Erratum to *Hilbert's Nullstellensatz is in the Polynomial Hierarchy*

Pascal Koiran
LIP, Ecole Normale Supérieure de Lyon – CNRS
46 allée d'Italie, 69364 Lyon Cedex 07
France
email: koiran@lip.ens-lyon.fr

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This erratum correct two mistakes (pointed out by Teresa Krick) which appear both in the journal paper and the DIMACS report [1].

- In the proof of Theorem 10, "The prime factors of π_n" should be replaced by "The prime factors of $\pi_n^{\pi_n} - 1$". The rest of the paper is not affected.
- In Theorem 8, the correct lower bound for $\pi_S(x)$ is:

  $$\frac{\pi(x) - 2^{(n\log\sigma)^{c_6}} x^{1/2} \log x}{L.2^{(n\log\sigma)^{c_4}}} - L.2^{(n\log\sigma)^{c_5}} x^{1/2}.$$

  This expression should also be substituted to the old lower bound in the proof of Theorem 1. The statement of that theorem and the rest of the paper are not affected.

Further Remarks

The paper relies in particular on the existence of good bounds on the complexity of algebraic solutions to systems of algebraic equations, and on the complexity of primitive elements. In [2] one can find better bounds for these problems than in [1]. However, these bounds are less general than those in [1] since they apply only to systems with isolated solutions. In the following we briefly sketch how these bounds can be adapted to arbitrary systems.

Let S be a satisfiable system and $V \subseteq \mathbb{C}^n$ the set of solutions. Let d be the dimension of this algebraic variety. It is well known that if H is a "generic" affine subspace of dimension $n - d$, $V \cap H$ is finite and nonempty. In fact, one can show there exists an H with this property which is defined by a system of linear equations with rational coefficients of polynomial size. Then one can apply the bounds of [2] to $V \cap H$.

References

[1] P. Koiran. Hilbert's Nullstellensatz is in the polynomial hierarchy. *Journal of Complexity*, 12(4):273–286, 1996. Long version: DIMACS report 96-27 (http://lip.ens-lyon.fr/~koiran).

[2] T. Krick and L. M. Pardo. A computational method for Diophantine approximation. In L. Gonzalez-Vega and T Recio, editors, *Algorithms in Algebraic Geometry and Applications (Proc. MEGA'94)*, Progress in Mathematics 143. Birkhauser, 1996. Preliminary version: Une Approche Informatique pour l'Approximation Diophantienne. *Compte-Rendus de l'Académie des Sciences*, Série I, 318(5):407-412, 1994.