Phase extraction in speckle interferometry by a circle fitting procedure in the complex plane

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In speckle interferometry (SI), temporal signals are amplitude- and frequency-modulated signals and exhibit a fluctuating background. Prior to phase computation, this background intensity must be eliminated. Here our approach is to build a complex signal from the raw one and to fit a circle through the points cloud representing its sampled values in the complex plane. The circle fit is computed from a set of points whose length is locally adapted to the signal. This procedure—new to our knowledge in SI—yields the background and the modulation depth and leads to the determination of the instantaneous frequency. The method, applied to simulated and experimental signals, is compared to empirical mode decomposition (EMD). It shows great robustness in the computation of the sought quantities in SI, especially with signals close to the critical sampling or, on the contrary, highly oversampled, situations where the background elimination by EMD is the most prone to errors.

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It is widely acknowledged that speckle interferometry (SI) techniques are powerful tools to characterize rough surface deformations, in static or quasi-static regimes, when small displacements are involved (typically below $10\,\mu$m or so). The case of dynamic regimes with likely large displacements (of the order of hundreds of micrometers) is still an actual research topic. The classical fringe analysis techniques, including the otherwise very efficient phase-shifting techniques [1], turn out to be unable to address this issue mainly because of decorrelations, which are unavoidable in any SI measurement. Decorrelations limit the range of measurement to the correlation volume, i.e., the speckle grain, and spoil the results by random errors. The temporal approach consists in analyzing the evolution of the interferometric signal of each pixel of the recording sensor. Some data manipulation is required to build the temporal pixel signals, but it allows us to get rid of the correlation limit, without of course eliminating the induced random phase errors. Genuine experimental temporal pixel signals are shown in Figs. 1 and 4.

The analysis of such signals with the Morlet wavelet transform gives outstanding results especially when implemented through a ridge tracking algorithm [2]. This method has shown a high noise rejection power, but the convergence of the algorithm might be an issue when the signal spectrum covers a large spectral bandwidth. To overcome this issue, a totally different approach has been followed in [3], resorting to empirical mode decomposition (EMD) [4] with the goal to put the signal into the *ad hoc* shape for subsequent phase extraction by the analytic method (AM), i.e., the use of the Hilbert transform as a quadrature operator. For SI signals, it boils down to eliminate the fluctuating background intensity. This method, data driven, efficient, and which implies very simple operations [$O(n)$ complexity], relies on the computation of the signal envelopes, based itself on the seeking of the signal extrema, a delicate operation particularly sensitive to sampling conditions and noise.

In this Letter we report a different strategy to exploit the analytic signal in the complex plane. The basic idea is to process directly the signal as it is, instead of forcing it to be centered. The first step is to build a complex valued signal $z[k]$ from the original signal $s[k]$ by means of the Hilbert transform as follows:

$$z[k] = s[k] + i\mathrm{HT}\{s[k]\}. \quad (1)$$

It is useful to remind that the use of the Hilbert transform as a quadrature operator is subject to requirements that are not fulfilled here. The goal is simply to place the signal in the complex plane and not to compute a phase at this early stage.

At each instant k, we consider a set of N_k points $Z_k = \{z[p]\}$, with p belonging to the interval $[k \ldots k + N_k]$. A circle is then fitted through the cloud of data points by the Kåsa method (see Fig. 1), described further in the text. From the computation of the osculating circle, a local determination of the background (the abscissa of the circle center a_k), the modulation depth (the circle radius R_k), and the instantaneous frequency (IF) is obtained. Similarly to what has been outlined in [5] and considered

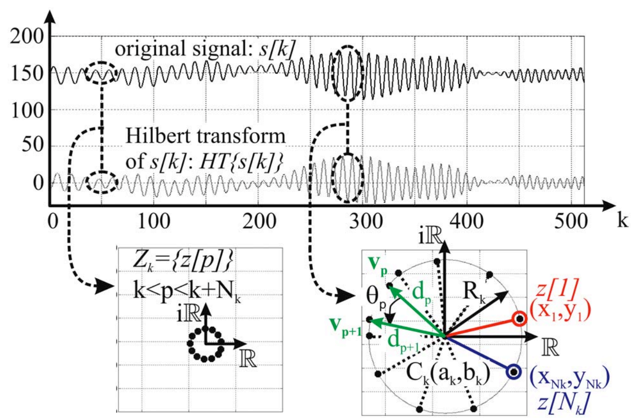

Fig. 1. (Color online) Principle of the method of the osculating circle to the signal trajectory in the complex plane.

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independently in [6], the IF, ν_k, said to be circular, can be calculated by the mean value of the angles between consecutive radii of the fitted circle, designated by θ_p in Fig. 1. The knowledge of the IF leads directly to the phase by numerical integration.

Adjusting a circle to a points cloud in a least-square sense is mathematically formalized as follows:

$$\min \sum_{p=1}^{N_k} \left\{ \sqrt{(x_p - a_k)^2 + (y_p - b_k)^2} - R_k \right\}^2, \tag{2}$$

where (x_p, y_p) are the data coordinates in the complex plane. The index k refers again to the frame number and reminds us of the fact that the computation is carried out at this instant and is thus valid for this very moment. The problem stated in Eq. (2) is nonlinear and can be solved by Gauss–Newton-like iterative methods. Minimizing the following quantity J_k is an alternative to Eq. (2):

$$J_k = \sum_{p=1}^{N_k} \{(x_p - a_k)^2 + (y_p - b_k)^2 - R_k^2\}^2. \tag{3}$$

It has been demonstrated in [7] that this latter formulation comes to a linear least-squares problem, simpler and faster to solve and, last but not least, more robust to outliers. The cancellation of the derivates of J_k with respect to a_k, b_k, and R_k gives immediately a 3×3 matrix equation whose inversion leads to the sought-after circle parameters. Kåsa derived the equations from this latter formulation in [8] and also proposed a thorough error analysis. A slightly different formulation is given in [9] for a significant gain of accuracy when the data are localized on a small circle arc, but at the cost of an increased computation load.

To minimize the fitting errors, the length of the data set, N_k, must be adjusted to the signal, so that n local periods are covered. The angle θ_p is computed for each couple of vectors $(\boldsymbol{v}, \boldsymbol{v}_{p+1})$ as follows (see Fig. 1):

$$\theta_p = \arctan(\det(\boldsymbol{v}, \boldsymbol{v}_{p+1}) / (\boldsymbol{v} \cdot \boldsymbol{v}_{p+1})). \tag{4}$$

The algebraic mean of the angles θ_p gives the average IF in the considered data set. The length of the data set is adjusted from one instant k to the next by

$$N_{k+1} = 2n\pi / \nu_k. \tag{5}$$

Choosing n is making a trade-off between temporal and spectral localizations, as usual. Moreover, it is possible to smooth further N_k with a moving average over its past values. In addition, carrying out the computation every u_k time samples may still reduce the computation load. Finally, SI signals, in the dynamic regime, feature unavoidable losses of modulation. In those regions of low signal-to-noise ratio (SNR), the extracted phase is spoiled by random errors and possibly completely meaningless. It is thus preferable to discard the points located too close from the circle center, according to an empirically set threshold (typically taken equal to five gray levels, which corresponds in our case to three times the standard deviation of the black signal of the camera), and

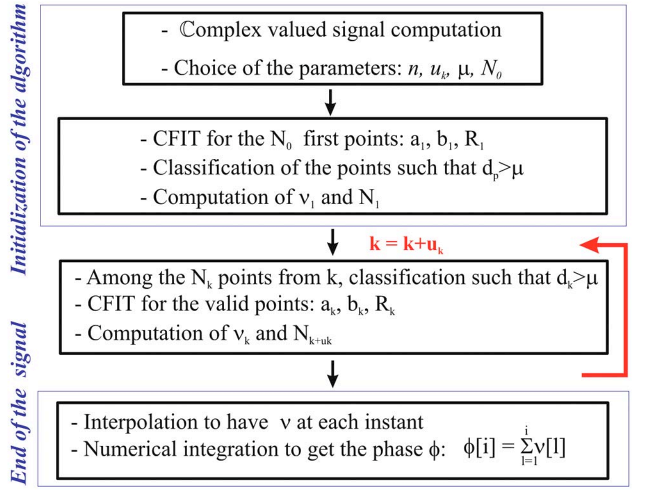

Fig. 2. (Color online) CFIT procedure of phase extraction in SI.

to keep all the algorithm parameters unchanged for the next instant. We incidentally mention that a solution to this modulation loss issue in SI has been proposed in [3,10]. The circle fit (CFIT) procedure is summarized in Fig. 2.

The CFIT procedure necessitates at least three points, and it is thus not possible to process critically sampled signals. In the same manner as what has been done in [11], the sampling influence on the EMD and the CFIT procedure is studied. To quantitatively evaluate the influence of sampling, we consider a single tone embedded in white noise, defined as follows:

$$s[k] = \cos(2\pi k / 2^r + \varphi_r) + \sigma[k], \tag{6}$$

where $1/2^r$ is the tone frequency, φ_r a random phase term uniformly distributed over $[-\pi, \pi]$, and σ the noise. The comparison criterion is the standard deviation of the quantity $\delta\phi$ defined below:

$$\delta\phi = \langle \phi_{\mathrm{ex}} - 2\pi k / 2^r \rangle_{\varphi_r}, \tag{7}$$

where ϕ_{ex} stands for the extracted phase either with the CFIT method or with the combination of EMD and AM.

The simulation is actually carried out for 20 realizations of the random phase term ϕ_r. Figure 3 illustrates the behavior of the two methods with respect to the

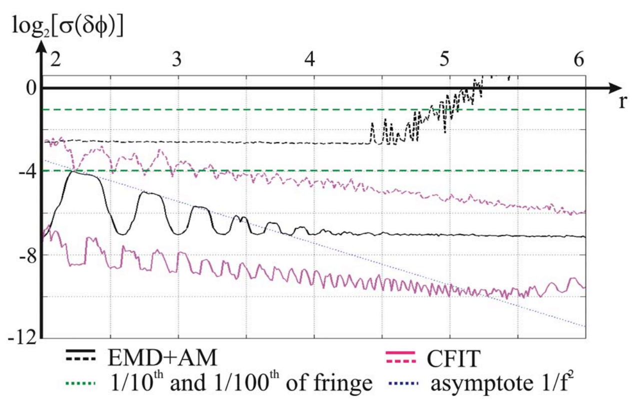

Fig. 3. (Color online) Phase error as a function of the sampling frequency.

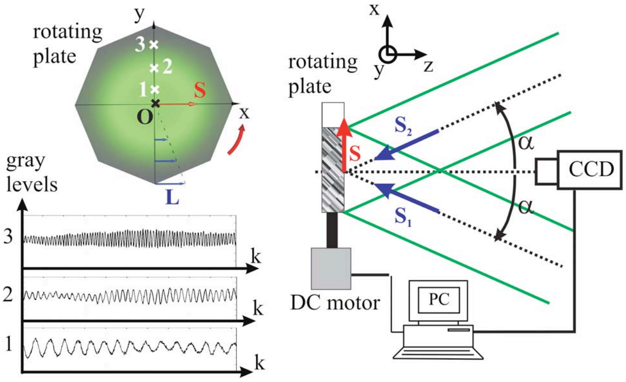

Fig. 4. (Color online) Leendertz SI setup with in-plane sensitivity vector **S**.

sampling frequency when the SNRs of 30 dB (solid curves) and 3 dB (dotted curves) are considered. For comparison purposes, we also placed in Fig. 3 the error corresponding to a tenth and a hundredth of fringe (green dashed curves). Figure 3 clearly evidences the potential benefits of CFIT.

The proposed technique has been further tested on experimental SI signals. A Leendertz-type setup with in-plane sensitivity is used to measure the continuous rotation of a rough metal plate around the z axis (see Fig. 4). Temporal signals located at different distances from the rotation center O are also shown. In such an experiment, the equiphase lines are parallel to the sensitivity vector, i.e., parallel to the x axis.

We compare in Fig. 5 the results obtained with the proposed method CFIT (thick blue line) and the combination of EMD and AM (thin black line). We observe a much better rendering of the central part of the plate with the CFIT technique, where the pixels experience few fringes of displacement. The proposed method allows us thus to extract the phase from nonstationary SI signals on a much wider bandwidth than the AM associated with EMD. On the computation load aspect, the combination of EMD with the AM keeps a slight advantage over the CFIT method, the latter being not yet optimized. To fix the ideas, processing 200 temporal pixel signals located along the dotted lines in Fig. 5 takes 7 s with the "EMD + AM" method (on a PC equipped with a Q9400 CPU and 3 GB of RAM) and 25% more with the CFIT technique, with the parameters u_k, μ, and n set to $N/2$, 5, and 2, respectively (parameters taken to obtain the results shown in Fig. 5).

As already noticed, a filtering technique based on the Delaunay triangulation has been proposed in [3] as a possible solution to the modulation loss issue specific to SI. Any phase extraction method has to cope with this issue, and the outlook is now to combine CFIT with the Delaunay interpolation method in order to optimize conjointly the spatial resolution and the accuracy of the phase extraction.

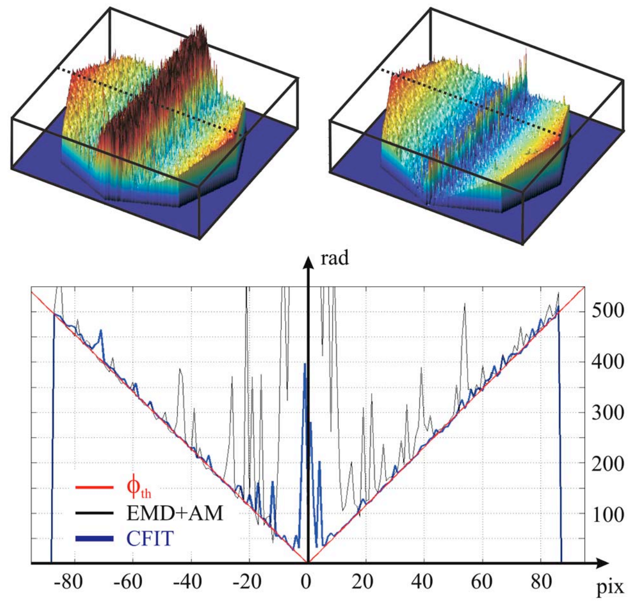

Fig. 5. (Color online) Phase maps corresponding to the total displacement (given in radians) and the cross sections (along dotted lines) shown with the theoretical phase.

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References

1. K. Creath, Appl. Opt. **24**, 3053 (1985).
2. M. Cherbuliez, P. Jacquot, and X. Colonna de Lega, Proc. SPIE **3813**, 692 (1999).
3. S. Equis and P. Jacquot, Opt. Express **17**, 611 (2009).
4. N. E. Huang, N. E. Huang, Z. Shen, S. R. Long, M. C. Wu, H. H. Shih, Q. Zheng, N. C. Yuen, C.-C. Tung, and H. H. Liu. Proc. R. Soc. A **454A**, 903 (1998).
5. D. Aboutajdine, M. Najim, and J. G. Postaire, in *Signal Processing, Theories and Applications, Proceedings of EUSIPCO-80, First European Signal Processing Conference, Lausanne, Switzerland, September 16–18, 1980*, M. Kunt and F. De Coulon, eds. (North-Holland, 1980), p. 57.
6. P. F. Pai, Adv. Adapt. Data Anal. **2**, 39 (2010).
7. I. D. Coope, J. Optim. Theory Appl. **76**, 381 (1993).
8. I. Kåsa, IEEE Trans. Instrum. Meas. **IM-25**, 8 (1976).
9. N. I. Chernov and G. A. Ososkov, Comput. Phys. Commun. **33**, 329 (1984).
10. S. Equis and P. Jacquot, Proc. SPIE **7387**, 738709 (2010).
11. G. Rilling and P. Flandrin, in *IEEE International Conference on Acoustics, Speech, and Signal Processing* (IEEE, 2006), p. 444.