

# Bootstrap for Multifractal Analysis

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Complementary Results - ICASSP 2006

## 1 Bias and EQM

$N_{MC} = 3000$ ,  $H = 0.8$ ,  $c_1 = 0.75$ ,  $c_2 = 0.08$ ;  $P = 200$

### 1.1 $2^9$ Samples

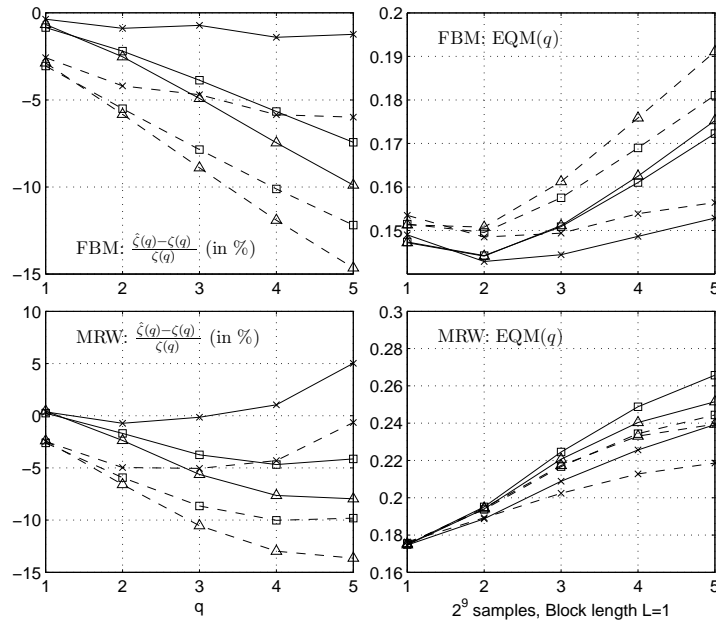


Figure 1: Relative bias (left column) and EQM (right column) of estimators  $\hat{\zeta}(q)$  for fBM (top row) and MRW (bottom row) for  $q \in \{1, 2, 3, 4, 5\}$ . Solid lines correspond to  $\hat{\zeta}_X(q)$ , dashed lines to  $\hat{\zeta}_X^B(q)$ , the symbols ( $\times$ ,  $\square$ ,  $\triangle$ ) to  $X = (G, B, A)$ .  $2^9$  samples, blocklength  $L = 1$ .

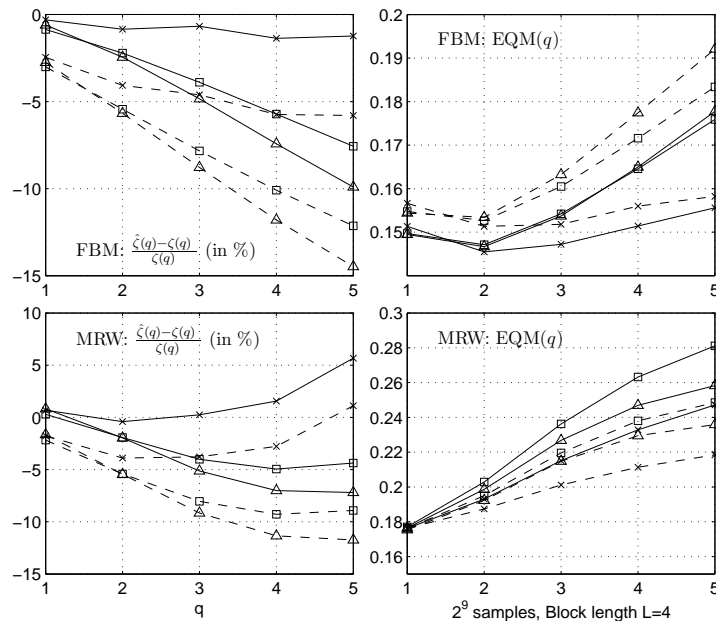


Figure 2: Relative bias (left column) and EQM (right column) of estimators  $\hat{\zeta}(q)$  for fBM (top row) and MRW (bottom row) for  $q \in \{1, 2, 3, 4, 5\}$ . Solid lines correspond to  $\hat{\zeta}_X(q)$ , dashed lines to  $\hat{\zeta}_X^B(q)$ , the symbols ( $\times$ ,  $\square$ ,  $\triangle$ ) to  $X = (G, B, A)$ .  $2^9$  samples, blocklength  $L = 4$ .

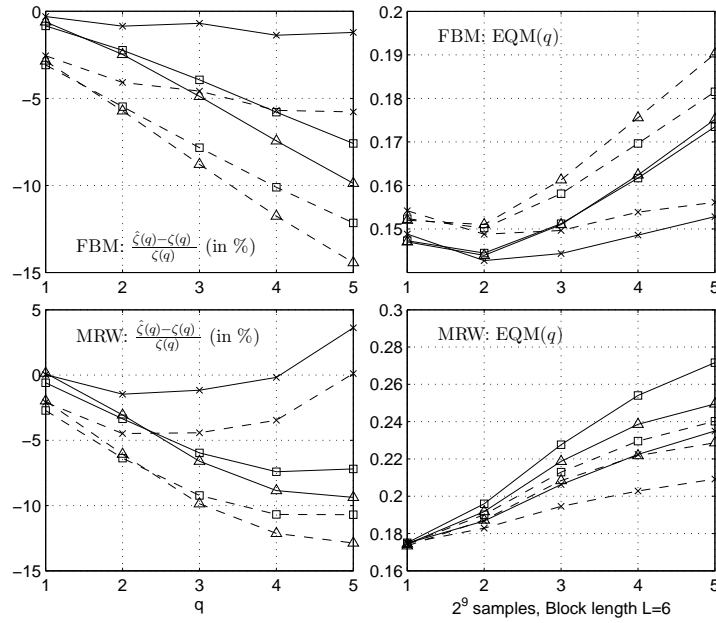


Figure 3: Relative bias (left column) and EQM (right column) of estimators  $\hat{\zeta}(q)$  for fBM (top row) and MRW (bottom row) for  $q \in \{1, 2, 3, 4, 5\}$ . Solid lines correspond to  $\hat{\zeta}_X(q)$ , dashed lines to  $\hat{\zeta}_X^B(q)$ , the symbols  $(\times, \square, \triangle)$  to  $X = (G, B, A)$ .  $2^9$  samples, blocklength  $L = 6$ .

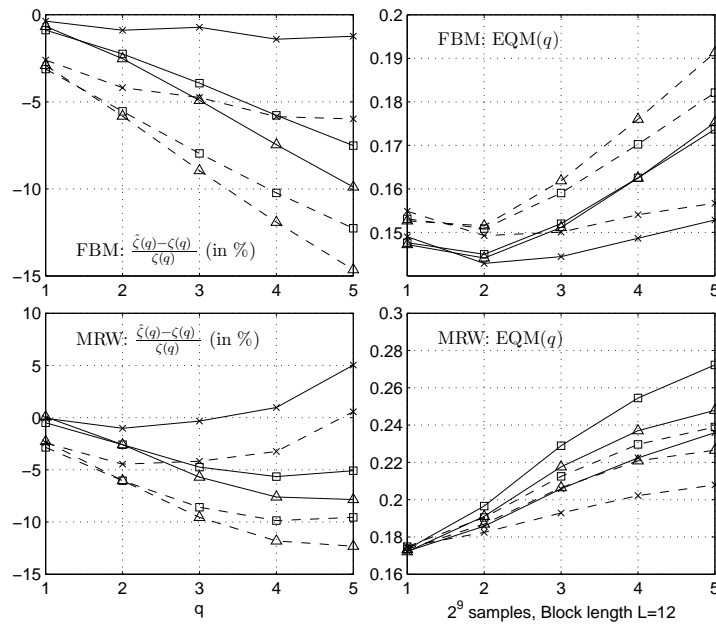


Figure 4: Relative bias (left column) and EQM (right column) of estimators  $\hat{\zeta}(q)$  for fBM (top row) and MRW (bottom row) for  $q \in \{1, 2, 3, 4, 5\}$ . Solid lines correspond to  $\hat{\zeta}_X(q)$ , dashed lines to  $\hat{\zeta}_X^B(q)$ , the symbols  $(\times, \square, \triangle)$  to  $X = (G, B, A)$ .  $2^9$  samples, blocklength  $L = 12$ .

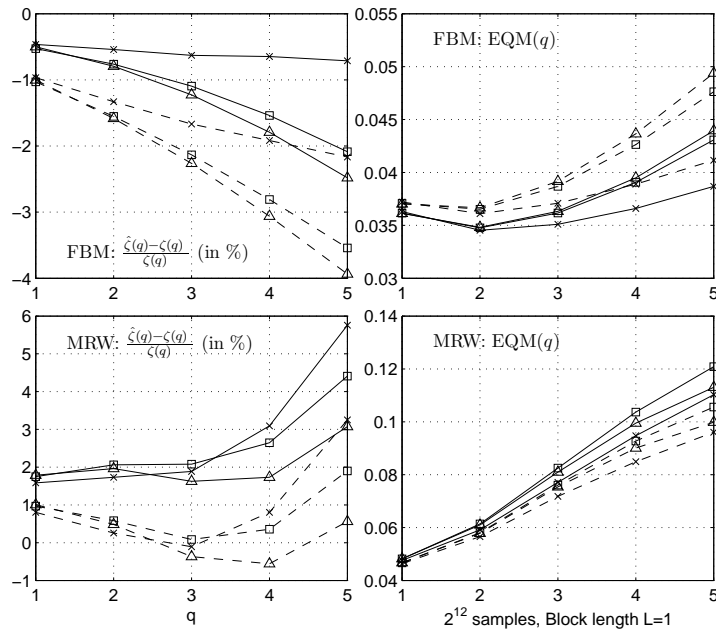
1.2  $2^{12}$  Samples

Figure 5: Relative bias (left column) and EQM (right column) of estimators  $\hat{\zeta}(q)$  for fBM (top row) and MRW (bottom row) for  $q \in \{1, 2, 3, 4, 5\}$ . Solid lines correspond to  $\hat{\zeta}_X(q)$ , dashed lines to  $\hat{\zeta}_X^B(q)$ , the symbols  $(\times, \square, \triangle)$  to  $X = (G, B, A)$ .  $2^{12}$  samples, blocklength  $L = 1$ .

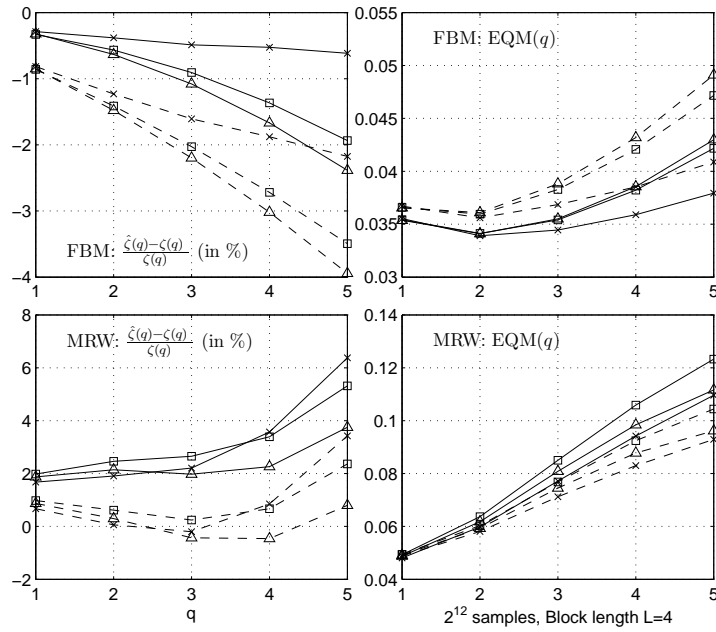


Figure 6: Relative bias (left column) and EQM (right column) of estimators  $\hat{\zeta}(q)$  for fBM (top row) and MRW (bottom row) for  $q \in \{1, 2, 3, 4, 5\}$ . Solid lines correspond to  $\hat{\zeta}_X(q)$ , dashed lines to  $\hat{\zeta}_X^B(q)$ , the symbols  $(\times, \square, \triangle)$  to  $X = (G, B, A)$ .  $2^{12}$  samples, blocklength  $L = 4$ .

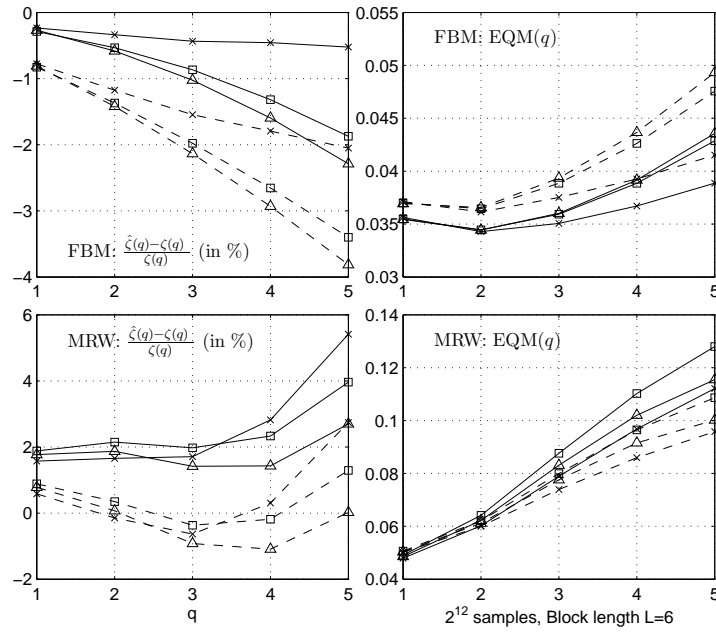


Figure 7: Relative bias (left column) and EQM (right column) of estimators  $\hat{\zeta}(q)$  for fBM (top row) and MRW (bottom row) for  $q \in \{1, 2, 3, 4, 5\}$ . Solid lines correspond to  $\hat{\zeta}_X(q)$ , dashed lines to  $\hat{\zeta}_X^B(q)$ , the symbols  $(\times, \square, \triangle)$  to  $X = (G, B, A)$ .  $2^{12}$  samples, blocklength  $L = 6$ .

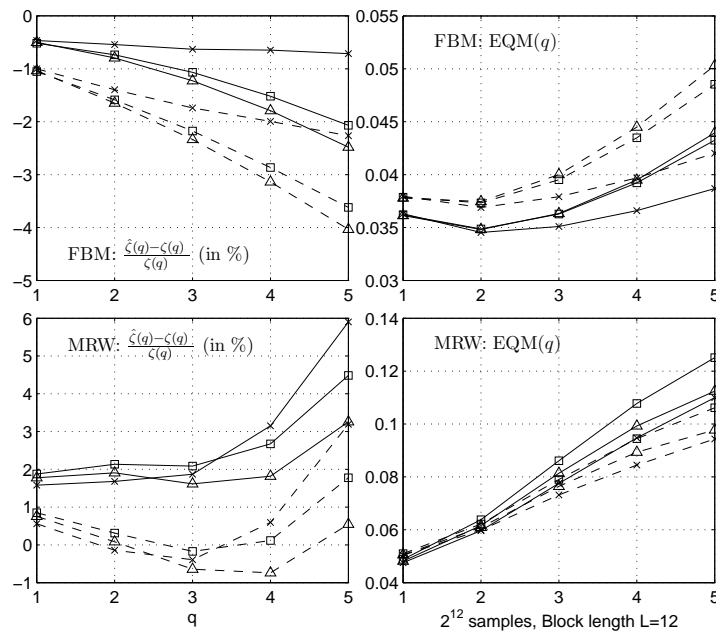


Figure 8: Relative bias (left column) and EQM (right column) of estimators  $\hat{\zeta}(q)$  for fBM (top row) and MRW (bottom row) for  $q \in \{1, 2, 3, 4, 5\}$ . Solid lines correspond to  $\hat{\zeta}_X(q)$ , dashed lines to  $\hat{\zeta}_X^B(q)$ , the symbols  $(\times, \square, \triangle)$  to  $X = (G, B, A)$ .  $2^{12}$  samples, blocklength  $L = 12$ .

1.3  $2^{15}$  Samples

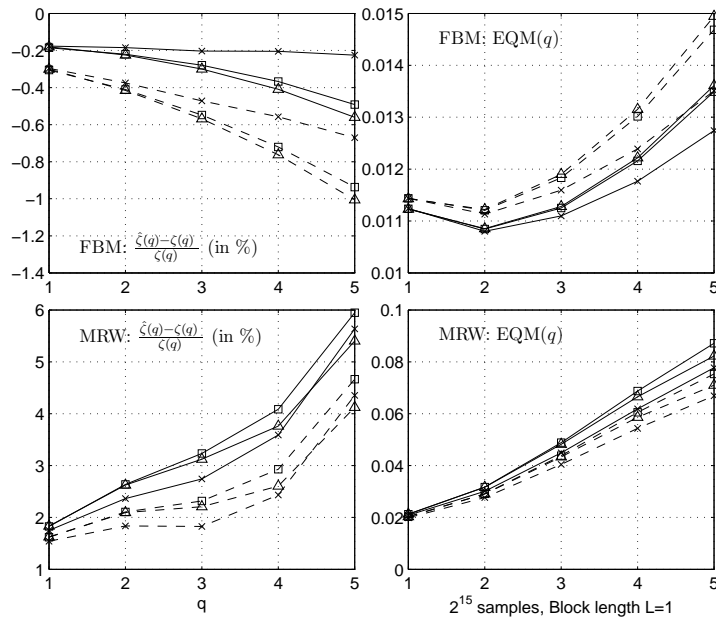


Figure 9: Relative bias (left column) and EQM (right column) of estimators  $\hat{\zeta}(q)$  for fBM (top row) and MRW (bottom row) for  $q \in \{1, 2, 3, 4, 5\}$ . Solid lines correspond to  $\hat{\zeta}_X(q)$ , dashed lines to  $\hat{\zeta}_X^B(q)$ , the symbols ( $\times, \square, \triangle$ ) to  $X = (G, B, A)$ .  $2^{15}$  samples, blocklength  $L = 1$ .

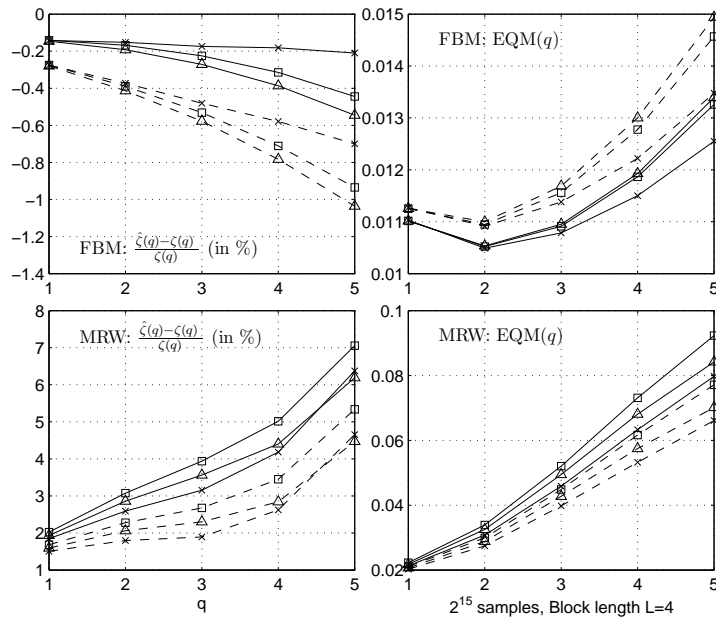


Figure 10: Relative bias (left column) and EQM (right column) of estimators  $\hat{\zeta}(q)$  for fBM (top row) and MRW (bottom row) for  $q \in \{1, 2, 3, 4, 5\}$ . Solid lines correspond to  $\hat{\zeta}_X(q)$ , dashed lines to  $\hat{\zeta}_X^B(q)$ , the symbols ( $\times, \square, \triangle$ ) to  $X = (G, B, A)$ .  $2^{15}$  samples, blocklength  $L = 4$ .

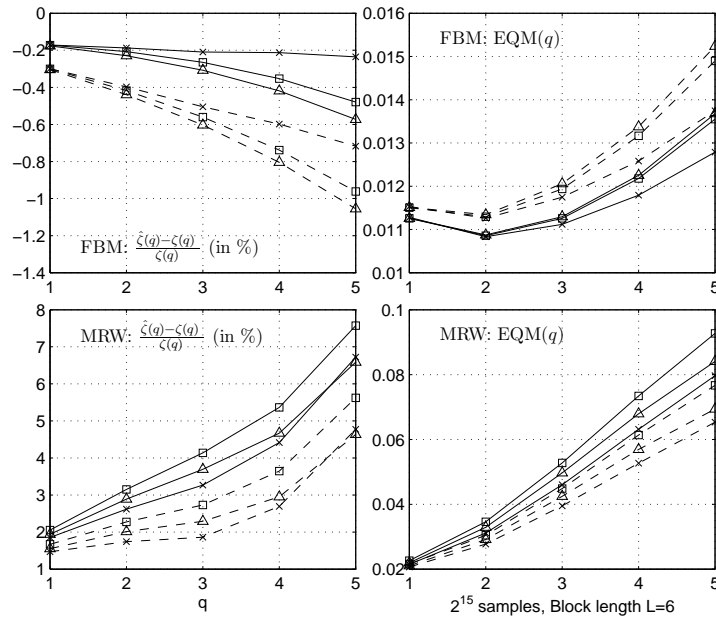


Figure 11: Relative bias (left column) and EQM (right column) of estimators  $\hat{\zeta}(q)$  for fBM (top row) and MRW (bottom row) for  $q \in \{1, 2, 3, 4, 5\}$ . Solid lines correspond to  $\hat{\zeta}_X(q)$ , dashed lines to  $\hat{\zeta}_X^B(q)$ , the symbols ( $\times, \square, \triangle$ ) to  $X = (G, B, A)$ .  $2^{15}$  samples, blocklength  $L = 6$ .

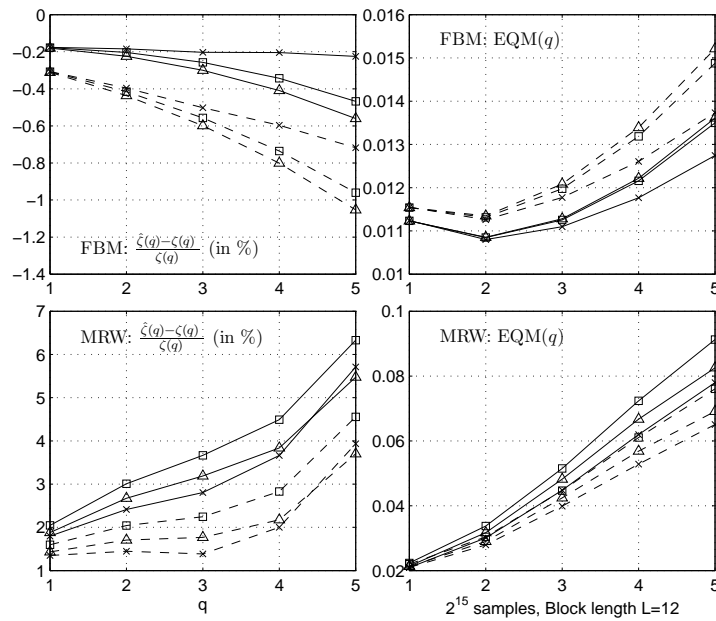


Figure 12: Relative bias (left column) and EQM (right column) of estimators  $\hat{\zeta}(q)$  for fBM (top row) and MRW (bottom row) for  $q \in \{1, 2, 3, 4, 5\}$ . Solid lines correspond to  $\hat{\zeta}_X(q)$ , dashed lines to  $\hat{\zeta}_X^B(q)$ , the symbols ( $\times, \square, \triangle$ ) to  $X = (G, B, A)$ .  $2^{15}$  samples, blocklength  $L = 12$ .

## 2 Empirical Coverage: FBM vs. MRW (varying block length)

$N_{MC} = 3000$ ,  $H = 0.8$ ,  $c_1 = 0.75$ ,  $c_2 = 0.08$ .

### 2.1 $2^9$ Samples

q	X	$\hat{\zeta}_X(q)$			$\hat{\zeta}_X^B(q)$			
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	87.4	91.5667	89.3333	89.5667	90.5333	88.7667	88.7
	B	87.7333	91.8333	89.6333	89.7333	91.1333	88.8333	88.9667
	A	87.7667	91.8667	89.6667	89.8	91.1667	88.8	88.9667
2	G	84.1333	91.4	87.4	85.2	90.0333	85.8667	83.3
	B	83.3	90.8667	86.9333	84.7333	89.6667	85.6333	83.0667
	A	83	90.7667	87.0667	84.6333	89.4	85.5	82.7333
3	G	82.8333	91.6	86.2333	81.3667	90.3333	85.3333	79.1
	B	79.5	90.6333	84.6667	79.0333	88.7667	82.4667	76.2667
	A	78.5	90.4	84.8	78.5333	87.9667	81.7333	75.0333
4	G	81.2333	92.0667	85.4667	76.4667	90.7333	84	74.2667
	B	75.7	89.4333	81.8333	72	87.7	79.1333	68.7
	A	73.8667	89.3667	81.2333	71.4667	86.1667	77.9667	66.9
5	G	81.9667	92.7333	84.6667	71.8667	91.8	83.5667	70.5
	B	73.3	88.7667	79.5	65.3	86.7	76.7667	62.6667
	A	69.8667	88.0333	78.4333	64.5333	84.7667	74.5333	59.2333
1	G	89.1	86.6	89.5	90.0667	86	89.2667	89.6667
	B	89.2333	86.5333	89.5667	89.9333	85.9	89.2333	89.4
	A	89.2667	86.6	89.4667	89.9667	86.0333	89.1333	89.4667
2	G	86.3	81.7333	87.4333	84.1667	81.9333	87.2333	84.1333
	B	85.3667	80.2667	86.7	83.3333	81.0333	86.6	83.2333
	A	84.9	80.6667	86.7333	83.6667	80.9	86.4667	83.1667
3	G	86.0333	78.2667	84.8667	77.4333	80	85.8	78.4333
	B	84.0333	75.7333	83.2	74.6333	77.4333	84.1	76.2
	A	83.4667	76.7667	83.5333	75.2333	77.2667	84	76.2
4	G	86.4333	76.5	83.3333	70.1333	80.1	85.4667	73.2667
	B	83.2667	72.7333	80.2	66.0667	76.2	81.9	69.1
	A	82.5333	74.5333	81.3333	67.5667	76.8	82.3667	69.5667
5	G	86.7	75.3667	81.9333	64.0333	79.6	84.7667	67.3
	B	83.5667	71.3	79.1333	58.7333	76.2	81.4	61.5667
	A	83.4333	73.9	80.9667	60.6667	77.1333	82.5	62.9667

Table 1: Empirical coverage (in %) of nominal 95% confidence intervals for fBM (top part) and MRW (bottom part):  $P = 200$ ,  $L = 1$ ;  $2^9$  samples

q	X	$\hat{\zeta}_X(q)$				$\hat{\zeta}_X^B(q)$		
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	86.7667	90.8	88.6	88.6667	89.7333	88	87.5
	B	87.4	90.8667	89.2333	88.8667	89.9	88.4667	87.8667
	A	87.5	91.0333	89.3667	88.8667	90.0333	88.5	87.9667
2	G	84.7	90.1667	87.6	84.1667	89.2	86.5333	83
	B	84.4	90	87.4667	83.6	89.1333	86.0667	82.6667
	A	84.3333	89.9	87.5667	83.3667	88.7667	85.7	82.5667
3	G	83.2333	90.6667	86.4667	79.6	90.2333	85.1333	78.7667
	B	80.8333	89.8333	84.7333	77.2667	88.3667	83.0667	75.8333
	A	80	90.1667	84.7	77.5	87.9	82.8	74.9667
4	G	82.6	91.8333	85.0667	74.7333	90.9	84.4333	73.7667
	B	77.5667	89.2	82.5	70.8667	87.9667	80.8	68.7667
	A	76.1333	89.4667	82.1333	70.8	86.3667	79.2	67.7
5	G	82	91.9333	85.1667	70.2	92.1	84.4333	70.2333
	B	74.6	88.2	79.9333	65	86.9667	76.8667	63
	A	71.8	87.9333	78.6333	64.5667	84.7333	75.5	60.2667
1	G	92.6667	86.2667	92.8333	90.8	86.3333	92.9333	90.5667
	B	92.8	86.2667	92.9667	90.7333	85.9333	92.9667	90.4333
	A	93.0333	86.4667	93.0333	90.7333	86.2333	93.1333	90.6
2	G	91.3	81.5667	90.9667	84.6	81.6	91.7	85.4333
	B	90.7667	78.8667	90.3667	83.4333	79.9333	91.1	84.1667
	A	90.8333	79.8	90.6333	84.0333	80.3	91.2667	84.6
3	G	91.1	77.5	89.3667	76.9667	79.8333	90.8667	80.1333
	B	89.3	73.6333	86.2	72.9667	76.1	89.0667	75.9333
	A	89.3	75.2	87.5333	75.0667	77.3	89.1667	77.4333
4	G	90.6	75.7	87.1667	68.8	79.5333	89.4	73.1
	B	88.1667	71.3	83.7667	64.6333	74.8333	86.7667	68.4667
	A	88.8333	73.5667	85.2	67	75.7	87.8333	69.9333
5	G	89.5333	74.1333	84.9667	60.9667	80.1667	88.2	66.6333
	B	87.7667	70.3333	82	57.2	74.5667	86.2333	61.7
	A	88.8	73.5333	84.8	60.4	76.4	86.8667	63.8667

Table 2: Empirical coverage (in %) of nominal 95% confidence intervals for fBM (top part) and MRW (bottom part):  $P = 200$ ,  $L = 4$ ;  $2^9$  samples



q	X	$\hat{\zeta}_X(q)$				$\hat{\zeta}_X^B(q)$		
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	87.7	91.6	89.6667	89.6	90.5	88.8333	88.6667
	B	87.9667	91.8	89.9	89.7667	91.1	89.2667	89.1667
	A	87.9667	91.9	90.0333	89.8667	91.2	89.2667	89.1667
2	G	85.0667	91.4	87.9	85.2	89.9	86.6333	83.4333
	B	83.9	90.6333	87.3	84.6667	89.5	86.2	83.0667
	A	83.6333	90.7667	87.5333	84.6667	89.3667	86.2333	82.8
3	G	83.9333	91.6	86.7333	81.3	90.4333	86.1	79.2667
	B	80.3333	90.4667	85.1667	78.6333	88.5	83.5	75.9333
	A	79.7333	90.4	85.3333	78.6333	87.5667	83.0333	74.7667
4	G	82.3333	92.0667	85.5667	76.4333	90.9333	84.9667	74.1333
	B	77.2333	89.1667	82.6333	71.7333	87.8	79.8333	69.2333
	A	75.3667	89.3667	82.3	71.5667	86.9333	78.6	67.3333
5	G	81.9667	92.7333	85.1	71.9	91.7333	84.6	70.1667
	B	74.1667	88.2667	79.2	65.2667	86.9333	76.5333	63.0333
	A	71.1667	88.0333	78.5667	64.6333	85.0333	74.9333	59.5
1	G	92.7333	85.9333	93.7	90.4	86.1333	93.5333	90.4333
	B	92.8667	86.1667	93.9	90.2333	86.3	93.6	90.5333
	A	92.8667	86.1	93.7667	90.5	86.5333	93.6667	90.5667
2	G	92.1	82.0667	92.2667	86.2	82.9	92.2333	86.8
	B	91.5667	80.6333	91.6	84.9	81.5	91.9	85.8667
	A	91.8333	81.1667	92.0333	85.5667	82.1	91.8333	86.2
3	G	91.6333	79.1667	90	79.8333	81.8667	90.9667	81.9667
	B	90.7667	74.9333	88.5667	75.7667	78.3	89.7667	78.6
	A	91.0333	76.5667	89.5	77.3667	78.7667	90.1667	79.3333
4	G	91.1667	77.1333	87.8333	71.5333	81.5333	90.1333	76.2
	B	89.5	72.1	85.6667	65.8667	76.5333	88.4333	71.2333
	A	89.5333	74.3333	87.4667	68.5	77.4667	89.1	72.7
5	G	90.6667	76.3667	86.2333	64.5	82.2	89.2333	70.1667
	B	89.4	70.9667	82.8667	58.8	76.2333	87.4667	63.8333
	A	89.8667	74.2667	85.6667	61.6333	77.9	87.8	66.2

Table 3: Empirical coverage (in %) of nominal 95% confidence intervals for fBM (top part) and MRW (bottom part):  $P = 200$ ,  $L = 6$ ;  $2^9$  samples

q	X	$\hat{\zeta}_X(q)$			$\hat{\zeta}_X^B(q)$			
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	87.4333	91.5667	89.5667	89.5667	90.6	88.8	88.4
	B	87.4667	91.7333	89.5667	89.6333	91.0667	88.7	88.7333
	A	87.6	91.8667	89.6333	89.8	91.1333	88.9	89.0333
2	G	84.7667	91.4	87.9333	85.2	89.9333	86.6	83.7667
	B	84.2333	90.7	87.0333	84.5667	89.4333	85.8667	82.6667
	A	84.1	90.7667	87.2667	84.6333	89.3667	86.1	82.5333
3	G	83.8667	91.6	86.8667	81.3667	90.4667	86.0333	78.8
	B	80.0667	90.2333	84.8667	78.7	87.8333	83.2333	75.8
	A	79.3	90.4	85.2	78.5333	87.4333	82.5	75.0667
4	G	82.5	92.0667	86.1667	76.4667	91.2667	85.6667	74.1333
	B	76.7333	89.1667	82.8333	71.6333	87.2667	79.7333	69.0667
	A	75.0667	89.3667	82.4333	71.4667	86.3	78.6667	66.5667
5	G	82.1	92.7333	85.1	71.8667	91.9	84.8667	69.7667
	B	73.7	88.3333	79.8667	64.9667	86.5667	77.1333	62.4333
	A	71.0333	88.0333	79.3	64.5333	84.8667	75	59.3
1	G	92.7667	86.5333	94	90.2333	86.3	93.5	90.2
	B	92.9	86.3667	94.0333	90.1667	86.4	93.6333	89.9333
	A	93.1667	86.9333	94.1667	90.3667	86.6	93.7667	90.3333
2	G	92.0333	82.1333	92.1667	84.9333	82.8333	92.7667	85.3
	B	91.4	79.9	91.2	83.4333	80.5667	91.8667	83.8333
	A	91.4333	80.9667	92	84.1333	81.3333	92.2	85.0667
3	G	92.2667	79.5	90.5	77.9333	81.6667	91.6333	80.3333
	B	90.1333	75.5667	87.9667	75.2667	77.4667	89.5333	77.9333
	A	90	77.2	88.9333	77.0667	78.3667	90.0667	78.9
4	G	91.6667	77.9	88.3	71.1	81.8	90.7333	74.4333
	B	89.5333	72.7333	85.0333	66.7333	76.5667	87.7667	70.5667
	A	90.2333	75.4667	87.0667	69.5667	77.1667	88.8333	71.8
5	G	91.0333	76.5	85.6333	63.9	81.9	89.2667	68.8
	B	89.2	71.4333	82.7667	58.6667	76.2333	86.9333	63.7667
	A	90.1	75.3	85.8333	61.6	78.0667	88.5	65.0333

Table 4: Empirical coverage (in %) of nominal 95% confidence intervals for fBM (top part) and MRW (bottom part):  $P = 200$ ,  $L = 12$ ;  $2^9$  samples

2.2  $2^{12}$  Samples

q	X	$\hat{\zeta}_X(q)$			$\hat{\zeta}_X^B(q)$			
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	86.3667	88.6333	87.7333	88.2	87.4333	87.2	87.1333
	B	86.3	88.8	88	88.1667	87.6333	87.1	87.2333
	A	86.3667	88.7667	88.0333	88.3	87.5667	87.3	87.4
2	G	83.2333	87.8333	86.6	85.8	86.2667	85	83.7667
	B	82.6667	87.5	86.0667	85.1333	86	84.3333	83.0667
	A	82.7	87.5	86	85.1333	85.8333	84.1333	82.7333
3	G	82.2	89	85.6	83.7667	86.6	83.8667	81.8
	B	79.9333	87.8333	84.9333	82.8333	84.8667	81.6333	79.7333
	A	79.4333	87.5667	84.9333	82.3667	84.2667	81.0667	79.2
4	G	80.8333	89.8667	85.1333	82.1667	87.6	83.2667	79.5
	B	76.2333	87.2333	82.4	79.3667	83.6	78.1333	74.8333
	A	74.9667	86.7	82.0333	78.3333	82.8	76.9667	73.6667
5	G	80.1333	90.9	85.0667	80.5667	88.5667	82.6333	77.8333
	B	72.5	86.8333	79.7333	74.4667	83.4	74.9333	70.0333
	A	70	86.2333	78.6	73.8	81.7333	73.1333	67.5333
1	G	90.7667	77.9667	90.2667	90.3667	79.4	91.3333	91.3333
	B	90.8667	77.3	90.1	89.7	79.1667	91.2333	91.4
	A	90.8	77.1333	89.9667	89.6667	79.2	91.1333	91.3333
2	G	89.3667	65.0333	88.0333	87.3	66.7667	89.4333	88.9667
	B	88.6667	63.5333	87.2333	86.1	65.9667	88.6	88.3
	A	88.8667	63.8333	87.5	86.3333	65.9333	88.7333	88.1333
3	G	88.3333	55.2	85.3333	82.3333	58.4	87.7667	85.6333
	B	87.1333	52.5333	83.2	80	56.7667	86.4	84
	A	87.5	53.5333	84.0333	80.8333	57.4333	86.7667	84.4333
4	G	86.7	48.5333	80.7333	73.7	53.7333	85.5	79.7333
	B	84.9333	46.0667	78.2	71	50.9	83.6	77.1333
	A	85.6667	47.4333	80.2	73.0667	52.6333	84.5333	78.2
5	G	82.7333	44.1	75.4	64.8333	51.1333	81.1333	71.1333
	B	81.2667	42.4667	72.9667	62.1	48.4667	79.7333	68.6333
	A	83.6667	44.8	76.0333	65.3333	50.6667	82	71.5

Table 5: Empirical coverage (in %) of nominal 95% confidence intervals for fBM (top part) and MRW (bottom part):  $P = 200$ ,  $L = 1$ ;  $2^{12}$  samples

q	X	$\hat{\zeta}_X(q)$				$\hat{\zeta}_X^B(q)$		
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	88.5333	89.4667	90.8	88.8667	88.3667	89.3667	87.9
	B	88.5333	89.5333	90.6667	88.9333	88.4	89.4333	88.2
	A	88.6667	89.5333	90.7	89.0333	88.4	89.7	88.1333
2	G	86.2667	88.6667	89.4	86.3667	86.4333	87.2667	84.6667
	B	86.0667	88.4667	89.3667	86.5333	86.2	87.1	83.9
	A	85.8	88.4	89.3667	86.3	86.2	86.7	83.7667
3	G	85.0667	89.7	89.1667	85.0333	86.9333	86.3	81.8
	B	83.0667	88.6667	87.5	83.5333	85.7333	84.8	80.4333
	A	82.3667	88.3667	87.2333	83.5	85.1	84.3667	79.7
4	G	84.1	90.8	88.6667	83.4	87.8667	85.7333	80.5333
	B	79.4	88.2	86.1333	80.8667	84.5	81.6	76.2333
	A	78.3	87.5667	85.7	80.4667	83.6	80.8333	74.7333
5	G	82.8667	91.4667	88.1667	81.3667	89.1	85.1	78.0667
	B	75.6333	88.3	84.1333	76.7333	83.3667	78.1	70.6333
	A	73.0667	87.5333	82.8667	76.0333	81.7667	76.2	68.6333
1	G	94.8667	77.0667	95.5333	89.5667	76.5	95.3	89.2
	B	94.7333	75.7	95.0667	89.0333	76.2333	95.0333	88.7333
	A	94.8	76.0667	95.2333	89.2333	76.4333	95.1	89
2	G	94.4333	64.6667	93.7333	86.4	66.2	94.4	87.9667
	B	93.7	61.8667	92.3667	84.6333	64.2333	94	86.5
	A	94.1333	63.2667	93.0667	85.5	65.4	94.2333	87.2
3	G	93.8	54.7333	90.5333	81.4	58.5	93.6667	84.9
	B	92.1	51.2667	88.6	77.9333	55.1	91.7	82.4667
	A	93.1333	53.5	89.6	79.6	56.7667	92.7667	83.9
4	G	91.6333	48.2333	85.7	72.7	54.0667	90.7	79.6
	B	90.1667	44.4667	82.7667	68.5667	50.2667	88.7333	75.5333
	A	91.3333	47.6	85.6333	72.2667	52.0667	90.2667	78.2667
5	G	87.5667	43.7667	79.7667	63.4	51.5	86.0667	71.5667
	B	85.9667	41.5333	76.9	59.5667	47.9	84.4333	67.2333
	A	88.7333	44.8667	81.6	63.9	50.9333	86.7	71.7667

Table 6: Empirical coverage (in %) of nominal 95% confidence intervals for fBM (top part) and MRW (bottom part):  $P = 200$ ,  $L = 4$ ;  $2^{12}$  samples

q	X	$\hat{\zeta}_X(q)$			$\hat{\zeta}_X^B(q)$			
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	88.2	89.2333	90.1667	88.8333	88.0333	89.3333	87.6667
	B	88.3667	89.3	90.6333	88.9667	88.4667	89.5	87.8333
	A	88.5667	89.3667	90.6333	88.9667	88.3333	89.6667	88.0667
2	G	87.4333	88.8667	89.4667	86.3667	86.8667	88.2667	84.5667
	B	86.8	88.7	89.4	86.4667	86.4667	87.9667	84.2667
	A	86.8	88.8	89.5667	86.7333	86.5	87.7333	84.3
3	G	84.5667	89.4333	88.0667	84.0333	86.5	86.3	81.4
	B	83.4333	88.4	87.3667	83.3333	85.0333	84.7333	80.2333
	A	82.5667	88.2333	87.4333	83.2	84.8	84.4333	79.4667
4	G	83.4333	90	87.4333	82.2667	86.8333	84.6333	79.6333
	B	79.6667	87.4667	84.9333	79.9333	83.8667	81.1333	75.7667
	A	78.6333	87.2667	84.8667	79.3	82.8333	80.1333	74.1667
5	G	82.3333	90.8667	87	80.4	88.7667	84.1667	77.3667
	B	74.7333	87.8333	82.7	76	82.9667	77.6667	70.6
	A	73	87.2	81.8667	75.0333	81.4667	75.8	67.8
1	G	95.9	77.4333	96.7333	89.6667	75.4	96.3667	88.4
	B	95.7667	76.0667	96.5333	88.8667	74.9667	96.1667	87.9333
	A	95.7667	76.5333	96.6667	89.1667	75.0667	96.3	88.2
2	G	95.3	64	94.9	86.5333	64.1333	94.9	87.5667
	B	94.6333	61.9333	93.8	84.1667	63.2	94.2333	86.0667
	A	94.9333	62.5333	94.2	85.3333	63.5333	94.4667	86.7
3	G	93.6	54.4	91.5	81.0667	56.9333	93.4667	84.9
	B	92.4	50.7	88.8667	77.7333	54.8333	92.4	82.3667
	A	93.1333	52.8667	90.3667	79.7	55.7333	92.7	83.5667
4	G	91.8667	47.7667	86.7333	72.8	52.1667	91	79.2
	B	90.2	45.0667	83.1667	68.3667	48.9667	89.2	75.5
	A	91.3	46.7333	86.0667	71.8	50.7667	90.2667	77.7667
5	G	88.0333	44.5	80.7	63.9	50.4333	86.8333	71.5667
	B	86.8	41.7	77.9333	60.1	48.0333	85.7	67.4333
	A	89.2	44.2667	81.9333	64.3	50.0667	87.7667	71.1333

Table 7: Empirical coverage (in %) of nominal 95% confidence intervals for fBM (top part) and MRW (bottom part):  $P = 200$ ,  $L = 6$ ;  $2^{12}$  samples

q	X	$\hat{\zeta}_X(q)$				$\hat{\zeta}_X^B(q)$		
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	87.0667	88.6333	89.7667	88.2	86.4	87.8	86.2
	B	86.9333	88.8	89.6667	88.1667	86.4667	87.8667	86.1333
	A	86.9333	88.7667	89.8	88.3	86.5	87.8333	86.3
2	G	84.3667	87.8333	88.5667	85.8	85.1	86.1333	82.4333
	B	83.8	87.6	87.9333	85.0667	84.7667	85.5	81.9333
	A	83.8333	87.5	87.8333	85.1333	84.3333	85.3667	81.7333
3	G	83.4	89	87.8	83.7667	86.0333	85.0667	80.5667
	B	81.7	87.6333	86.4	82.6333	84.6	83.1	78.8333
	A	81.3	87.5667	86.6333	82.3667	83.9	82.7667	78.1
4	G	82.6333	89.8667	86.7	82.1667	86.7667	83.7333	78.7333
	B	77.7333	86.8667	84.1333	79.2	83	80.1667	74.2333
	A	76.6	86.7	83.8	78.3333	82.1667	79.0667	73.1667
5	G	81.4	90.9	86.4333	80.5667	88.6333	83.1333	77.1667
	B	73.2667	86.8	81.8667	74.5	82.3	75.9667	68.9333
	A	71.2	86.2333	81.0333	73.8	80.7333	74.4333	66.4667
1	G	97.7667	77.0667	98.0667	89.6333	75.4	97.6333	87.5333
	B	97.6333	76.2667	97.8333	89.2667	74.7333	97.4667	86.9
	A	97.7	76.7	97.9	89.5	74.9667	97.5333	87.3333
2	G	96.8	65.2333	96.4333	86.9333	65.5667	96.5667	86.5
	B	96.2667	62.5333	95.6	84.7	63.9333	95.9667	85.6333
	A	96.4667	63.7	96.1	85.8667	64.8	96.2667	86.2
3	G	94.8667	56	92.5333	81.8	58	94.4667	84
	B	94.1	52.2667	91.1333	78.3	55.4	93.4333	82.1667
	A	94.5333	53.6	91.8	80.4667	56.9	93.9333	82.9333
4	G	92.3	49.7667	88.2	73.7333	54.1667	91.6333	78.8333
	B	90.9	45.4	86.3333	69.4	50.7	89.9333	76.1667
	A	92.4667	49	88.3333	73.1333	52.9	91.4	77.8667
5	G	88.3667	45	82.9333	65	51.3667	87.5667	72.2667
	B	87	42.5333	80.4	60.7	48.6	86.1	68.5333
	A	89.5	46.7	84.0667	65.6333	51.7333	88.5333	72

Table 8: Empirical coverage (in %) of nominal 95% confidence intervals for fBM (top part) and MRW (bottom part):  $P = 200$ ,  $L = 12$ ;  $2^{12}$  samples

2.3  $2^{15}$  Samples

q	X	$\hat{\zeta}_X(q)$			$\hat{\zeta}_X^B(q)$			
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	87.2667	88.6	88.5333	88.5667	88.3333	88.3333	88.2333
	B	87.2	88.7333	88.6333	88.6	88.3	88.3667	88.2333
	A	87.3	88.6667	88.7333	88.7	88.3667	88.4333	88.3333
2	G	85.3	87.7667	86.9667	86.8333	86.6333	85.9333	85.8667
	B	85.0667	87.4	86.8667	86.7667	86.2333	85.6333	85.5333
	A	84.9	87.4	86.8333	86.7	86.3	85.6667	85.5
3	G	84.3333	88	86.5667	86.2667	86.6	85.3	84.6333
	B	83.0667	87.2333	86.1667	85.7333	85.7333	84.3667	83.6
	A	82.6	87.2	86.2333	85.9	85.6	84.0667	83.4667
4	G	83.4333	88.4667	86.7	85.4667	87.4667	84.4333	83.7667
	B	80.9333	87.6333	85.4333	84.4	85.1667	82.2333	81.5
	A	80.5667	87.5	85.2667	84.4	84.7333	81.8667	80.7667
5	G	82.7333	90.6333	86.5333	85.0333	88.3	84.1	82.6
	B	78.5	88.1333	84.3667	82.8333	84.6333	80.0667	78.8
	A	77.0667	88.2	84.1667	82.6333	83.8667	79.2333	78.0667
1	G	83.9667	59.5667	81.1667	81.2667	62.4333	83.3667	83.4667
	B	83.2	58.5	80.2	80.2667	61.4667	82.3	82.4
	A	83.0333	58.4	80.0667	80.2	61.4333	82.2667	82.3333
2	G	84.9667	40.8333	80.6667	80.4667	45.2	84.4	84.4667
	B	82.9333	38.7	78.0667	78	42.9	82.2	82.0667
	A	82.9333	38.9	78.1	78.1333	42.9667	82.2333	82.1667
3	G	85.0333	29.8333	80.0667	79.0667	33.8333	84.3333	83.7333
	B	81.9	27.9	76.1333	74.9333	31.2667	81.4	80.7667
	A	82.3333	27.9	76.7333	75.7	32.0667	81.8667	81.0333
4	G	82.3667	24.3333	75.1333	71.9333	27.4667	81.4	78.0667
	B	79.0333	22.0667	70.3667	67.5333	25.2333	77.1667	73.6
	A	79.9333	22.8667	72.0667	68.6667	25.9	78.0333	75.3667
5	G	76.2333	20.4667	69.0333	62.7333	23.4333	75.1333	69.8667
	B	72.1333	18.3333	64.9667	58.8667	21.9	71.6	64.6667
	A	74.7333	19.5	66.9	61.0667	22.9333	73.8333	67

Table 9: Empirical coverage (in %) of nominal 95% confidence intervals for fBM (top part) and MRW (bottom part):  $P = 200$ ,  $L = 1$ ;  $2^{15}$  samples

q	X	$\hat{\zeta}_X(q)$				$\hat{\zeta}_X^B(q)$		
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	90.0333	88.9667	91.4667	89	88.1	90.8333	88.1333
	B	90.2333	89.1667	91.6333	89.1333	88.2667	90.8667	88.2
	A	90.1667	89.1667	91.6	89.1333	88.3	90.7667	88.2667
2	G	89.6333	89.1	91.1333	88.5	87.2333	90.0333	86.9
	B	89.3667	88.8667	90.9333	88.4333	87.3	89.9	86.7
	A	89.2	88.9333	90.8667	88.3333	87.1667	89.7667	86.5667
3	G	88.1667	89.2333	91.1333	87.7667	87.1333	89.4	85.7333
	B	87.6333	88.8	90.7333	87.3333	86.8333	88.9333	84.5667
	A	87.2333	88.9667	90.8	87.3333	86.3	88.6	84.0333
4	G	87.1667	89.6667	90.2333	87.1	88	88.2	84.6
	B	85.4	88.6333	89.5667	85.5667	85.9667	86.2333	82.6667
	A	84.4333	88.6667	89.2333	85.2667	85.2667	85.9333	81.6
5	G	85.7667	90.8333	90	86.6	88.6667	87.0667	83.3
	B	81.5333	89.3	87.9	83.9	85.6667	83.5333	79.8333
	A	80.8	88.8667	87.3	83.2667	84.8	82.8667	78.4667
1	G	93.8	57.7333	93.1	80.0333	61.3667	94.0667	82.0667
	B	93.1667	54.7333	91.9667	77.5667	58.6	92.9667	80.1667
	A	93.5	56.5667	92.5	78.7333	60.2	93.5	81.1667
2	G	93.1	39.3333	89.1667	78.9	45.0333	92.5333	83.8667
	B	90.7333	35.9	86.4	74.4	41.3667	90.1667	80.2333
	A	91.8667	37.1	87.6	76.6667	43.2333	91.2	81.9
3	G	90.6667	28.1	85.3667	76.6667	33.7333	90.5333	82.4667
	B	87.3333	24.5333	80.5333	71.7667	30	87.1333	78.5333
	A	88.8333	26.3	82.4	73.8	31.1333	88.5333	80.2667
4	G	87.2	21.5667	79.7	70.4667	26.3333	86.4	78.1
	B	82.6	18.7333	74.4333	63.8333	22.5667	81.7333	72.5667
	A	85.2	19.5333	76.9333	67.2667	24.2333	84.2667	75.6333
5	G	80.7333	17.6667	72.0667	60.6	22.2333	80.4	69.6667
	B	75.6	16.1	67.1	54.3667	19.1	75.0333	63.8333
	A	79.2667	16.8667	70.9333	58.8667	20.9667	78.5667	67.7667

Table 10: Empirical coverage (in %) of nominal 95% confidence intervals for fBM (top part) and MRW (bottom part):  $P = 200$ ,  $L = 4$ ;  $2^{15}$  samples



q	X	$\hat{\zeta}_X(q)$				$\hat{\zeta}_X^B(q)$		
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	89.9333	88.1667	90.8667	87.9667	87.6333	90.5667	87.4
	B	89.9333	88.4667	91.0333	88.1667	87.7	90.6	87.6333
	A	89.9333	88.3333	91.0667	88.1333	87.7333	90.5667	87.5667
2	G	88.1333	86.7667	90.5667	85.9333	86.2667	89.0333	85.4
	B	87.9333	86.8	90.3333	86.0333	86.2	88.9333	85.4667
	A	87.8667	86.9	90.4	85.9667	85.9333	88.8	85.3333
3	G	86.6667	87.7	90.0333	85.9667	86.1	87.9	84.1333
	B	86.0333	87.6333	89.3667	85.4667	85.2	87.1	83.5333
	A	85.7333	87.5667	89.3667	85.4333	85	86.7333	82.9
4	G	86.3	89.2	89.2667	86.3333	86.4667	87.2	83.1
	B	83.9333	87.9333	88.3667	84.9667	84.7333	85.3	80.9667
	A	83.3667	87.4667	88.0333	84.1667	84.3667	84.7	80.4333
5	G	84.6	90.4333	89.4	85.8333	87.5333	86.5667	82.4667
	B	79.9	88.3333	87.1333	83.6	84.7	82.3333	78.3333
	A	79.1	88.0333	86.7667	82.8	83.4	81.4667	76.6333
1	G	96.1	55.9667	96.0333	79.2667	60.4333	95.9667	81
	B	95.1333	52.9333	95.0333	76.4	57.8333	95.3333	79.3333
	A	95.7	54.6667	95.4333	78.3333	59.6	95.7333	80.2
2	G	94.8333	38.5	91.6667	77.8	44.6667	94.5667	84
	B	92.8667	34.8667	88.7333	73.1667	40.7	92.5667	79.9333
	A	93.8333	36.5333	90.0333	75.8	42.8667	93.7333	81.9667
3	G	92	29.7667	86.8333	75.7333	33.9667	91.8	83.3333
	B	89.1	26.6	82.1	70.1	31.1333	88.2333	77.8
	A	90.2667	28.1333	84.4667	72.4667	32.0667	89.9	80.2667
4	G	87.8667	24.1667	79.6333	68.9667	28.8333	86.6667	77.4333
	B	82.8667	21.2333	74	62.7	26.1333	82.1333	71.0667
	A	85.5	22.8333	76.8333	65.7667	27.6667	84.8	74.2333
5	G	80.5667	20.0667	71.8333	60.4	25.2	80.4333	69.0333
	B	75.5	16.8	66.7	55.3667	22.1667	75.0333	63.0667
	A	79.1	18.6333	70.7	58.6333	24.1333	78.7	66.8

Table 11: Empirical coverage (in %) of nominal 95% confidence intervals for fBM (top part) and MRW (bottom part):  $P = 200$ ,  $L = 6$ ;  $2^{15}$  samples

q	X	$\hat{\zeta}_X(q)$				$\hat{\zeta}_X^B(q)$		
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	89.6667	88.6	91.4	88.5667	87.4667	90.2	87.4667
	B	89.7	88.7333	91.5	88.6333	87.6	90.1333	87.5667
	A	89.7	88.6667	91.4	88.7	87.6333	90.1	87.7
2	G	88.1	87.7667	90.3667	86.8333	86.1333	88.9	85.1667
	B	87.7333	87.6	90.2333	86.5667	85.8667	88.8	84.9667
	A	87.6333	87.4	90.1	86.7	85.7	88.7333	84.9667
3	G	86.6333	88	89.5667	86.2667	85.9667	87.7667	83.5667
	B	86	87.4	89.3333	85.6	85.7333	87.4667	82.9
	A	85.2	87.2	89.1333	85.9	85.0667	87.0667	82.2667
4	G	85.6333	88.4667	89.2667	85.4667	86.3333	86.6667	83.2333
	B	83.6	87.4667	88.1333	84.5	84.8	85	81
	A	82.7333	87.5	87.7667	84.4	84.3	84.3	80.0667
5	G	84.9	90.6333	89.2667	85.0333	87.6667	85.9667	82.2
	B	81	88.3667	86.5667	82.8333	84.6	82.5	78.3
	A	80.1	88.2	86.4	82.6333	83.7333	81.7333	77.0667
1	G	97.7667	57.4333	98.7667	79.9333	57.6667	98.1333	80.1333
	B	97.5333	53.3333	98.2	76.1333	55.5	97.7333	78.2333
	A	97.7	55.7667	98.6	78.8333	57.0667	97.9667	79.3
2	G	97.2	40.4	95.5	79.3667	43.9667	96.9	82.9
	B	95.9333	36.2667	93.4667	73.4	40.7333	95.5333	79.6
	A	96.4333	38.9667	94.6	76.9667	42.2333	96.2333	81.6333
3	G	94.9333	30.6667	90.7	76.8	33.3333	94.7	82.8
	B	92.3667	27.2667	86.4	70.7	30.6667	91.6333	78.2333
	A	93.9667	29.0333	88.2667	73.8333	31.6	93.2333	80.4667
4	G	90.5	23.9333	84.4	70.7333	27.2333	89.9333	77.8
	B	87.2	20.5667	78.9333	64.1667	25.1667	86.1	72
	A	88.9	22.4667	82.0667	67.8	26.3333	88.2	75.4
5	G	84.4	19.7	76	62.0667	24.5	83.9	69.9333
	B	79.6667	16.5333	71	56.6333	21.0667	78.8	63.7
	A	82.8667	18.4667	74.7	60.2667	23.1667	82.6	67.7667

Table 12: Empirical coverage (in %) of nominal 95% confidence intervals for fBM (top part) and MRW (bottom part):  $P = 200$ ,  $L = 12$ ;  $2^{15}$  samples

### 3 Empirical Coverage: MRW (varying number of BS samples)

#### 3.1 $2^9$ Samples

q	X	$\hat{\zeta}_X(q)$			$\hat{\zeta}_X^B(q)$			
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	92.7333	85.0667	92.2	89.1667	85.9333	92.8667	89.2
	B	92.8667	84.8333	92.4333	88.5667	86.4	92.8333	89.1333
	A	92.9	85.1667	92.3333	88.9	86.5333	92.8333	89.3
2	G	92.2333	80.4333	90.6333	83.5	81.9	91.6667	84.9333
	B	91.5	78.9667	89.5667	82.4667	80.6667	91.0333	84.0333
	A	91.7667	79.6333	89.9667	83.2667	81.1333	91.2	84.2333
3	G	92.1	77.3667	88.8333	76.5333	80.6667	90.4	80.1667
	B	90.5667	73.8	86.9	74.1667	77.0333	88.9667	76.6333
	A	90.9667	75.1667	88.0333	75.8	77.9333	89.3	77.5
4	G	91.6333	76.3667	86.2333	69.1333	80.2	89.4333	73.7
	B	89.5	71.3667	83.1333	65.2667	75.3667	87.1333	69.2333
	A	89.9667	73.7667	85.5333	67.9667	76.6667	88.1333	70.6333
5	G	90.5667	75.1667	83.7667	61.7667	80.8333	87.5667	67.7
	B	88.8	70.8	81.5667	57.3	75.3333	86.1667	63.1
	A	89.9667	73.7333	85.1333	60.4667	77.3333	87.6667	65.2
1	G	92.7333	85.9333	93.7	90.4	86.1333	93.5333	90.4333
	B	92.8667	86.1667	93.9	90.2333	86.3	93.6	90.5333
	A	92.8667	86.1	93.7667	90.5	86.5333	93.6667	90.5667
2	G	92.1	82.0667	92.2667	86.2	82.9	92.2333	86.8
	B	91.5667	80.6333	91.6	84.9	81.5	91.9	85.8667
	A	91.8333	81.1667	92.0333	85.5667	82.1	91.8333	86.2
3	G	91.6333	79.1667	90	79.8333	81.8667	90.9667	81.9667
	B	90.7667	74.9333	88.5667	75.7667	78.3	89.7667	78.6
	A	91.0333	76.5667	89.5	77.3667	78.7667	90.1667	79.3333
4	G	91.1667	77.1333	87.8333	71.5333	81.5333	90.1333	76.2
	B	89.5	72.1	85.6667	65.8667	76.5333	88.4333	71.2333
	A	89.5333	74.3333	87.4667	68.5	77.4667	89.1	72.7
5	G	90.6667	76.3667	86.2333	64.5	82.2	89.2333	70.1667
	B	89.4	70.9667	82.8667	58.8	76.2333	87.4667	63.8333
	A	89.8667	74.2667	85.6667	61.6333	77.9	87.8	66.2
1	G	93.8	87.2333	93.4333	91.2333	87.5333	93.8667	91.3667
	B	93.8	86.9	93.8	91.0667	87.2667	94.0333	91.2667
	A	93.8	87.2667	93.5667	91.5	87.5667	93.9667	91.3333
2	G	92.8667	82.0667	92.0667	86.0667	83.5667	92.5667	87.1
	B	92.3333	80.9	91.3667	84.1333	81.7333	92.2333	85.4667
	A	92.4667	81.7	91.6	85.2	82	92.2667	86.0667
3	G	92.1667	79.3333	90.6	78.6667	81.7333	91.7	81.8667
	B	91.1333	76.6	88.0333	75.8333	77.9	90.4667	78.0333
	A	91.4333	77.7	89.2	77.6333	79.3667	90.6	79.3
4	G	91.6	77.8	88	71.4667	81.5667	90.6333	75.2333
	B	90.2	73.7	85.7667	66.7333	76.5667	88.8	70.5
	A	90.7667	75.8667	87.6333	69.1667	78.5667	89.5	72.1
5	G	90.2	76.3	86	63.2	81.2333	89.2667	69.6
	B	89.5	72.3667	83.7	59.1333	76.8	87.9	64.1
	A	90.7	75.8	86.3667	62.0333	79.0667	88.7333	66.2333

Table 13: Empirical coverage (in %) of nominal 95% confidence intervals for MRW:  $P = 100$  (top part) -  $P = 200$  (center part) -  $P = 1000$  (bottom part),  $L = 6$ ;  $2^9$  samples

3.2  $2^{12}$  Samples

q	X	$\hat{\zeta}_X(q)$			$\hat{\zeta}_X^B(q)$			
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	95.9333	76.6333	96.3	89.1667	75.2667	95.6667	88.4333
	B	95.8	75.7333	96.2	88.4333	74.5667	95.4	88.1
	A	95.9	75.9667	96.2333	88.9	74.8667	95.6333	88.2333
2	G	95.5	63.9	94.4	86.7667	64.3333	95.1667	87.2
	B	94.9333	62.1	93.2333	84.5	63.3333	94.2	86.3
	A	95.0333	62.6	93.9667	85.9	63.6667	94.5667	86.6333
3	G	94.6	54.9667	91.6333	81.1	57.9333	92.9	85.1333
	B	93.2333	51.7333	89.6333	77.3	55	92.0333	82.5333
	A	93.7333	53	90.7333	79.3667	55.8667	92.3333	83.5
4	G	92.8333	48.5667	87.5333	73.1667	53.2667	90.9	79.1333
	B	91.3667	46.4667	84.4	69.6	50.0667	89.3333	75.9333
	A	92.2333	47.7	87.2	72.6	52.1667	90.4333	77.5667
5	G	89.6667	44.8333	81.4	64.2667	51.5333	86.9667	71.9667
	B	89.0667	41.8333	79	61.1	47.9667	86	68.1
	A	90.8333	45.1333	83.0333	65.2333	51.1667	88	71.0333
1	G	95.9	77.4333	96.7333	89.6667	75.4	96.3667	88.4
	B	95.7667	76.0667	96.5333	88.8667	74.9667	96.1667	87.9333
	A	95.7667	76.5333	96.6667	89.1667	75.0667	96.3	88.2
2	G	95.3	64	94.9	86.5333	64.1333	94.9	87.5667
	B	94.6333	61.9333	93.8	84.1667	63.2	94.2333	86.0667
	A	94.9333	62.5333	94.2	85.3333	63.5333	94.4667	86.7
3	G	93.6	54.4	91.5	81.0667	56.9333	93.4667	84.9
	B	92.4	50.7	88.8667	77.7333	54.8333	92.4	82.3667
	A	93.1333	52.8667	90.3667	79.7	55.7333	92.7	83.5667
4	G	91.8667	47.7667	86.7333	72.8	52.1667	91	79.2
	B	90.2	45.0667	83.1667	68.3667	48.9667	89.2	75.5
	A	91.3	46.7333	86.0667	71.8	50.7667	90.2667	77.7667
5	G	88.0333	44.5	80.7	63.9	50.4333	86.8333	71.5667
	B	86.8	41.7	77.9333	60.1	48.0333	85.7	67.4333
	A	89.2	44.2667	81.9333	64.3	50.0667	87.7667	71.1333
1	G	96.2	77.1	96.5	89.6	76.1	96.2333	88.8667
	B	96.1333	76.4	96.3333	88.9667	75.5667	96.1667	88.5667
	A	96.1333	76.7667	96.4333	89.3667	75.9667	96.2667	88.9
2	G	95.4333	64.0333	95.0333	86.4667	64.9667	95.3667	87.7333
	B	95.1	61.6667	94.2	84.6667	63.2	94.9333	86.2333
	A	95.3	62.8	94.7	85.4333	63.8	95.1667	87.1
3	G	93.8	54.5667	91.4	81.5667	57.7	93.2333	84.8333
	B	92.6	52.0333	89.1	78.5	54.9333	91.9333	82.7333
	A	93.2	53.4333	90.2333	80.2333	56.3333	92.5667	83.4333
4	G	91.6667	48.5333	86.9667	74.2667	54	90.9	79.9333
	B	90.2	45.5333	84.5333	69.7	51.0667	89.1333	75.9
	A	91.5667	47.8	86.6333	72.7333	52.7	90.4	78.4
5	G	88.0333	45.0333	81.7667	64.6	51.9333	87.2667	73.0667
	B	86.8	42.1	79.2667	60.7333	48.3	85.7333	67.8333
	A	89.1333	45.4333	82.7667	65.1333	51.4	87.7667	71.3

Table 14: Empirical coverage (in %) of nominal 95% confidence intervals for MRW:  $P = 100$  (top part) -  $P = 200$  (center part) -  $P = 1000$  (bottom part),  $L = 6$ ;  $2^{12}$  samples

3.3  $2^{15}$  Samples

q	X	$\hat{\zeta}_X(q)$			$\hat{\zeta}_X^B(q)$			
		CI <sub>B</sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>	CI <sub>V<sub>G</sub></sub>	CI <sub>V<sub>B</sub></sub>	CI <sub>V<sub>A</sub></sub>
1	G	96.9667	56.2	95.6	80.8667	59.8	95.5667	82
	B	96.3	53.3	94.4333	78.6	57.2333	95	80.3
	A	96.6333	54.7	95.2	79.7333	58.7333	95.3667	81.4
2	G	95.9667	38.6333	92.4	79.4	43.7667	94.7333	84.5667
	B	94.2667	34.4	89.7	74.5667	40.1333	93	80.9333
	A	95.0333	36.5333	91.0333	77.3	41.8	94	82.7667
3	G	93.7	28.4	87.3	75.9667	34.3667	92.2	83.1
	B	91.4	25	82.2333	70.0667	30.8333	89.0333	77.9667
	A	92.4	26.3667	84.6333	73	32.6333	90.1333	80.3667
4	G	90.1	22.7	80.2667	69.3667	27	87.5667	77.8
	B	85.9667	19.9333	75.3	63.4333	23.6667	82.4333	71.9
	A	87.9667	21.2	78.2333	66.7333	25.4333	85.0667	74.8333
5	G	83.3	18.8667	72.5333	59.3667	22.6	80.1	69.1667
	B	78.5	16.6667	66.8333	53.2667	19.9667	75.1667	62.4667
	A	82.1	18.5333	70.7667	57.3333	21.4667	79.1	66.7
1	G	96.1	55.9667	96.0333	79.2667	60.4333	95.9667	81
	B	95.1333	52.9333	95.0333	76.4	57.8333	95.3333	79.3333
	A	95.7	54.6667	95.4333	78.3333	59.6	95.7333	80.2
2	G	94.8333	38.5	91.6667	77.8	44.6667	94.5667	84
	B	92.8667	34.8667	88.7333	73.1667	40.7	92.5667	79.9333
	A	93.8333	36.5333	90.0333	75.8	42.8667	93.7333	81.9667
3	G	92	29.7667	86.8333	75.7333	33.9667	91.8	83.3333
	B	89.1	26.6	82.1	70.1	31.1333	88.2333	77.8
	A	90.2667	28.1333	84.4667	72.4667	32.0667	89.9	80.2667
4	G	87.8667	24.1667	79.6333	68.9667	28.8333	86.6667	77.4333
	B	82.8667	21.2333	74	62.7	26.1333	82.1333	71.0667
	A	85.5	22.8333	76.8333	65.7667	27.6667	84.8	74.2333
5	G	80.5667	20.0667	71.8333	60.4	25.2	80.4333	69.0333
	B	75.5	16.8	66.7	55.3667	22.1667	75.0333	63.0667
	A	79.1	18.6333	70.7	58.6333	24.1333	78.7	66.8
1	G	96.2333	56.9667	96	79.8667	60.4	96.3	82.1333
	B	95.5667	54.5	95.4	77.1333	57.7333	95.5	80.1
	A	95.8333	55.9667	95.7333	78.6	59.2667	95.7667	81.1667
2	G	94.5333	41.1	91.7667	78.7667	45.7333	94.6333	84.2
	B	92.8667	37.0333	89.5	74.2333	42.4667	92.6	80.0333
	A	93.7667	39.0333	90.7333	76.7	44.1333	93.6667	82.5333
3	G	92.4333	29.9667	87.6333	77.3	36	92.1333	83.5
	B	89.3	26.4	82.9	71.8333	31.7667	89	79.1667
	A	91.0667	28.0333	85.1	74.5	33.6667	90.7	81.2333
4	G	88.3667	24.0667	81.2667	71	28.8	88.0667	79.1333
	B	84.0333	20.9333	76.3333	64.5333	25.0333	83.5333	73.1333
	A	86.1667	22.2	78.8	68.1	26.4333	86.0667	76.5
5	G	81.8	19.2333	74.0333	60.4333	24.9667	81.4333	71.0667
	B	77	16.4333	68.6333	54.5	21	77.0667	63.5667
	A	80.1	18.1	72.2667	58.5333	22.8333	80.3333	68.4667

Table 15: Empirical coverage (in %) of nominal 95% confidence intervals for MRW:  $P = 100$  (top part) -  $P = 200$  (center part) -  $P = 1000$  (bottom part),  $L = 6$ ;  $2^{15}$  samples