr) and [pablo.jensen@ens-lyon.fr](mailto:pablo.jensen@ens-lyon.fr)

Received 4 November 2013

Accepted for publication 27 January 2014

Published 11 March 2014

Online at [stacks.iop.org/JSTAT/2014/P03010](http://stacks.iop.org/JSTAT/2014/P03010)

[doi:10.1088/1742-5468/2014/03/P03010](https://doi.org/10.1088/1742-5468/2014/03/P03010)

**Abstract.** We argue that when a social convergence mechanism exists and is strong enough, one should expect the emergence of a well-defined ‘field’, i.e. a slowly evolving, local quantity around which individual attributes fluctuate in a finite range. This condensation phenomenon is well illustrated by the Deffuant–Weisbuch opinion model for which we provide a natural extension to allow for spatial heterogeneities. We show analytically and numerically that the resulting dynamics of the emergent field is a noisy diffusion equation that has a slow dynamics. This random diffusion equation reproduces the long-ranged, logarithmic decrease of the correlation of spatial voting patterns empirically found in Borghesi and Bouchaud (2010 *Eur. Phys. J. B* **75** 395) and Borghesi *et al* (2012 *PLoS One* **7** e36289). Interestingly enough, we find that when the social cohesion mechanism becomes too weak, cultural cohesion breaks down completely, in the sense that the distribution of intentions/opinions becomes infinitely broad. No emerging field exists in this case. All these analytical findings are confirmed by numerical simulations of an agent-based model.

**Keywords:** critical phenomena of socio-economic systems, interacting agent models

---

**Contents**

<b>1. Introduction</b>	<b>2</b>
<b>2. The model</b>	<b>3</b>
<b>3. Local equilibrium and phase transitions</b>	<b>5</b>
<b>4. Spatial heterogeneities</b>	<b>8</b>
<b>5. Numerical simulations</b>	<b>9</b>
5.1. Single city dynamics . . . . .	9
5.2. Spatial correlations . . . . .	11
5.3. The influence of changing places . . . . .	12
<b>6. Conclusion</b>	<b>13</b>
<b>Acknowledgments</b>	<b>13</b>
<b>Appendix</b>	<b>14</b>
<b>References</b>	<b>14</b>

---

**1. Introduction**

The understanding that unremarkable individual elements can give rise, through interactions, to remarkable collective emergent phenomena is arguably the most important contribution of statistical physics to science. These collective effects are often so ordinary that we do not think of them as surprising, like for example the rigidity of a solid, which is still one of the most remarkable properties of interacting atoms<sup>6</sup>. Some exotic states of matter are more astonishing, such as superfluidity or superconductivity, liquid crystals, etc. However, this concept reaches out far beyond physics and is relevant to our understanding of a host of situations, ranging from avalanches [4], the functioning of the brain [5], traffic jams [6] and bird flocks [7], to all kinds of social phenomena, like crowd motions [8], fads [9], bubbles, panic and crashes, see e.g. [10]–[14] for recent reviews.

Recently, two of us (CB and JPB) analyzed the spatial correlations of the turnout rate in many elections and in different European countries [1, 2]. We found in all cases a slow, *logarithmic decay* of the correlation function as a function of distance. This logarithmic behavior was very recently confirmed on US data by another group [15]. This empirical regularity not only means that the behavioral pattern of individuals is far from random, but also suggests that some universal underlying mechanism must be at play for such a non-trivial characteristic pattern to appear.

<sup>6</sup> On this point, please read the deeply insightful remarks of Anderson [3]. One particularly relevant quote in the present context is the following. *We are so accustomed to [this] rigidity property that we don't accept its almost miraculous nature, that is an ‘emergent property’ not contained in the simple laws of physics, although it is a consequence of them.*

Intrigued by this logarithmic correlation function, we proposed in [1] an interpretation in terms of a space dependent collective field, which we called a ‘cultural’ field, that determines the local propensity of individuals to vote. We argued that this field ‘transcends individuals while being shaped, shared, transported and transformed by them’, and conjectured [1] that the spatio-temporal evolution of this field is governed by a noisy diffusion equation (a.k.a. the Edwards–Wilkinson equation [16]),<sup>7</sup> which naturally leads to logarithmically decaying correlation functions. Still, this proposal was only based on analogies and plausibility, and a detailed, agent-based justification of the existence of such a field, and its diffusive character, was lacking. The arguments put forward in [1] were targeted at the statistical physics community but were deemed unconvincing in other circles, in particular sociologists, who in fact strongly balked at the name ‘cultural’, if not at the concept itself. There are two points of possible contention. First, physicists’ notion of a field is quite different from its counterpart in, for example, Bourdieu’s sociology. Although in both cases the field is supposed to induce some agents’ actions, in Bourdieu’s meaning of the word [18] the field designates a much more heterogeneous structure, a system of social positions structured by power relationships in which different actors occupy different positions and try to seize social capital (such as money or influence) according to their specific position in the ‘field’. Here, the field is rather a scalar quantity, slowly changing in space. Second, the concept of a ‘field’ as an autonomous entity automatically inducing agents’ actions, as if they were passive puppets, is criticized by some sociologists [19].

The aim of this work is precisely to provide a well-defined, agent-based micro-foundation to the notion of field (which we rename henceforth ‘intention field’, to avoid any possible confusion induced by the word ‘cultural’). As we were working on these ideas, [15] appeared on arXiv with related, but different, ideas. Here, we investigate a model of interacting agents that exhibits a transition between a socially cohesive phase, where the concept of an emergent intention field has a precise meaning and obeys a diffusion equation, and a socially disconnected phase where no emergent quantity can be defined. The model we consider can be viewed as a spatially heterogeneous version of the Deffuant–Weisbuch model [20]. When simulated on the actual map of French towns, our model is found to reproduce *quantitatively* the shape of the correlation function measured on data. Our model, furthermore, allows us to discuss several interesting questions, such as the role of population migrations on the integrity of the intention field.

## 2. The model

We assume that each potential voter  $i$  in city  $\alpha$  (located in space around  $\vec{R}_\alpha$ ) has, at time  $t$ , a propensity to vote (or ‘intention’)  $\phi_{\alpha,i}(t)$ . The probability that this potential voter will actually vote on election day is chosen to be the logistic function, as is standard in choice theory [21],

$$p = \frac{1}{1 + e^{-\phi}}. \quad (1)$$

<sup>7</sup> A similar proposal was actually made in 2000 by Schweitzer and Holyst in a different context, and without the noise component [17].

Now, we posit, as in many previous studies [13, 20], [22]–[24], that the intention  $\phi$  of each individual evolves through interactions with others. (This can, of course, be generalized to other situations beyond voting, and the intentions can also be thought of as ‘opinions’).

More precisely, we postulate the following rules, which are quite general and plausible. Between  $t$  and  $t + dt$ , each individual  $i$  from city  $\alpha$  can meet an individual  $j$  from city  $\beta$ , and his intention at the end of the meeting will have evolved towards that of individual  $j$  by some quantity, possibly zero. The probability (per unit time) that these two individuals meet *and* that the encounter leads to a change of intention is<sup>8</sup>

$$w(i, \alpha; j, \beta) = f(|R_\alpha - R_\beta|) G[|\phi_{\beta,j} - \phi_{\alpha,i}|], \quad (2)$$

where  $f(r)$  is a decreasing function (over a scale of a few kilometers) and  $G$  is a decreasing function of its argument, so as to model, in the spirit of the Deffuant–Weisbuch model [20], the fact that far away opinions have a smaller probability of being in the same social circle, or that such encounters leave each participant with unchanged opinion. The quantity  $dt \sum_{\beta,j} f(|R_\alpha - R_\beta|) G[|\phi_{\beta,j} - \phi_{\alpha,i}|]$  is the total probability of an encounter for individual  $i$  between  $t$  and  $t + dt$ . In practice, a reasonable order of magnitude is probably one (or a few) per week.

The result of this encounter is that both intentions will evolve as<sup>9</sup>

$$\begin{aligned} \phi_{\alpha,i} &\rightarrow \phi_{\alpha,i} + \gamma(\phi_{\beta,j} - \phi_{\alpha,i}) + \eta, \\ \phi_{\beta,j} &\rightarrow \phi_{\beta,j} + \gamma(\phi_{\alpha,i} - \phi_{\beta,j}) + \eta', \end{aligned} \quad (3)$$

where  $0 \leq \gamma \leq \frac{1}{2}$  measures how intentions get closer together, much like in other opinion formation models, and  $\eta, \eta'$  describe all sources of noise due to other influences, which vary both in time and across agents, with some distribution  $Q(\eta)$ , for example Gaussian. The component of the  $\eta$ s common to all agents, independently of the city where they live, can be reabsorbed in a shift of all  $\phi_{\alpha,i}$  by a time dependent contribution  $\bar{\phi}(t)$ , which is irrelevant in the following discussion. We will assume henceforth that the remaining idiosyncratic  $\eta$ s are independent, identically distributed variables, both in time and across agents. (In a refined version of the model, it might be interesting to allow  $\eta$  to be correlated for individuals of the same city, in particular to account for the empirical dependence of the variance of  $\phi$  on the city size, reported in [2].)

These rules allow one to write a Boltzmann-like equation for the probability  $P_t(\phi|\alpha)$  that a given individual in city  $\alpha$  has intention  $\phi$ :

$$\begin{aligned} \frac{\partial P_t(\phi|\alpha)}{\partial t} &= -W_t(\phi) P_t(\phi|\alpha) + \sum_{\beta} f(|R_\alpha - R_\beta|) \\ &\times \int \int \int d\eta d\psi d\psi' Q(\eta) P_t(\psi|\alpha) P_t(\psi'|\beta) G(|\psi - \psi'|) \\ &\times \delta(\phi - \psi - \gamma(\psi' - \psi) - \eta), \end{aligned} \quad (4)$$

<sup>8</sup> We assume for simplicity that all cities have the same population size, and identical number of individuals and probabilities.

<sup>9</sup> Many variations are of course possible, such as choosing  $\gamma$  to be a random variable, but we believe that our results are robust to most changes. Note in particular that the rule given by equation (3) does not conserve the total intention at each encounter.

with

$$W_t(\phi) = \sum_{\beta} f(|R_{\alpha} - R_{\beta}|) \int d\psi' P_t(\psi'|\beta) G(|\phi - \psi'|). \quad (5)$$

By inspection, one checks that the above Boltzmann equation preserves the total probability in each city (at this point we do not consider the possibility that people may change places—but see below).

In the spirit of the Chapman–Enskog expansion which allows one to derive (macroscopic) Navier–Stokes equations from the (microscopic) Boltzmann equation [25], we will argue that under some conditions, a local equilibrium is quickly reached, of the form

$$P_{eq}(\phi|\alpha) = P^*(\phi - \varphi_{\alpha}), \quad (6)$$

where  $\varphi_{\alpha} = \int d\phi \phi P_{eq}(\phi|\alpha)$  is the (time dependent) average of the intention of the individuals in city  $\alpha$ , for which a diffusion equation can be established. The emergence of a slow ‘intention’ field  $\varphi_{\alpha}$  (which is what we called the cultural field in [1]) is the analogue of the emergence of the macroscopic velocity field  $\vec{U}(\vec{r}, t)$  in the Boltzmann equation. In this case, the distribution of velocities quickly reaches a local equilibrium shape (the Maxwell distribution) centered around  $\vec{U}(\vec{r}, t)$  that obeys a (slow) hydrodynamic equation. As with the intention/opinion field, each molecule in the hydrodynamic flow interacts with the velocity field  $\vec{U}(\vec{r}, t)$  such that the local equilibrium is maintained, while the latter is an emergent, collective variable (the local velocity average) that does not ‘belong’ to any molecule in particular, but acquires an autonomous, slow dynamics that allows it to persist on long time scales and develop long-range correlations (whereas the residual velocity of each molecule/individual decorrelates on fast time scales).

### 3. Local equilibrium and phase transitions

Let us first look for a homogeneous solution of the Boltzmann equation, with  $\varphi_{\alpha} = \varphi^*$ , which can, at this point, be set to zero without loss of generality:

$$P_{eq}(\phi|\alpha) = P^*(\phi). \quad (7)$$

In order to keep the calculations tractable, we assume that the  $\eta$ s are Gaussian variables of zero mean and variance  $\Sigma^2$ , and that the interaction kernel  $G$  is also Gaussian with a range  $\zeta$ :

$$G(u) = e^{-u^2/2\zeta^2}, \quad (8)$$

meaning that individuals with intentions/opinions that are too distant from each other ( $>\zeta$ ) cease to interact, much as in the original model of Deffuant and Weisbuch [20]. We believe that these specific choices do not change the qualitative conclusions below.

We will also write

$$\sum_{\beta} f(|R_{\alpha} - R_{\beta}|) = F, \quad (9)$$

which could depend on  $\alpha$  for non-translation-invariant geographies, but this would only change the local equilibration time and not the shape of the equilibrium. Note that  $F$  has the dimension of an inverse timescale.

We look for a Gaussian solution:

$$P^*(\phi) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\phi^2/2\sigma^2}. \quad (10)$$

Inserting into the definition of  $W(\phi)$  yields

$$W^*(\phi) = F \frac{\zeta}{\sqrt{\zeta^2 + \sigma^2}} e^{-\frac{\phi^2}{2(\zeta^2 + \sigma^2)}}. \quad (11)$$

Hence,  $W^*(\phi)P^*(\phi)$  is proportional to a Gaussian of zero mean and variance  $\sigma^2(\zeta^2 + \sigma^2)/(\zeta^2 + 2\sigma^2)$ .

Now, look at the second term on the right-hand side of the equation:

$$\int \int \int d\eta d\psi d\psi' Q(\eta) P^*(\psi) P^*(\psi') G(|\psi - \psi'|) \delta(\phi - \psi - \gamma(\psi' - \psi) - \eta), \quad (12)$$

which is clearly also a Gaussian in  $\phi$  with zero mean and variance:

$$\Sigma^2 + \frac{\sigma^2}{1 + 2\Gamma} (1 + \Gamma - 2\gamma(1 - \gamma)), \quad \Gamma = \frac{\sigma^2}{\zeta^2}. \quad (13)$$

Equating this expression with the above result leads to an equation for our unknown  $\sigma^2$ :

$$\sigma^2 = \frac{1}{2} \frac{\Sigma^2 \zeta^2}{\gamma(1 - \gamma)\zeta^2 - \Sigma^2}, \quad (14)$$

which, obviously, only makes sense if the denominator is positive. Therefore, a stationary (Gaussian) solution only exists provided the idiosyncratic noise is small enough:

$$\Sigma^2 < \Sigma_c^2 = \zeta^2 \gamma(1 - \gamma). \quad (15)$$

We show in the appendix that for larger idiosyncratic noise there is indeed *no* stationary state to the Boltzmann equation: the variance of  $P(\phi)$  diverges without bound (in the absence of any other mechanism limiting the growth of  $\phi$ ) and there is therefore no emergent average opinion/intention.

Below this value, on the other hand, the opinion-sharing interaction between individuals is strong enough to keep the dispersion of intentions finite. The above inequality means that for this to happen the idiosyncratic evolution of intentions (of strength  $\Sigma$ ) must be small compared to the convergence effects: larger  $\gamma$  leads to a stronger reduction of heterogeneities at each encounter, while  $\zeta$  large means that even far away opinions have a chance to meet and converge. Our numerical results below fully confirm these analytic findings.

The above transition between confined and dispersed intentions, i.e. between consensus and dissent, is in fact generic. It has been noted in the context of the Deffuant–Weisbuch model in [22] (see also [23, 24]). It is also present in a variation of the above model, where intentions evolve not only after an encounter, but further diffuse continuously in time,



due to influences unrelated to encounters, like all sorts of books, news media, personal observations and experiences, etc which contain a part common to all agents (which can be discarded as noted above) and a part that is agent specific. This last contribution adds a diffusion term in the Boltzmann equation of the form  $D\partial^2 P(\phi)/\partial\phi^2$ . We again find a phase transition in this case, although it is now a *first order transition*, where  $\sigma^2$  jumps discontinuously from a finite value when the diffusion constant  $D$  is smaller than a certain threshold  $D_c \propto \gamma(1 - \gamma)F\zeta^2$  to infinity for larger idiosyncratic noises. The transition discussed above, on the other hand, is second order, with  $\sigma^2$  diverging continuously as  $\Sigma^2$  reaches the threshold value  $\Sigma_c^2$ , see equation (14).

In any case, the conclusion of this analysis is that when the idiosyncratic noise (called—perhaps ironically—‘free will’ in [22]) is small enough, the basic message of the Deffuant–Weisbuch model holds: there is convergence of the propensities, intentions or opinions of the different individuals towards the average initial opinion of the population, up to a random variable of zero mean (for infinite size populations) and variance given by  $\sigma^2$ . The *condensation of opinions* emphasized by Weisbuch *et al* is essentially what is needed for a collective variable (the average  $\varphi_\alpha$ ) to emerge and play a special role in the long term evolution of the system. In this sense, our model is a spatially heterogeneous extension of the noisy Deffuant–Weisbuch model. Beyond the threshold, there is no convergence anymore and the concept of an intention field (the average intention or opinion) evaporates and becomes meaningless. This happens when the social interactions are no longer strong enough to maintain cultural cohesion.

The convergence of  $P(\phi)$  towards its equilibrium, condensed shape is however non-trivial and needs to be discussed. When the width of the initial distribution  $\sigma_0$  is not too broad compared to the interaction distance  $\zeta$ , the time  $\tau$  needed for the initial distribution to converge towards its asymptotic form  $P^*(\phi)$  is set by a *renormalized* microscopic timescale, given by

$$\tau^{-1} = F \frac{\zeta}{\sqrt{\zeta^2 + \sigma^2}}. \quad (16)$$

In the consensus phase far from the transition  $\Sigma^2 \ll \zeta^2\gamma(1 - \gamma)$ , one has also  $\sigma^2 \ll \zeta^2$ , and therefore  $\tau \approx F^{-1}$ , where  $F$  is simply the total ‘encounter rate’ for each individual, which is, as we said above, roughly one per week. Therefore, we expect convergence to happen quickly, over weeks or perhaps months in a well-connected society, where individuals share their thoughts. If, on the other hand, the system approaches its deconfining phase transition,  $\sigma^2 \gg \zeta^2$  and the equilibration time diverges as  $\tau \approx F^{-1}\sigma/\zeta$ .

The situation becomes quite interesting when the initial condition becomes broad, i.e.  $\sigma_0 \gg \zeta$ . In this case, the convergence takes place, in the condensed phase, in two steps. First, after a time of order  $F^{-1}$ , several sharp peaks emerge, corresponding to a mixed phase where sub-populations are socially cohesive, but the population as a whole is fragmented [20]. Then, a very slow process sets in, through which the number of these peaks progressively decreases and the width of  $P(\phi)$  finally decreases to its equilibrium value  $\sigma^2$ . However, the time needed for this to happen can be estimated to be  $\sim e^{\sigma_0^2/\zeta^2} \ln N$ , which can be so long that equilibrium concepts become irrelevant, and several ‘sub-cultures’ survive in the population. This picture is fully confirmed by our numerical results, see figure 2 below.



#### 4. Spatial heterogeneities

At this point, we borrow from the standard Chapman–Enskog approach to the Boltzmann equation [25] and assume that (a) local equilibration towards a space dependent, shifted distribution  $P^*(\phi - \varphi_\alpha)$  is fast and (b) the field  $\varphi_\alpha$  varies smoothly and slowly in space, in such a way that  $\varphi_\alpha - \varphi_\beta$  is small whenever two individuals from the two cities have an appreciable probability to interact (i.e. when  $f(|R_\alpha - R_\beta|)$  is large).

The time evolution of  $\varphi_\alpha$  can then be computed from the Boltzmann equation, only retaining contributions linear in  $\varphi_\alpha - \varphi_\beta$  and discarding terms of higher order in the difference. The final result reads

$$\frac{\partial \varphi_\alpha}{\partial t} = \frac{\zeta}{\sqrt{\zeta^2 + 2\sigma^2}} \left( 1 + \left( \gamma - \frac{1}{2} \right) \frac{\zeta^2}{\zeta^2 + 2\sigma^2} \right) \sum_\beta f(|R_\alpha - R_\beta|) (\varphi_\beta - \varphi_\alpha), \quad (17)$$

which is precisely the diffusion equation proposed in [1, 15]. This allows one to obtain the value of the diffusion constant in terms of the microscopic parameters of the model. If the range of the function  $f$  is  $\xi$ , we find, in order of magnitude, that the diffusion constant is given by  $\xi^2/\tau$ , where  $\tau$  is the equilibration time discussed above. The relaxation time of the  $\varphi$  field, in a country of linear size  $L$ , is therefore  $\sim \tau(L/\xi)^2$ , which is much larger than the local equilibration time itself provided  $L \gg \xi$ . For France, for example,  $L \sim 500$  km while  $\xi$  is maybe 5 km or so, so that the ratio of time scales is  $\sim 10\,000!$  It is precisely this separation of time scales that justifies the very idea of a ‘hydrodynamical’ description in terms of a collective field.

In the above derivation, we have not taken into account the noise term that describes city-specific contributions to the intentions. This would add an additive random term to the above equation, which, as shown in [1], eventually leads to the logarithmic spatial correlation profile found empirically.

The microscopic model considered up to now discards several effects that would be quite easy to reintegrate. One interesting effect is immigration of individuals with significantly different cultures. This can be seen as adding to the Boltzmann equation a term of the form

$$\left. \frac{\partial P_t(\phi|\alpha)}{\partial t} \right|_{\text{imm}} = \nu [\rho(\phi) - P_t(\phi|\alpha)], \quad (18)$$

where  $\nu$  is the immigration rate and  $\rho(\phi)$  is the distribution of intention (/opinion) of the incoming population. If  $\nu$  is small enough, convergence towards a local, single peak equilibrium distribution will still hold. Although the local average  $\varphi_\alpha$  evolves towards the average intention of the new population, cultural cohesion is maintained. This is similar to the observation by Grauwlin and Jensen in [26], where they show that for small enough population turnover, ‘structures may last longer than individual entities’. This is precisely saying that there exists a slowly evolving local quantity around which individual attributes (propensity, intention, opinion, etc) fluctuate in a finite range. For larger values of  $\nu$ , on the other hand, one may expect a phase transition akin to what happens in the original Deffuant–Weisbuch model, where a multimodal distribution of intentions or opinions sets in. In this case, a description in terms of a unique field is clearly not warranted.

Another effect is that people can change places, adding a true transport term in the above Boltzmann equation, of the form

$$\left. \frac{\partial P_t(\phi|\alpha)}{\partial t} \right|_{\text{tr}} = - \sum_{\beta} W_{\alpha \rightarrow \beta} P_t(\phi|\alpha) + \sum_{\beta} W_{\beta \rightarrow \alpha} P_t(\phi|\beta), \quad (19)$$

where  $W_{\alpha \rightarrow \beta}$  is the probability per unit time that an inhabitant of city  $\alpha$  moves to city  $\beta$ . This extra term may lead to two different effects. An obvious one is to increase the effective diffusion constant in equation (17) above, or possibly even change the diffusion structure by allowing long-range, Lévy-like jumps (for example from Paris to Montpellier, which has become a popular move recently)<sup>10</sup>. However, we believe, on the basis of the numerical results reported below, that this effect is much smaller than the diffusion induced by the contact process investigated above, at least in European countries where internal mobility is still quite small. Indeed, whereas contact interaction takes place on the scale of weeks, changing places rather happens on the scale of several years. Still, this could play an stronger role in the USA for example.

The other effect of the extra term (19) is similar to the immigration process above. If the moving process is sufficiently frequent, mixing between different populations may be strong enough to impede the convergence towards any local equilibrium. In this case, the whole construction above, based on the emergence of a local field, should break down. This, on the other hand, is not unexpected when mixing becomes so strong that the notion of locality becomes moot. We will investigate this case numerically in section 5.

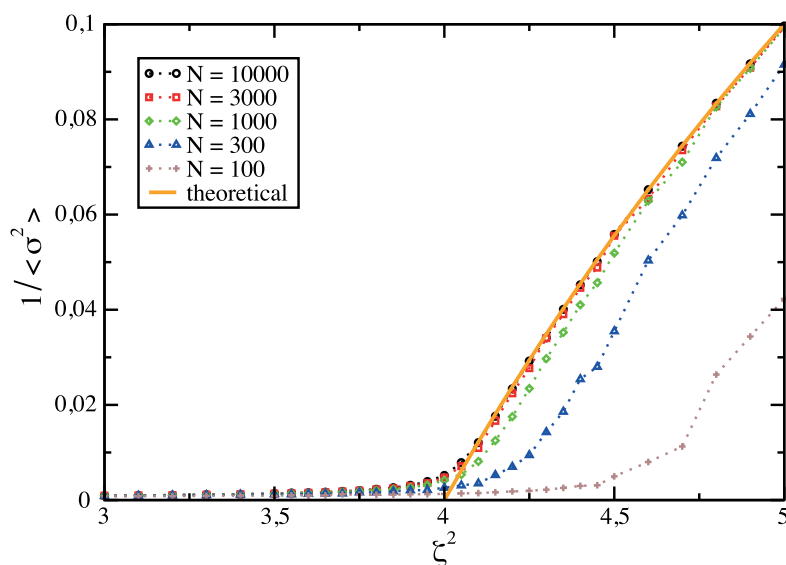
## 5. Numerical simulations

We now present numerical simulations of our model, with three aims in mind. First, we want to confirm the existence of the second order condensation transition that we obtained analytically above, and study the equilibration dynamics. Second, we want to investigate the spatial pattern generated by our model when simulated with the real positions of French towns, and a short-range interaction kernel  $f(r)$ , and demonstrate that the logarithmic behavior of the correlation function indeed appears. Finally, we investigate the role of moving places, and show that beyond a certain frequency of long-range moves, the whole correlation pattern vanishes.

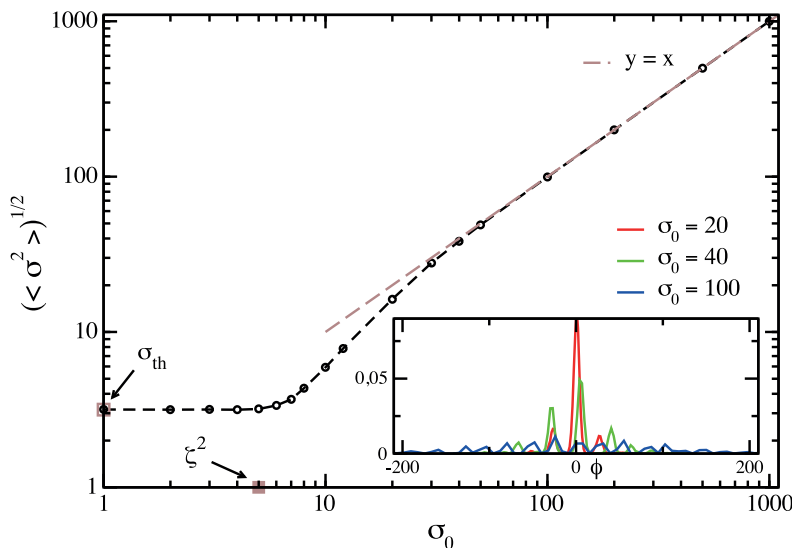
### 5.1. Single city dynamics

We first consider the case where all  $N$  agents live in the same city. For each iteration, a pair of individuals  $i, j$  is chosen at random and they change their opinions/intentions  $\phi_i$  and  $\phi_j$  according to equation (3) with the probability given by equation (2), with  $f(r=0) = 1$ . We choose  $\gamma = 0.5$  and fix the idiosyncratic noise variance  $\Sigma^2 = 1$ . We study the system as a function of the width of the interaction kernel  $\zeta$ , see equation (8). Note that the critical value for  $\gamma = 0.5$  and  $\Sigma^2 = 1$  is  $\zeta_c^2 = 4$ .

<sup>10</sup> Here we discard the possibility that the destination of the move might be correlated with the initial opinion of the individual. However, much as the  $G$  function decays as the opinion ‘distance’ increases, it might well be that the transition rate  $W_{\alpha \rightarrow \beta}$  depends on  $\phi - \phi_\beta$ .



**Figure 1.** The homogeneous model, corresponding to single city dynamics. We plot  $\langle \sigma^2 \rangle^{-1}$  as a function of  $\zeta^2$  for different system sizes  $N$ . The theoretical value given by equation (14), and corresponding to  $N = \infty$ , is also plotted. The agreement is excellent, once finite size effects are accounted for.



**Figure 2.** The role of the initial conditions. We now plot  $\langle \sigma^2 \rangle^{1/2}$  as a function of the initial width of the distribution  $\sigma_0$ , measured after a time  $t = 40\,000$ . For  $\sigma_0 < \sim \zeta$ , the result agrees with the equilibrium theoretical prediction, equation (14), as expected. However, for  $\sigma_0 \gg \zeta$ , the distribution fragments into sub-peaks (as shown in the inset), but the overall width of the distribution hardly shrinks with time. The time needed for convergence becomes exceedingly long in this case, and equilibrium may lose its relevance.

From the calculations above, we predict a transition towards a condensed phase where the variance  $\sigma^2$  of the opinion/intention becomes finite when  $\zeta^2 > \zeta_c^2 = 4$ . Figure 1 shows  $\langle \sigma^2 \rangle^{-1}$  as a function of  $\zeta^2$  for different system sizes  $N$ , and with initial condition  $\sigma_0^2 = 1$ . The average of  $\sigma^2$  is made over 50 realizations long after the transients. These curves are compared with the theoretical prediction, equation (14), with very good agreement when  $N$  is sufficiently large. In particular, our numerical results are in full agreement with the existence of a second order transition for  $\zeta_c^2 = 4$ , beyond which a socially cohesive situation sets in.

When the initial condition becomes broader, however, we observe the fragmentation of the population discussed above and first reported in [20]. The width of the distribution  $P(\phi)$  obtained after a long, but finite, time  $t = 40\,000$  is shown in figure 2.

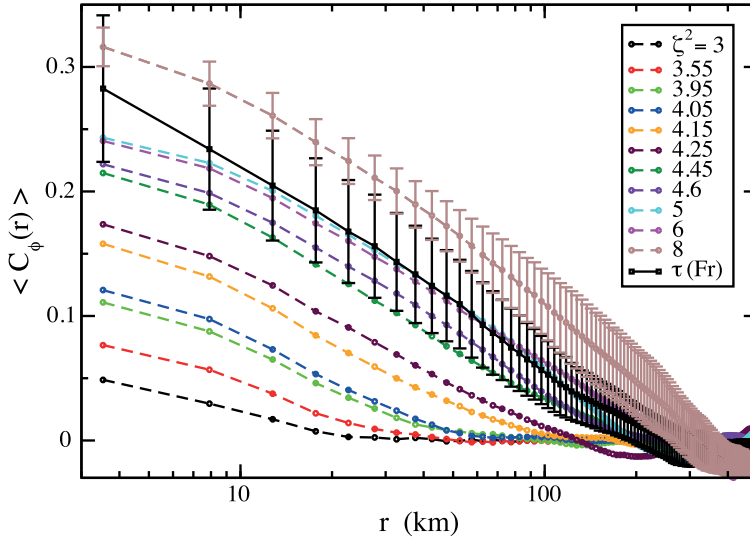
## 5.2. Spatial correlations

Spatial heterogeneities are taken into account via the real positions of the *mairies* (town-halls) of the 36 000 French mainland municipalities. We consider that all agents living in a given municipality have exactly the same spatial position: that of the *mairie* [1, 2]. Moreover, in order to test spatial effects we assume that each municipality has the same number of agents,  $N = 1000$ . This is quite small in view of figure 1, but this already corresponds to a simulation with 36 million agents, and we found little difference with the case  $N = 100$ . However, when computing the total variance of the  $\phi$ s over all cities, we already find a result very close to the theoretical prediction, equation (14). This comes from the fact that agents of nearby cities interact, leading to a large effective value of  $N$ . In any case, our main focus here is to demonstrate the emergence of a logarithmic dependence of the spatial correlation function.

We choose the spatial interaction kernel  $f(|R_\alpha - R_\beta|)$  as an exponential,  $f(r) = \exp(-r/\xi)$ . The characteristic length  $\xi$  is chosen to be the same as in [1, 2]:  $\xi = 4.5$  km. Figure 3 shows, for different values of  $\zeta^2$ , the average spatial correlation, defined exactly as in [1, 2] for the empirical data:

$$C_\phi(r) = \frac{\langle (\phi_\alpha - m)(\phi_\beta - m) \rangle_{|r_{\alpha\beta}=r}}{\langle (\phi_\alpha - m)^2 \rangle}, \quad (20)$$

where the average is taken over all cities  $\alpha, \beta$  at a certain distance  $r$  (within a certain  $dr = \pm 2.5$  km) and  $m = \langle \phi_\alpha \rangle$  is the (national) average value of  $\phi$ . The correlation function is furthermore averaged over 50 realizations, after equilibration is reached. We also show the average spatial correlation of the actual logarithmic turnout rate, for the same 36 000 municipalities over the 20 French national elections studied in [2]. As expected, the overall level of  $C_\phi(r)$  increases with  $\zeta^2$ . More interestingly, we see a clear logarithmic dependence of  $C_\phi(r)$  as a function of  $r$ , which extrapolates to zero for  $r \sim 300$  km, exactly as for the empirical results. We see that when the model is well into the cohesive phase ( $\zeta^2 \approx 5-6$ ), the value and shape of  $C_\phi(r)$  are well reproduced, without any extra fitting parameter. The main message is that our spatial extension of the Deffuant–Weisbuch model is indeed able to account for the logarithmic decay of spatial correlations. This is in line with the conclusions of [15] as well.



**Figure 3.** The average spatial correlation  $\langle C_\phi(r) \rangle$  of  $\phi$  as a function of  $r$  for different values of  $\zeta^2$ , with  $N = 1000$  agents in each city. We show for comparison the average spatial correlation of the empirical logarithmic turnout rate, for the same 36 000 municipalities over 20 French national elections. The logarithmic behavior of the empirical correlation function is very well accounted for with our agent-based model in its socially cohesive phase:  $\zeta^2 \approx 5-6$ .

### 5.3. The influence of changing places

Up to now, we have considered exchange of opinion or intention between agents living relatively close by, since we assume that the interaction kernel  $f(r)$  decays on the scale of a few kilometers. This is probably reasonable for most everyday life encounters. Still, this model neglects vacations or business travel, where people can meet with family, friends or colleagues who live very far apart. This could be modeled by adding a slowly decaying tail to  $f(r)$ . Another effect is that people occasionally change places; these moves can be long-ranged and are permanent. Intuitively, it is clear that the effect of these moves is to strongly mix opinions/intentions; this could destroy the logarithmically correlated spatial patterns discussed above.

In order to investigate this effect, we have added the following ingredient to our numerical simulation. At each time step, each agent is given a certain probability  $\pi$  to move. If he/she decides to move, the new town is chosen randomly among the 36 000 municipalities. This means that most moves in this model are long-ranged, since the average distance of a move is of the order of  $L$ , the size of the country. The move is in fact a swap, so that the number of inhabitants per city is fixed throughout the simulation. The correlation function  $C_\phi(r)$  is then measured for different values of  $\pi$ . We find that whenever  $\pi < \pi_c$ , the correlation remains approximately logarithmic, but with an amplitude and a range that become smaller and smaller:

$$C_\phi(r) \approx A(\pi) [\ln L^*(\pi) - \ln r], \quad r < L^*(\pi); \quad C_\phi(r) \approx 0, \quad r > L^*(\pi), \quad (21)$$

where both  $L^*(\pi)$  and  $A(\pi) \ln L^*(\pi)$  decrease when  $\pi$  increases, and appear to vanish continuously and simultaneously for  $\pi = \pi_c$ . Beyond  $\pi_c$ , the system is completely

homogenized and  $C_\phi(r > 0)$  is zero. We find, however, that the value of  $\pi_c$  is quite large: it corresponds to  $\approx 0.03$  per agent and per time step, which, as we argued, should correspond to a week in physical time. Actual rates of moving, especially long-range moves, are much smaller than this, probably once every 20 years (but a fraction of the population actually never moves, so assuming a unique  $\pi$  for all agents is not a good approximation). This would correspond to  $\pi = 10^{-3}$ , a value much smaller than the homogenization threshold  $\pi_c$ . It is therefore reasonable to neglect these moves in accounting for the spatial pattern of election turnouts.

## 6. Conclusion

In conclusion, we have argued that if an opinion/intention convergence mechanism exists and is strong enough, one should expect the emergence of a slowly evolving, local quantity around which individual attributes fluctuate in a finite range. We identify this quantity with the ‘cultural field’ proposed in [1]. The opinion condensation mechanism is well illustrated by the Deffuant–Weisbuch model; our mathematical analysis builds upon this model and allows for spatial heterogeneities. We show analytically and numerically that the resulting dynamics of the emergent field is a noisy diffusion equation that has a slow dynamics, much slower than the time needed to reach local equilibrium, as required for a hydrodynamical description in terms of collective variables. This random diffusion equation reproduces the long-ranged, logarithmic decrease of the correlation of spatial voting patterns empirically found in [1]. Interestingly enough, we find, again theoretically and numerically, that when social cohesion mechanisms are too weak (i.e. when individuals do not communicate enough and/or are not influenced by the opinion of others), cultural cohesion can break down completely, in the sense that no local equilibrium exists and the distribution of intentions/opinions becomes infinitely broad. In this case, obviously, the notion of ‘field’ becomes meaningless. Reversing the argument, though, we view the empirical results of [1, 2, 15] on voting patterns as strong quantitative evidence that the western populations studied there are in a culturally cohesive phase, at least as far as voting habits are concerned.

The ‘neighborhood effect’ on voting patterns was conjectured in political geography papers as early as 1969 [27], and substantiated by some quantitative studies that suggest ‘conversion by conversation’ [28]. A fuller understanding of voting patterns needs to account for the strong dependence of voting attitudes on people’s intrinsic characteristics such as social status or age (see for example [29]). Since social status is known to be spatially correlated as well [30], further studies are needed to disentangle the respective effects of social status and ‘conversion by conversation’ in the spatial correlations observed. Extension of these ideas to other types of behavioral patterns would also be very interesting.

## Acknowledgments

This work sprung out of a one-day meeting between physicists and sociologists at Sciences-Po, Paris, on 24 May 2013, entitled ‘Le tout est-il plus ou moins que la somme de ses parties?’. We thank Bruno Latour and Juliette Stehlé for useful comments and suggestions.

## Appendix

We establish here an equation for the dynamical evolution of the variance of the intentions  $\phi$ , valid for a Gaussian interaction kernel  $G(u)$ . We start from the Boltzmann equation, (4), and multiply both sides by  $\phi$  and integrate over  $\phi$  to get (assuming  $\langle \phi \rangle = 0$ )

$$\begin{aligned} \frac{\partial \langle \phi^2 \rangle_t}{\partial t} &= F \Sigma^2 \int \int d\psi d\psi' P_t(\psi) P_t(\psi') G(|\psi - \psi'|) + F \gamma (1 - \gamma) \\ &\quad \times \int \int d\psi d\psi' (\psi - \psi')^2 P_t(\psi) P_t(\psi') G(|\psi - \psi'|). \end{aligned} \quad (\text{A.1})$$

Going to Fourier space with the corresponding  $\hat{P}_t(k)$  and  $\hat{G}(k)$ , one has

$$\int \int d\psi d\psi' (\psi - \psi')^2 P_t(\psi) P_t(\psi') G(|\psi - \psi'|) \equiv - \int \frac{dk}{2\pi} \hat{P}_t(k) \hat{P}_t(-k) \frac{\partial^2 \hat{G}(k)}{\partial k^2}. \quad (\text{A.2})$$

Using the fact that  $G(u)$  is Gaussian allows one to obtain

$$- \int \frac{dk}{2\pi} \hat{P}_t(k) \hat{P}_t(-k) \frac{\partial^2 \hat{G}(k)}{\partial k^2} = \zeta^2 \int \frac{dk}{2\pi} \hat{P}_t(k) \hat{P}_t(-k) \hat{G}(k), \quad (\text{A.3})$$

where the second term is exactly  $\zeta^2 \Phi(t)$ , where  $\Phi(t)$  is the rate of efficient encounters per unit time:

$$\Phi(t) = \int \frac{dk}{2\pi} \hat{P}_t(k) \hat{P}_t(-k) \hat{G}(k) \equiv \int \int d\psi d\psi' P_t(\psi) P_t(\psi') G(|\psi - \psi'|). \quad (\text{A.4})$$

Therefore, finally,

$$\frac{\partial \langle \phi^2 \rangle_t}{\partial t} = F \Phi(t) [\Sigma^2 - \gamma(1 - \gamma)\zeta^2], \quad (\text{A.5})$$

which shows that whenever  $\Sigma^2 > \gamma(1 - \gamma)\zeta^2$ , the variance of  $\phi$  grows without bound and no stationary state can be reached.

## References

- [1] Borghesi C and Bouchaud J-P, *Spatial correlations in vote statistics: a diffusive field model for decision-making*, 2010 *Eur. Phys. J. B* **75** 395
- [2] Borghesi C, Raynal J-C and Bouchaud J-P, *Election turnout statistics in many countries: similarities, differences, and a diffusive field model for decision-making*, 2012 *PLoS One* **7** e36289
- [3] Anderson S P, 1997 *Basic Concepts in Condensed Matter Physics* (Reading, MA: Addison-Wesley)
- [4] Sethna J, Dahmen K and Myers C, *Crackling noise*, 2001 *Nature* **410** 242
- [5] see e.g. de Arcangelis L, Perrone-Capano C and Herrmann H J, *Self-organized criticality model for brain plasticity*, 2005 *Phys. Rev. Lett.* **96** 028107  
Tagliazucchi E and Chialvo D, *The collective brain*, 2011 *Decision Making, A Psychophysics Application of Network Science* ed P Grigolini and B J West (Singapore: World Scientific) p 57
- [6] Helbing D, *Traffic and related self-driven many-particle systems*, 2001 *Rev. Mod. Phys.* **73** 1067
- [7] Bialek W, Cavagna A, Giardina I, Mora T, Silvestri E, Viale M and Walczak A M, *Statistical mechanics for natural flocks of birds*, 2012 *Proc. Nat. Acad. Sci.* **109** 4786
- [8] Helbing D, Johansson A and Al-Abideen H Z, *Dynamics of crowd disasters: an empirical study*, 2007 *Phys. Rev. E* **75** 046109



- [9] Michard Q and Bouchaud J-P, *Theory of collective opinion shifts: from smooth trends to abrupt swings*, 2005 *Eur. Phys. J. B* **47** 151  
 Borghesi C and Bouchaud J-P, *Of songs and men: a model for multiple choice with herding*, 2007 *Qual. Quant.* **41** 557
- [10] Schelling T, 1978 *Micromotives and Macrobehaviour* (London: W W Norton)
- [11] Buchanan M, 2007 *The Social Atom* (New York: Bloomsbury Press)
- [12] Ball P, 2012 *Why Society is a Complex Matter* (Berlin: Springer)
- [13] Castellano C, Fortunato S and Loreto V, *Statistical physics of social dynamics*, 2009 *Rev. Mod. Phys.* **81** 591
- [14] Bouchaud J P, *Crises and collective socio-economic phenomena: simple models and challenges*, 2013 *J. Stat. Phys.* **151** 567
- [15] Fernandez-Gracia J, Suchecki K, Ramasco J J, San Miguel M and Eguluz V M, Is the Voter Model a model for voters? 2013 arXiv:1309.1131
- [16] Edwards S F and Wilkinson D R, *The surface statistics of a granular aggregate*, 1982 *Proc. R. Soc. A* **381** 17
- [17] Schweitzer F and Holyst J A, *Modelling collective opinion formation by means of active brownian particles*, 2000 *Eur. Phys. J. B* **15** 723  
 Schweitzer F, 2002 *Coordination of Decisions in a Spatial Model of Brownian Agents (Economics with Heterogeneous Interacting Agents (WEHIA))* (Berlin: Springer)
- [18] Bourdieu P and Wacquant L, 1992 *An Invitation to Reflexive Sociology* (Chicago, IL: University of Chicago Press and Polity)
- [19] Latour B, *Reassembling the social*, 2005 *An Introduction to Actor-Network-Theory* (Oxford: Oxford University Press)
- [20] Deffuant G, Neau D, Amblard F and Weisbuch G, *Mixing beliefs among interacting agents*, 2001 *Adv. Complex Syst.* **3** 87
- [21] Anderson S P, De Palma A and Thisse J F, 1992 *Discrete Choice Theory of Product Differentiation* (Cambridge, MA: MIT Press)
- [22] Pineda M, Toral R and Hernandez-Garcia E, *Diffusing opinions in bounded confidence processes*, 2011 *Eur. Phys. J. D* **62** 109
- [23] Pineda M, Toral R and Hernandez-Garcia E, *Noisy continuous-opinion dynamics*, 2009 *J. Stat. Mech.* P08001
- [24] Lallouache M, Chakrabarti A S, Chakraborti A and Chakrabarti B K, *Opinion formation in kinetic exchange models: Spontaneous symmetry-breaking transition*, 2010 *Phys. Rev. E* **82** 056112
- [25] see e.g. Chapman S and Cowling T G, 1971 *The Mathematical Theory of Non-uniform Gases* 3rd edn (Cambridge: Cambridge University Press)
- [26] Grauwil S and Jensen P, *Opinion groups formation and dynamics: structures that last from non lasting entities*, 2012 *Phys. Rev. E* **85** 066113
- [27] Cox K R, *The voting decision in a spatial context*, 1969 *Progress in Geography* vol 1, ed C Board *et al* (London: Edward Arnold)  
 Taylor P J and Johnston P J, 1979 *Geography of Elections* (London: Penguin)
- [28] MacAllister I, Johnston R J, Pattie C J, Tunstall H, Dorling D F L and Rossiter D J, *Class dealignment and the neighbourhood effect: miller revisited*, 2001 *Br. J. Political Sci.* **31** 41
- [29] Andersen R and Heath A, *Social class and voting: a multi-level analysis of individual and constituency differences*, Centre for research into elections and social trends, 2000 *Working Paper* 83
- [30] Jargowsky P A, *Take the money and run: economic segregation in US metropolitan areas*, 1996 *Am. Soc. Rev.* **61** 984