Erratum to "Elimination of Constants from Machines over Algebraically Closed Fields"

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This erratum corrects mistakes in the statement and the proof of Lemma 2. The rest of the paper is not affected.

Lemma 2 *Let $P \in \mathbb{Z}[X_1, \ldots, X_p]$ be a degree d polynomial with coefficients bounded by M in absolute value. Let $\alpha = (\alpha_1, \ldots, \alpha_p)$ be any sequence of integers satisfying $\alpha_1 \geq M+1$ and $\alpha_k \geq 1 + M(d+1)^{k-1}\alpha_{k-1}^d$ for $k \geq 2$. Then if P is not identically zero, $P(\alpha) \neq 0$.*

Proof. For $p = 1$, this follows from the standard bound on polynomial roots. Assume by induction that the result holds for $p-1$, and write $P(x) = \sum_{j=0}^{d} P_j(x_1, \ldots, x_{p-1}) x_p^j$. If $P \not\equiv 0$, one of the P_j's must be non-zero. By induction hypothesis, such a P_j takes a non-zero value at $(\alpha_1, \ldots, \alpha_{p-1})$. consider the polynomial $Q(x) = \sum_{j=0}^{d} P_j(\alpha_1, \ldots, \alpha_{p-1}) x_p^j$. Since P_j is of degree at most d, it cannot have more than $(d+1)^{p-1}$ monomials. Therefore the coefficients of Q are bounded by $M(d+1)^{p-1}\alpha_{k-1}^d$. The result now follows from the case $p = 1$. $\square$