STATISTICAL ANALYSIS AND MODELLING OF COMPUTER NETWORK TRAFFIC: SCALING, SELF-SIMILARITY, MULTIFRACTALITY?

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OUTLINE

• I. INTERNET AND SCALING
  — I.1 INTERNET,
  — I.2 SCALING,

• II. ANALYSIS TOOLS: MULTiresOLUTION ANALYSIS
  — II.1 AGGREGATION, INCREMENT,
  — II.2 WAVELET AND MULTIResolution ANALYSIS,

• III. MODEL1: SELF-SIMILARITY, LONG MEMORY AND RANDOM WALKS
  — III.1 RANDOM WALKS, SELF SIMILARITY AND LONG MEMORY,
  — III.2 SELF SIMILARITY, LONG MEMORY AND WAVELETS,
  — III.3 HURST PARAMETER ESTIMATION,
  — III.4 ROBUSTNESS TO NON-STATIONARITY,
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• IV. MODEL2: MULTIPLICATIVE CASCADES AND MULTIFractal
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• V. MODEL3: CLUSTER POINT PROCESS AND SCALING
  — V.1 CLUSTER POINT PROCESS AND SCALING,
  — V.2 CLUSTER POINT PROCESS AND COMPUTER TRAFFIC.
***HETEROGENITY:***

Geography, Topology, Hardware, Protocols, Applications, Nature of Data...
INFORMATION FLOWS MODELLING

• **Why?**
  – to **Design Networks**,  
  – to **Estimate Dimensions of Components**: Buffer Sizes, Server Capacities, Link Rates,...  
  – to **Design Protocols**,  
  – to **Ensure Quality Of Services**,  
  – to **Avoid Congestion, Delays, Losses**,  
  – to **Design Control Of Admission Policies**,  
  – to **Design Tarification Policies**,  
  – ...  

• **How?**
  – by **Collecting Information**, by **Performing Measurements**,  
  – by **Performing Statistical Analysis Of Data And Time Series**,  
  – by **Connecting Analyses To Network Mechanisms**.
• **PASSIVE MEASUREMENTS:** Look at packets on a link ...

... and collect

- **TIME ARRIVALS,**
- **PACKET LOAD,**
- **PROTOCOL INFORMATION,**
- **APPLICATION INFORMATION,**
- ...

• **MODELLING:** Marked Point Process \( \{ t_i, S_i \}_{i \in I} \)
  - \( t_i \): **ARRIVAL TIMES,**
  - \( S_i \): **MARKS (PACKET LOAD, ...)**

  \( \Rightarrow \text{HUGE COLLECTION OF DATA} \)
• **AGGREGATING ARRIVALS**: Number of Packets per Bin of Size $\Delta$.

**Bin Size**

- $W_{\Delta}(i) =$ Number of Packets in Bin Number $i$,
- $W_{\Delta}(i) = \sum_{\forall k, t_i \leq t_k < t_{i+1}} S_k$.

**Discrete Time Time Series**: $\{W_{\Delta}(i)\}_{i \in I}$ (Regular Sampling)
 PACKET LEVEL (AGGREGATED TIME SERIES)
From packets to flows

- **IP Flow**: Set of packets with Identical 5-tuple

<table>
<thead>
<tr>
<th>IP protocol</th>
<th>Source Address</th>
<th>Destination Address</th>
<th>Source Port</th>
<th>Destination Port</th>
</tr>
</thead>
</table>

Time
FROM PACKET TO FLOW (OR CONNECTION)

- **IP Flow**: Set of packets with Identical 5-tuple

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Flow Start

Flow Duration

Time

Time
Flow Level

- Identifying IP Flows:

- Marked Point Process: \( \{t_i, S_i\}_{i \in I} \),
  - \( t_i \): Arrival Times,
  - \( S_i \): Marks (Duration, total load, number of packets, ...)

\[9\]
FLOW LEVEL: FLUCTUATIONS OF THE NUMBER OF ACTIVE FLOWS
SESSION LEVEL

- **IDENTIFYING SESSIONS:**
  A GROUP OF FLOWS CORRESPONDING TO A SAME ACTIVITY OR A SAME INTERNAUT.

- **CONFIDENTIALITY ISSUES:** ?
STATISTICAL ANALYSIS

- **TELEPHONE-BASED ANALYSIS:**
  - Poisson Point Processes, Renewal Point Processes,
  - Gaussian Processes (Central Limit Theorem),
  - Short-Range Dependencies (Markov-Type Processes).

- **FAILURE:**
  - In Modelling,
  - In Predicting Performance,
  - In Enabling Relevant Network Design (Rooter, Buffer Sizes,...),
  - In Providing Relevant Quality Of Services Policies.
    ⇒ Not Solved by a Simple Over-Dimensioning Argument!

- **IRREGULARITIES IN TIME SERIES ?**
SCALING : DILATION ?

Trafic (WAN) Internet

temps (s)

nb connexions

Trafic (WAN) Internet

temps (s)

nb connexions
SCALING : AGGREGATION ?

\[ \delta = 12 \text{ms} \]

\[ \delta = 12 \times 8 \text{ms} \]

\[ \delta = 12 \times 8 \times 8 \text{ms} \]

\[ \delta = 12 \times 8 \times 8 \times 8 \text{ms} \]
SCALING : SPECTRUM ?

Trafic (LAN) Ethernet −−− Densite Spectrale de Puissance

Log\(_{10}\) (Frequence (Hz))

Log\(_{10}\) (DSP) −−− Nombre Octets
**DEFINITION:** NO CHARACTERISTIC SCALE (NON PROPERTY).
(OFTEN COMES WITH NON GAUSSIAN, NON STATIONARY, NON LINEAR)

**EVIDENCE:** THE WHOLE RESEMBLES ITS PART, AND VICE VERSA.

**ANALYSIS:** RATHER THAN FOR A CHARACTERISTIC SCALE,
LOOK FOR A RELATION, A MECANISM, A CASCADE BETWEEN SCALES.
**UBIQUITY ?**

**Trafic (LAN) Ethernet**

**Trafic (WAN) Internet**

**Turbulence de Jet, R_λ ~ 580**

**Vitesse (m/s)**

**Dissipation**

**Nombre Octets**

**nb connexions**

**temps (s)**

**temps (s)**

**temps (s)**
- Hydrodynamic Turbulence,
- Physiology, Biological Rythms (Heart beat, walk),
- Geophysics (Faults Repartition, Earthquakes),
- Hydrology (Water Levels),
- Statistical Physics (Long Range Interactions),
- Thermal Noises (semi-conductors),
- Information Flux on Networks, Computer Network Traffic,
- Population Repartition (local: cities, global: continent),
- Financial Markets (Daily returns, Volatility, Currencies Exchange Rates),
- ...
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TOOL 1: AGGREGATION

COMPARE DATA AGAINST A BOX, THEN VARY \( a \)

\[ T_X(a, t) = \frac{1}{aT_0} \int_t^{t+aT_0} X(u) du \]

AVERAGE

WORKS ONLY FOR POSITIVE TIME SERIES, DENSITY
TOOL 2: INCREMENT

COMPARE DATA AGAINST A DIFFERENCE OF DELTA FUNCTIONS, THEN VARY $a$

$$T_X(a, t) = X(t + a\tau_0) - X(t)$$

DIFFERENCE

INCREMENTS OF HIGHER ORDERS OR GENERALISED $N$-VARIATIONS

- Order 2: $T_X(a, t) = -X(t + 2a\tau_0) + 2X(t + a\tau_0) - X(t)$,
- Order $N$: $T_X(a, t) = \sum_{p=0}^{N} (-1)^p a_p X(t + p a\tau_0)$,

where $\sum_{p=0}^{N} (-1)^p a_p p^k \equiv 0, k = 0, \ldots, N - 1$. 

[21]
**Tool: MultiResolution Analysis**

- **MultiResolution Quantities:**

  \[ X(t) \rightarrow T_X(a, t) = \langle f_{a, t} | X \rangle, \quad f_{a, t}(u) = \frac{1}{a} f_0\left(\frac{u-t}{a}\right) \]

  **Aggregation**
  \[
  f_0(u) = (\beta_0)
  = \frac{1}{aT_0} \int_t^{t+aT_0} X(u) du
  
  **In increments**
  \[
  f_0(u) = (I_0)
  = X(t + a\tau_0) - X(t)
  
  **Box, Average**

  ![Graph of X(t) vs temps]

  ![Graph of X(t + a\tau_0) - X(t) vs temps]
**Tool: MultiResolution Analysis**

- **Multiresolution Quantities:**
  \[ X(t) \rightarrow T_X(a,t) = \langle f_{a,t} | X \rangle, \quad f_{a,t}(u) = \frac{1}{a} f_0(\frac{u-t}{a}) \]

- **Choices for Mother Functions:** \( f_0, \)

  **Aggregation**
  \[ f_0(u) = (\beta_0) = \frac{1}{aT_0} \int_t^{t+aT_0} X(u)du \]
  **Box, Average**

  **Increments**
  \[ f_0(u) = (I_0) = X(t + a\tau_0) - X(t) \]
  **Difference**

  **Wavelets**
  \[ f_0(u) = \psi_{0,N} = \int X(u) \frac{1}{a} \psi_0(\frac{u-t}{a}), \]
  **Average, Difference**
**Tool: Wavelet Transforms**

- **Mother-Wavelet and Wavelet "Basis":**
  \[ \int \psi_0(u)du = 0, \quad \psi_{a,t}(u) = \frac{1}{|a|} \psi_0\left(\frac{u-t}{a}\right). \]

- **Continuous Wavelet Transform:**
  \[ T_X(a, t) = \langle X, \psi_{a,t} \rangle \]

- **Modulus Maxima vs Discrete Wavelet Transforms:**
  **Skeleton:**
  
  **Maxima Lines**

  **Dyadic Grid:**
  \[ d_X(j, k) = T_X(a = 2^j, t = 2^j k) \]
WAVELETS AND SCALING: KEY INGREDIENTS

- **Dilation Operator**, \( \frac{1}{|a|} \psi_0 \left( \frac{t}{|a|} \right) \)
- **Dilation**, \( a = 1 \), \( a = 2 \), \( a = 4 \)

- **Number of Vanishing Moments**, \( N \geq 1 \),
  \[ \int t^k \psi_0(t) dt \equiv 0, \quad k = 0, 1, \ldots, N - 1. \]
TOOL: MultiResolution Analysis

- **MultiResolution Quantities:**
  \[ X(t) \rightarrow T_X(a, t) = \langle f_{a,t} | X \rangle, \quad f_{a,t}(u) = \frac{1}{a} f_0\left(\frac{u-t}{a}\right) \]

- **Choices for Mother Functions:** \( f_0, \)

  **Aggregation**
  \[
  f_0(u) = (\beta_0)^N \\
  = \frac{1}{aT_0} \int_t^{t+aT_0} X(u)du 
  \]
  **Box, Average**

  **Increments**
  \[
  f_0(u) = (I_0)^N \\
  = X(t + a\tau_0) - X(t) 
  \]
  **Difference**

  **Wavelets**
  \[
  f_0(u) = \psi_{0,N} \\
  = \int X(u) \frac{1}{|a|} \psi_0\left(\frac{u-t}{a}\right), 
  \]
  **Average, Difference**
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MODEL 1: RANDOM WALKS AND SELF SIMILARITY

- **RANDOM WALK:** \( X(t + \tau) = X(t) + \delta \tau X(t) \)  
  Steps or Increments

- **STATISTICAL PROPERTIES OF THE STEPS:**
  - **A1:** Stationary,
  - **A2:** Independent,
  - **A3:** Gaussian,
    - Ordinary Random Walk, Ordinary Brownian Motion,
    - \( \mathbb{E} X(t)^2 = 2D|t| \), Einstein relation,
    - \( \mathbb{E} X(t)^q = 2D|t|^{q/2} \), \( q > -1 \).

- **ANOMALIES:**
  - \( \mathbb{E} X(t)^2 = 2D|t|^{\gamma} \),
  - \( \mathbb{E} X(t)^2 = \infty \).

- **SELF SIMILAR RANDOM WALKS:**
  - **B1:** Stationary,
  - **B2:** Self Similarity
MODEL 1: SELF-SIMILARITY

- **DEFINITION:** \( \{\delta_\tau X(t)\}_{t \in \mathbb{R}} \overset{\text{fdd}}{=} \{a^H \delta_{\tau/a} X(t/a)\}_{t \in \mathbb{R}}, \)

\( \forall a > 0, \text{ Dilation Factor}, \quad 1 > H > 0 : \text{SELF-SIMILARITY EXPONENT} \)

- **INTERPRETATIONS:** **SCALING**!
  - Covariance under Dilation (Change of Scale),
  - The Whole and the SubPart (Statistically) Undistinguishable,
  - No Characteristic Scale of Time.

- **IMPLICATIONS:**
  - Additive Structure, Random Walks
  - Power Laws: \( \mathbb{E}|X(t + a\tau_0) - X(t)|^q = C_q|a|^{qH}, \)
  - A Single Scaling Exponent \( H \),
  - \( \forall a > 0, \forall q \geq 1, \)
  - Non Stationarity,
  - Long Memory, Long Range Correlation, when \( 1 > H > 1/2 \).
• **P1:** \( \{d_X(j, k), k \in \mathbb{Z}\} \) **Stationary** Sequences for each Scale \(2^j, N \geq 1\).

• **P2:** **Self-Similarity**

\[
\{X(t)\} \quad \overset{fdd}{\Rightarrow} \quad \{a^H X(t/a)\} \Rightarrow \{d_X(0, k)\} \quad \overset{fdd}{=} \quad \{2^{-jH}d_X(j, k)\}
\]

- **P2bis:** **Marginal Dist.** \( P_j(d) = \frac{1}{a} P_{j'}(\frac{d}{a}), \quad a = \left(\frac{2^{j'}}{2^j}\right)^H \) **Dilation**.

• **P3:** \( \{d_X(j, k)\} \) **Short Range Dependent** if \( N > H + 1/2 \).

\[
|2^j k - 2^j k'| \to +\infty, \quad |\text{Cov } d_X(j, k)d_X(j', k')| \leq D |2^j k - 2^j k'|^{2(H-N)}.
\]

**Dilation and** \( N \geq 1 \).*
Wavelets and Long Range Dependence

H = 0.15
Haar

H = 0.5
Daubechies2

H = 0.95
**Wavelets and \( H \)-Self-Similar Processes with Stationary Increments and Finite Variance**

- **P1:** \( \{d_X(j, k), k \in \mathbb{Z}\} \) Stationary Sequences for each Scale \( 2^j, N \geq 1 \).

- **P2:** Self-Similarity
  \[ \{X(t)\} \overset{fdd}{=} \{a^H X(t/a)\} \Rightarrow \{d_X(0, k)\} \overset{fdd}{=} \{2^{-jH} d_X(j, k)\} \]

  - **P2bis:** Marginal Dist.
    \[ P_j(d) = \frac{1}{a} P_{j'}\left(\frac{d}{a}\right), \quad a = \left(\frac{2^{j'}}{2^j}\right)^H \]
    Dilation.

- **P3:** \( \{d_X(j, k)\} \) Short Range Dependent if \( N > H + 1/2 \).
  \[ |2^j k - 2^{j'} k'| \to +\infty, \quad |\text{Cov} d_X(j, k)d_X(j', k')| \leq D|2^j k - 2^{j'} k'|^{2(H-N)}, \]
  Dilation and \( N \geq 1 \).
  \[ \Rightarrow \text{Idealisation:} \quad d_X(j, k) \text{ Independent Variables} . \]
  \[ \Rightarrow \text{Interpretations:} \quad X(t) = \sum_k a_X(J, k) \varphi_{J,k}(t) + \sum_{j,k} d_X(j, k) \psi_{j,k}(t). \]

  \[ \Rightarrow \text{Implications:} \quad \mathbb{E}|d_X(j, k)|^q = \mathbb{E}|d_X(0, 0)|^q 2^{jqH}, \quad \forall q / \mathbb{E}|d_X(0, 0)|^q < \infty. \]
**Estimation of the Scaling (Hurst) Param. H**

**Principles:**

- **Ideas:** \( P2 \) and \( P1 \) \( \Rightarrow \) \( E|d_X(j, k)|^q = E|d_X(0, 0)|^q 2^j q^H \)
  \( \Rightarrow \) \( \log_2 E|d_X(j, k)|^q = j q H + \log_2 E|d_X(0, 0)|^q, \) \( \text{cste}_q \)

- **Problems:** Estimate \( E|d_X(j, k)|^q \) from a Single Finite Length Observation?

- **Solution:** \( P1 \) and \( P3 \) \( \Rightarrow \)Statistical Averages \( \Rightarrow \) Time Averages,
  \( S_q(j) = (1/n_j) \sum_{k=1}^{n_j} |d_X(j, k)|^q \)

**Log-Scale Diagrams:** \( \log_2 S_q(j) \) vs \( \log_2 2^j = j \)
**LOG-SCALE DIAGRAMS:** $\log_2 S_q(j)$ vs $\log_2 2^j = j$

- **SYNTHETIC DATA:** Fractional Brownian Motion, $q = 2$.

\[ \alpha = 2.57 \]
\[ 1 \leq j \leq 10 \]
• **Definitions**: Let $X$ be a 2nd stationary process with,
- Correlation Function: $c_X(\tau) = \mathbb{E}X(t)X(t + \tau)$
- Spectrum: $\Gamma_X(\nu)$

$$
c_X(\tau) = c_\tau |\tau|^{-\beta}, \quad 0 < \beta < 1, \quad \tau \to +\infty
$$

$$
\Gamma_X(\nu) = c_f |f|^{-\alpha}, \quad 0 < \alpha < 1, \quad \nu \to 0
$$

with $\alpha = 1 - \beta$ and $c_f = 2(2\pi) \sin((1 - \gamma)\pi/2)c_\tau$.

• **Consequences**:
- $\sum_{A}^{+\infty} c_X(\tau) d\tau = +\infty$, $A > 0$,
- No Characteristic Scale,
- Aggregation: $T_X(a, t) = \frac{1}{aT_0} \int_{t}^{t+aT_0} X(u) du$,

$$
\Rightarrow \text{Var} T_X(a, t) \sim a^{\alpha - 1}, \quad a \to +\infty
$$

- Self.-Sim. Proc. (with $H > 1/2$) are Long Range Dep. (with $\alpha = 2H - 1$).
Wavelets and Long Range Dependence

- **Wavelet based Spectral Analysis:**
  Let $X$ be a 2nd Order stationary process,
  Let $\psi$ have central frequency $\nu_0$ and bandwith $\Delta\nu_0$.

  $$
  E|d_X(j, k)|^2 = \int \Gamma_X(\nu)|\Psi(2^j \nu)|d\nu \\
  \approx 2^{-j} \Gamma_X(2^{-j}\nu_0) \text{ within bandwith } 2^{-j}\Delta\nu_0.
  $$

- **Let $X$ be Long Range Dependent:**
  - **Power Law:** $E|d_X(j, k)|^2 \sim C 2^j(\alpha - 1)$, $j \to +\infty$,
  - **Decorrelation:** $Ed_X(j, k)d_X(j, k') \sim C|k - k'|^{-1-2N}$, $|k - k'| \to +\infty$,
  Short Range Dependence as soon $N > \alpha/2$.

- **Analysis:** $ \log_2(1/n_j \sum_{k=1}^{n_j} |d_X(j, k)|^2) \text{ vs } \log_2 2^j = j$
  $\Rightarrow$ Log-Scale Diagram with $q = 2$. 

[36]
LOG-SCALE DIAGRAMS: $\log_2 S_2(j)$ vs $\log_2 2^j = j$

- **SYNTHETIC DATA**: FARIMA(P,d,Q), long memory parameter: $\alpha = 2d$.

\[ \alpha = 0.55 \]
\[ c_f = 4.7 \]
\[ 4 \leq j \leq 10 \]
Wavelets and 2nd-Order Scaling: Estimation

- **Dyadic Grid (Discrete Wavelet Transform):**

- **Structure Function (Time Average):**
  \[ Y_2(j) = \frac{1}{2} \log_2 S_2(2^j) = \frac{1}{2} \log_2 \frac{1}{n_j} \sum_{k=1}^{n_j} |d_x(j, k)|^2 \]

- **Definition (Weighted Least Squares):**
  \[ \hat{H} = \sum j w_j \log_2 S_2(j) \]
  \[ w_j = \frac{B_0^j - B_1^j}{a_j (B_0 B_2^j - B_1^j)}, \quad B_{k} = \sum j j^k / a_j \]
  with \( a_j \) arbitrary numbers

- **Performance:** What are the statistical performance of this estimator when applied to self-similar or long-range dependence processes?
WAVELETS AND 2ND-ORDER SCALING: ESTIMATION

- **ASSUME**:  
  - $i$) $X$ Gaussian,  
  - $ii$) Idealisation: exact independence.

- **BIAS**:  
  $$ \mathbb{E} \log_2 S_2(j) = \log_2 \mathbb{E} S_2(j) + \Gamma'(n_j/2) - \log_2(n_j/2). $$
  
  $$ \Rightarrow \mathbb{E} \hat{H} = H + \sum_j w_j g_j. $$

- **VARIANCE**:  
  $$ \text{Var} \hat{H} \simeq \left( (\log_2(e))^2 \left( \sum_j w_j^2 2^j \right) \right) / n, \text{ min.} $$
  
  if $a_j = \text{Var} \log_2 S_2(j)$.

- **ACTUAL PERFORMANCE**:  
  - **NEGligible Bias**,  
  - **A PRIORI KNOWN APPROX. Conf. INTERVAL**,  
  - **CLOSE TO MLE IN GAUSSIAN CASE**.

- **CONCEPTUAL AND PRACTICAL SIMPLICITY**:  
  DWT AND LINEAR FIT!

- **COMPUTATIONAL ISSUES**:  
  LOW COST, ON-LINE, REAL-TIME, ON-THE-FLY.
**ETHERNET DATA**

- **BELLCORE ETHERNET PAUG**: LAN, 10BASET, RATE = 0.138 MBPS, \( \simeq 1 h \).
\[ W_\Delta (\Delta = 128 \text{ ms}) \]

- **CONSISTENT WITH SELF-SIMILARITY MODELLING**: 
  - \( H \simeq 0.8 \), **OVER 12 OCTAVES (3 DECADES)**,
  - **GENERIC (AGGREGATED TS, ARRIVAL LISTS, LOADS, POINT PROCESS)**.

[40]
**ETHERNET DATA**

- **BELLCORE ETHERNET pAUG**: LAN, 10BaseT, Rate = 0.138Mbps, \( \sim 1h \).
  - GENERIC (AGGREGATED TS, ARRIVAL LISTS, LOADS, POINT PROCESS).

![Graphs](image-url)
Scaling vs Non-Stat.: Robustness vs Superimposed Trends:

\[ Y(t) = X(t) + T(t) \Rightarrow d_Y(j, k) = d_X(j, k) + d_T(j, k) \]

- If \( T(t) \) Polynomial of degree \( P \), then \( d_T \equiv 0 \) when \( N > P \),
- If \( T(t) \) smooth trend, then the \( d_T \) decrease as \( N \) increases.

Vary \( N \)!
SCALING VS NON-STAT.: ROBUSTNESS VS SUPERIMPOSED TRENDS:

Ethernet Data

Logsacle Diagram, N=2

Full trace: $\alpha = 0.60$
Part I: $\alpha = 0.62$
Part II: $\alpha = 0.58$
SCALING VS NON-STAT.: CONSTANCY OF SCALING:

Constancy along time of Scaling laws
INTERNET DATA: SCALING AND BiSCALING.

- **AUCKIV:** WAN IP, LINK = OC3, RATE = 2.5MBPS, DURATION ≃ 3h.

- **BiSCALING:**
  - SCALING OVER TWO RANGES OF SCALES (12 OCTAVES ALL TOGETHER),
  - SEPARATION TIME AROUND 1S,
  - GENERIC (DIFFERENT NETWORKS, LINKS, RATES...).
INTERNET DATA: SCALING AT COARSE SCALES

- **AuckIV:**

- **Coarse Scales:** $\geq 1$s, **Consistent with Self-Similarity,** $\zeta(q) \sim qH$,
  - $H \sim 0.8$, **Consistent with Long Range Dependence,**
  - **Rather Generic** (different networks, links, rates...),
  - **File Size Distributions** have **Heavy Tails.**
INTERNET DATA: SCALING AT FINE SCALES

- AuckIV:

- FINE SCALES: ≤ 1S, NOT CONSISTENT WITH SELF-SIMILARITY,
  - \( \zeta(q) \neq qH \),
  \[ \Rightarrow \text{CLAIM: MULTIFRACTAL MODELS?} \]
FROM SELF-SIMILARITY . . .

- **SELF-SIMILARITY:**
  \[ E|d_X(j, k)|^q = C_q(2^j)^{qH} \]
  - Power Laws,
  - \( \forall 2^j \), (All Scales),
  - \( \forall q, \quad /E|d_X(j, k)|^q < +\infty \),
  - A single parameter \( H \),
  - Additive Structure.
... TO MULTIFRACTAL.

- Self-Similarity:

$$\mathbb{E}|d_X(j, k)|^q = C_q(2^j)^{qH}$$

- Power Laws,
- \(\forall 2^j\), (All Scales),
- \(\forall q\), \(\mathbb{E}|d_X(j, k)|^q < +\infty\),
- A single parameter \(H\),
- Exponents Linear in \(q\): \(\zeta(q) = qH\),
- Additive Structure.

- Multifractal

$$\mathbb{E}|d_X(j, k)|^q = C_q(2^j)^{\zeta(q)}$$

- Power Laws,
- \(\forall 2^j < L\), (All Scales below the Integral Scale),
- \(\forall q\), ???,
- A whole collection of scaling parameters \(\zeta(q) \neq qH\),
- Additive Structure.

- ?
OUTLINE

• I.  **INTERNET AND SCALING**
  – I.1  INTERNET,
  – I.2  SCALING,

• II.  **ANALYSIS TOOLS: MULTiresOLUTION ANALYSIS**
  – II.1  AGGREGATION,_INCREMENT,
  – II.2  WAVELET AND MULTIResolution ANALYSIS,

• III.  **MODEL1: SELF-SIMILARITY, LONG MEMORY AND RANDOM WALKS**
  – III.1  RANDOM WALKS, SELF SIMILARITY AND LONG MEMORY,
  – III.2  SELF SIMILARITY, LONG MEMORY AND WAVELETS,
  – III.3  HURST PARAMETER ESTIMATION,
  – III.4  ROBUSTNESS TO NON-STATIONARITY,
  – III.5  SELF-SIMILARITY AND COMPUTER TRAFFIC,

• IV.  **MODEL2: MULTIPLICATIVE CASCADES AND MULTIFractal**
  – IV.1  MULTIPLICATIVE CASCADES, MULTIFractal PROCESSES,
  – IV.2  MULTIFractal PROCESSES AND WAVELETS,
  – IV.2  MULTIFractal PROCESSES AND COMPUTER TRAFFIC,

• V.  **MODEL3: CLUSTER POINT PROCESS AND SCALING**
  – V.1  CLUSTER POINT PROCESS AND SCALING,
  – V.2  CLUSTER POINT PROCESS AND COMPUTER TRAFFIC.
**Model 2: Multiplicative Cascades**

**Definition:**
- Split dyadic intervals $I_{j,k}$ into two,
- I.I.D. multipliers $W_{j,k}$
- $Q_J(t) = \prod_{(j,k): 1 \leq j \leq J, t \in I_{j,k}} W_{j,k}$,

**Implications:**
- Cascades, multiplicative structure,
- Power laws,
- $\mathbb{E} \left( 1/2^j \int_{k2^j}^{(k+1)2^j} \tau_0 X(u) du \right)^q = C_q |2^j| \zeta_q$,
- Multiple exponents $\zeta_q = -\log_2 \mathbb{E} W^q$, non-linear in $q$,
- Fine scales $a = 2^j \to 0$, $a \ll L$ integral scale,
- No characteristic scale of time beyond an integral scale,
- Non stationarity,
- Local Holder exponent,
- Multi-fractal sample paths, multi-fractal spectrum $D(h)$.
## Model 2: Multiplicative Cascades

<table>
<thead>
<tr>
<th>Yaglom, Mandelbrot</th>
<th>Barral, Mandelbrot</th>
<th>Schmmitt et al., Bacry et al., Chainais et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mandelbrot’s Cascade (CMC)</td>
<td>Compound Poisson Cascade (CPC)</td>
<td>Infinitely Divisible Cascade (IDC)</td>
</tr>
<tr>
<td>- IID $W$,</td>
<td>- IID $W$,</td>
<td>- Continuous Infinitely Divisible Measure $M$,</td>
</tr>
<tr>
<td>- Dyadic Grid,</td>
<td>- Point Process,</td>
<td></td>
</tr>
</tbody>
</table>

\[ Q_r(t) = \prod W_{j,k}, \]
\[ \varphi(q) = -\log_2 E W^q, \]
\[ A(t) = \lim_{r \to 0} \int_0^t Q_r(u)du, \]

For a range of $q$s, $E|A(t + \alpha \tau_0) - A(t)|^q = c_q |\alpha|^{q+\varphi(q)}$,

Resolution Depth $<\text{Scale}<\text{Integral Scale}$, $a_m = r < \alpha < a_M = L$. 

\[ \prod W_{j,k}, \]
\[ = -q(1 - E W) + 1 - E W^q, \]
\[ \exp \int dM(t', r'), \]
\[ = \rho(q) - q\rho(1), \]
MODEL 2: MULTI-FRACTAL PROCESSES

**Density:**

\[ Q_r(t) = \prod W_{j,k} \]

\[ \mathbb{E} \left( \frac{1}{a} \int_{t}^{t+a\tau_0} Q_r(u) du \right)^q = c_q a^{\varphi(q)}, \]

**Measure:**

\[ A(t) = \lim_{r \to 0} \int_{0}^{t} Q_r(u) du, \]

\[ \mathbb{E} |A(t + a\tau_0) - A(t)|^q = c_q |a|^{q\varphi(q)}, \]

**Fractional Brownian Motion in Multifractal Time:**

\[ V_H(t) = B_H(A(t)), \]

\[ \mathbb{E} |V_H(t + a\tau_0) - V_H(t)|^q = c_q |a|^{qH+\varphi(q)}, \]

**Multifractal Random Walk:**

\[ Y_H(t) = \int_{t}^{t} Q_r(s) dB_H(s), \]

\[ \mathbb{E} |Y_H(t + a\tau_0) - Y_H(t)|^q = c_q |a|^{qH+\varphi(q)}. \]
MULTIFRACTAL ANALYSIS

● PRINCIPLES:

- IDEAS: \( P2 \) and \( P1 \) ⇒ \( \mathbb{E}|d_X(j, k)|^q = \mathbb{E}|d_X(0, 0)|^q 2^j \zeta(q) \)
  ⇒ \( \log_2 \mathbb{E}|d_X(j, k)|^q = j \zeta(q) + \log_2 \mathbb{E}|d_X(0, 0)|^q \)

- PROBLEMS: Estimate \( \mathbb{E}|d_X(j, k)|^q \) from a SINGLE FINITE LENGTH OBSERVATION?

- SOLUTION: \( P1 \) and \( P3 \) ⇒ STATITICAL AVERAGES ⇒ TIME AVERAGES,
  \( S_q(j) = (1/n_j) \sum_{k=1}^{n_j} |d_X(j, k)|^q \)

● LOG-SCALE DIAGRAMS: \( \log_2 S_q(j) \) vs \( \log_2 2^j = j \)
**LOG-SCALE DIAGRAMS:** \( \log_2 S_q(j) \) vs \( \log_2 2^j = j \)

- **SYNTHETIC DATA:** MULTIFRACTAL RANDOM WALKS.
MultiResolution Estimators

- **Time Average at Given Resolution** $\alpha$:
  \[ S_n(a_j, q; f_0) = \frac{1}{n} \sum_{k=1}^{n} |T_X(a_j, t_{j,k}; f_0)|^q \]

- **Weighted Linear Fits in log-log Plots:**
  \[ \log S_n(a, q; f_0) \text{ versus } \log a \]

- **Dyadic Grid (Discrete Wavelet Transform):**
  $a_j = 2^j$, $t_{j,k} = k \cdot 2^j$,

- **What Are the Performance of Such Estimators?**
  When Applied to MultiFractal Processes.

\[ Y_{j,q,n} = \log_2 S_n(2^j, q; f_0) \text{ versus } \log_2 2^j = j, \]

\[ \hat{\zeta}(q, n) = \sum_{j=j_1}^{j_2} w_{j,q} Y_{j,q,n}. \]
METHODOLOGY.

• **Numerical Synthesis of Processes:**
  – Accumulate \(nbreal\) numerical replications with length \(n\) samples.

• **Apply Scaling Exponents Estimators:**
  – Compute \(\hat{\zeta}(q, n)_{(l)}\) for each replication,
  – Average over repl. to obtain the statistical performance of \(\hat{\zeta}(q, n)\)

• **Asymptotic Behaviours:**
  – The cascade depth increases for a given number of Integral Scales.
  – … ,

\[
\begin{array}{c}
1 \\
\vdots \\
0 \\
\end{array}
\]
\[
\begin{array}{c}
(0, r) \\
\vdots \\
(1, r) \\
\end{array}
\]
\[
\begin{array}{c}
(0, 1) \\
\vdots \\
(1, 1) \\
\end{array}
\]

\[
\begin{array}{c}
(t, r) \\
\vdots \\
(t, 1) \\
\end{array}
\]

\[
\begin{array}{c}
(t, 0) \\
\vdots \\
(t, 1) \\
\end{array}
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\vdots \\
(t, 1) \\
\end{array}
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(t, 0) \\
\vdots \\
(t, 1) \\
\end{array}
\]

\[
\begin{array}{c}
(t, 1) \\
\vdots \\
(t, 1) \\
\end{array}
\]

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METHODOLOGY.

- **Numerical Synthesis of Processes:**
  - Accumulate $nbreal$ numerical replications with length $n$ samples.

- **Apply Scaling Exponents Estimators:**
  - Compute $\hat{\zeta}(q, n)_l$ for each replication,
  - Average over repl. to obtain the statistical performance of $\hat{\zeta}(q, n)$

- **Asymptotic Behaviours:**
  - The cascade depth increases for a given number of Integral Scales.
  - The number of Integral Scales increases for a given cascade depth,
LINEARISATION EFFECT: $\hat{\zeta}(q) (1/7)$

$q > q_o$, $\hat{\zeta}(q, n) = \alpha_o + \beta_o q$, $q_o, \alpha_o, \beta_o$ ARE RV.
LINEARISATION EFFECT: LEGENDRE TRANSF. (2/7)

\[ D(h) = d + \min_q(qh - \zeta(q)), \] (d EUCLIDIEN DIMENSION OF SPACE).

ACCUMULATION POINTS: \( D_o(h_o) \), WITH \( D_o = d - \alpha_o, h_o = \beta_o \),
\( D_o, h_o \) ARE RV.
• Given resolution, increasing number of integral scales,

• Given number of integral scales, increasing resolution,
**LINEARISATION EFFECT: CONJECTURE (4/7)**

- **Critical Points:**
  \[
  \begin{align*}
  D_\pm^* &= 0, \\
  D(h_\pm^*) &= 0, \\
  h_\pm^* &= (d\zeta(q)/dq)_{q=q_\pm^*}.
  \end{align*}
  \]

- **Results:**
  \[
  EI : \begin{cases}
  \hat{\zeta}(q, n) = d - D_o^- + h_o^- q \quad \rightarrow \quad d - D_\pm^* + h_\pm^* q, \quad q \leq q_\pm^- , \\
  \hat{\zeta}(q, n) &\rightarrow \zeta(q), \quad q_-^- \leq q \leq q_\pm^+, \\
  \hat{\zeta}(q, n) = d - D_o^+ + h_o^+ q \quad \rightarrow \quad d - D_\pm^* + h_\pm^* q, \quad q_\pm^+ \leq q.
  \end{cases}
  \]

  \[
  EII&III : \begin{cases}
  \hat{\zeta}(q, n) = d - D_o^+ + h_o^+ q \quad \rightarrow \quad \zeta(q), \quad 0 < q \leq q_\pm^+, \\
  \hat{\zeta}(q, n) &\rightarrow \zeta(q), \quad q_\pm^+ \leq q.
  \end{cases}
  \]

- **Illustration:**

![Graph](image_url)
LINEARISATION EFFECT: COMMENTS (5/7)

• WHEN DOES THE LINEARISATION EFFECT EXIST?

— FOR ALL TYPES OF CASCADES: CMC, CPC, IDC,
— FOR ALL TYPES OF PROCESSES: $Q_r, A, V_H, Y_H$,
— FOR ALL NUMBERS OF VANISHING MOMENTS: $N \geq 1$,
— FOR ALL MRA-BASED ESTIMATORS: WAVELETS, INCREMENTS, AGGREGATION,
— CAN BE WORKED OUT FOR $q < 0$,
— EXTENDS TO DIMENSION HIGHER THAN $d > 1$. 
EXTENSION: STANDARD WT VERSUS WTMM (1/3).
EXTENSION: 2D MULTIPLICATIVE CASCADE (2/3).
EXTENSION: 3D MULTIPLICATIVE CASCADE (3/3).

3D CMC (LOG NORMAL), EI(1) COMPARED TO A 2D SLICE.
• **When does the Linearisation Effect exist?**
  - For all types of cascades: CMC, CPC, IDC,
  - For all types of processes: $Q_r, A, V_H, Y_H$,
  - For all numbers of vanishing moments: $N \geq 1$,
  - For all MRA-based estimators: Wavelets, Increments, Aggregation,
  - Can be worked out for $q < 0$,
  - Extends to dimension higher than $d > 1$.

• **What the Linearisation Effect is not:**
  - A low performance estimation effect.
  - A finite size effect: The critical parameters do not depend on $n$,
    be it the number of integral scales,
    or the depth (or resolution) of the cascades.
  - A finiteness of moments effect,
    \[-q_c^- < 0 < 1 < q_c^+, \ q - 1 + \varphi(q) = 0,\]
    \[-q_c^- < q_*^- < 0 < 1 < q_*^+ < q_c^+,
\]

• **What the Linearisation Effect might be:**
  - Multiplicative Martingales?
  - Ossiander, Waymire 00, Kahane, Peyrière 75, Barral, Mandelbrot 02.
**LINEARISATION EFFECT: PICTURE (6/7)**

- **TWO POWER-LAWS, TWO FUNCTIONS OF** $q$:

  - **BARE CASCADE**:
    \[
    \mathbb{E}Q_r(t)^q = r^{\varphi(q)}, \quad q \in \mathcal{R}.
    \]

  - **DRESSED CASCADE**:
    \[
    \frac{1}{n_a} \sum_k T_{Q_0}(t_k, a; \beta_0)^q = c_q |a|^{\zeta(q)}, \quad q \in [q_c^-, q_c^+] \\
    \frac{1}{n_a} \sum_k T_{Q_0}(t_k, a; \beta_0)^q = \infty, \quad \text{ELSE},
    \]

  WITH:
  \[
  \begin{align*}
  \zeta(q) &= 1 + q h_{-}^*, \quad q \in [q_c^-, q_*^-], \\
  \zeta(q) &= \varphi(q), \quad q \in [q_*^-, q_*^+], \\
  \zeta(q) &= 1 + q h_{+}^*, \quad q \in [q_*^+, q_c^+].
  \end{align*}
  \]

- **CONFUSION BETWEEN** $\varphi(q)$ AND $\zeta(q)$:

  - **MULTIPLICATIVE CASCADE**: $\varphi(q), \quad q \in \mathcal{R}$,
  - **SCALING EXPONENTS**: $\zeta(q), \quad q \in [q_c^-, q_c^+]$. 

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LINEARISATION EFFECT: CONSEQUENCES (7/7)

- **RECAST THE USUAL GOALS:**

  - **ESTIMATE THE INTEGRAL SCALE AND THE RESOLUTION OF THE CASCADE,**
    \[ \Rightarrow \text{ i.e., FIND A SCALING RANGE } [a_m, a_M] \]
  - **ESTIMATE THE CRITICAL PARAMETERS** \( D^\pm, h^\pm, q^\pm \),
  - **ESTIMATE THE** \( \zeta(q) \) **FOR** \( q \in [q^\pm, q^\pm] \),
ESTIMATION OF $q^*_*$: MAIN IDEAS

Asymptotic Modelling 1: LINEAR FIT OF $\hat{\zeta}(q)$ FOR $q \geq q_{as}$

$A_1(q) = \alpha + \beta q$

Asymptotic Modelling 2: QUADRATIC FIT OF $\hat{D}(h)$ FOR $\hat{D} \geq 0.5$

$A_2(q) = C_0 + C_1 q + C_2 q^2$

$\hat{q}^*_*: \text{when } \hat{\zeta}(q) \text{ shifts from } A_1(q) \text{ to } A_2(q)$

$\left| \hat{\zeta}(\hat{q}^*_*) - A_1(\hat{q}^*_*) \right| = \left| \hat{\zeta}(\hat{q}^*_*) - A_2(\hat{q}^*_*) \right|$

- Numerical Validation: CPC DENSITY, WAVELET ANALYSIS
LINEARISATION EFFECT: CONSEQUENCES (7/7)

**Recast the Usual Goals:**

- Estimate the Integral Scale and the Resolution of the Cascade,
  \[ \Rightarrow \text{i.e., Find a Scaling Range } [a_m, a_M] \]
- Estimate the Critical Parameters \( D^\pm, h^\pm, q^\pm, \)
- Estimate the \( \zeta(q) \) for \( q \in [q^-_*, q^+_*], \)

**Issues:**

- Discrimination of MF Models based on \( \hat{\zeta}(q, n), \)
  \[ ??? \text{Log-Normal versus Log-Poisson} ??? \]
- Discrimination between monoFractal versus MultiFractal.

**Answers:**

- No Need - No Point - No meaning - to use \( q \)s above \( q^+_* \).
NEGATIVE $q$s (1/4)

- **Difficulties?**
  - **Structure Functions** $S_q(j) = \frac{1}{n_j} \sum_{k=1}^{n_j} |d_X(j, k)|^q$
  - **Wavelet Coefficients** $\Rightarrow d_X(j, k) \approx 0$,
  - **Numerical Instability?**
  - **Finiteness**: $\mathbb{E}|d_X(j, k)|^q < \infty$?
  - **Theory**: Weak Hölder Exponent vs Exact Hölder Exponent?

- **Solutions?**

---

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AGGREGATION: \( T_x(a, t) = \frac{1}{aT_0} \int_{t}^{t+aT_0} X(u) du \)

⇒ APPLIES ONLY TO POSITIVE DATA MODELLED BY CONSERVATIVE MEASURE
⇒ DOES NOT SOLVE THEORETICAL ISSUES
NEGATIVE $q$S - SOLUTION 2 (3/4)

- **WT MODULUS MAXIMA** (Arneodo et al.)

\[ L_X(a, t_k) = \text{SUP}_{a' < a} |T_X(a', t_k(a'))| \]

⇒ SOLVES $q < 0$,
⇒ SOLVE THEORETICAL ISSUES?
⇒ COMPUTATIONALLY EXPENSIVE!
**NEGATIVE $qS$ - SOLUTION 3 (4/4)**

- **WAVELET LEADERS:** (JAFFARD ET AL.)

$$d_X(j, k) \rightarrow L_X(j, k) = \sup_{j' < j} d_X(j', 2^{-j'})$$

⇒ **Solves $q < 0$: MultiFractal Spectrum over its Entire Range,**
⇒ **Theor. Issues: MultiFractal Spectrum for Oscillating (Chirp-type) Singularities,**
⇒ **Computationally Efficient and Excellent Statistical Performance,**
⇒ **Straighforward Extension to Higher Dimensions.**
INTERNET DATA: SCALING AT FINE SCALES

• **AuckIV:**

![Graphs showing scaling behavior for different values of q](image)

- **Fine Scales:** ≤ 1s, claimed to be consistent with multi-fractality but ...

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INTERNET DATA: SCALING AT FINE SCALES

- **Estimated Scaling Exponents:** \( \zeta(q)/q = f(q) \),

- **Conclusions:** According to our works,
  - Weak evidence for multifractal at fine scales,
  - Cannot discriminate between self-similarity and multifractality,
  - Even call into question scaling . . . ?
MODEL 3: CLUSTER POINT PROCESS

• DEFINITION:
  – **Ingredient 1:** Poisson Renewal Process, $\lambda_F, \{t_F(i)\}_{i \in I}$, *Flow Arrivals*,
  – **Ingredient 2:** Number of Point per Cluster, $P$, $\mathbb{E}P = \mu_P$, *Pkts per Flow*,
  – **Ingredient 3:** Gamma Renewal Process, $\lambda_A, \gamma_A, \{A(l)\}_{l \in P}$, *Pkts in a Flow*,
  – **Result:** $X(t) = \sum_i \sum_{p=1}^{P(i)} \delta(t - t_F(i)) - \sum_{l=0}^{p-1} A_i(l)$,

• INTUITIONS:
  – **Back to Point Processes — Packet Arrival Processes**,
  – **Flow and Packet Levels**,  
  – **Physical Parameters Relevant to Traffic Modelling**, 
  – **Heavy Tails ($\beta$) in Pkt Numbers $P$ — Long Range Dependence $H = 2 - \beta$. 

[78]
Cluster Point Process and Scaling?

- **Fit**: Adjust CPP Params $\lambda_F, \mu_P, \beta, \lambda_A, \gamma_A$ to Data,

Data

<table>
<thead>
<tr>
<th>$q$</th>
<th>$q = 0.5$</th>
<th>$q = 1$</th>
<th>$q = 4$</th>
<th>$q = 6$</th>
</tr>
</thead>
</table>

- **Gamma Renewal**: Fit Marginals,
- **Heavy Tail in Pkt Nb → Long Range Dependence at Coarse Scale**, 
- Reproduce the Two Ranges and the Separation Position,
- **Mimic Scaling (MultiFractal)** at Fine Scales, BUT,
- Theoretically **Not Scaling** at Fine Scales, BUT,
- Actual Practical Analysis Tools Cannot Discriminate.
CONCLUSIONS

- **Scaling in Computer Network Traffic?**
  - Two ranges of scales,
  - Long range dependence at coarse scales,
  - Scaling at fine scales? Controversial!
  - If scaling at fine scales: MultiFractal? Controversial!
  - Valid at packet and flow levels,
  - Cluster point process,
  - Future evolution (Internet is a living beast)?

- **Analysing Scaling in Data? Think Wavelet**
  - Conceptual adequation,
  - Practical simplicity,
  - Robustness.

- **MultiFractal Analysis?**
  - A very intricate issue!!

- **Theoretical Analysing of Scaling?**
  - Goodness of fit: Scaling or not scaling,
  - Goodness of fit: Self-similarity vs MultiFractal,
  - Parameter estimation.
REFERENCES AND RESOURCES

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