

Multifractal analysis and authentication of Jackson Pollock paintings

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ABSTRACT

Recent work has shown that the mathematics of fractal geometry can be used to provide a quantitative signature for the drip paintings of Jackson Pollock. In this paper we discuss the calculation of a related quantity, the “entropy dimension” and discuss the possibility of its use as a measure or signature for Pollock’s work. We furthermore raise the question of the robustness or stability of the fractal measurements with respect to variables like mode of capture, digital resolution, and digital representation and include the results of a small experiment in the step of color layer extraction.

Keywords: fractal dimension, multifractal dimension, stylometry

1. INTRODUCTION

Mathematics, computer science, and statistics are beginning to play a larger role in the field of art history and connoisseurship. This is largely due to advances in imaging technology which now make possible the relatively inexpensive high resolution digital capture of a work of art. Once transformed to numbers, a work of art becomes a rich source of data, open to the methods of modern mathematics and computational analysis. In this setting it is natural to ask if resident in the numbers that represent a work on a computer (henceforth called the *digital representation* of the work) lies a numerical or mathematical signature characteristic of the creator of the work, i.e., is there perhaps a way to quantify the style of an artist? In the case of the literary arts there has, over the past hundred years or so, been great progress on this question, and an active field of *stylometry* now exists, which uses methods of advanced statistical analysis as applied to the numbers derived from various kinds of word counts as a means of identifying an author or deriving a quantitative measure of stylistic or intellectual development over a writer’s career. In the visual arts we are at the beginnings of such a field and make no mistake, it is coming (see, e.g.,¹⁻³).

To date, the most striking result toward a visual stylometry has been the work of physicist Richard Taylor in his use of the mathematics of *fractal geometry* for the study of the drip paintings of Jackson Pollock. Taylor has used fractals to track the development of Pollock’s work⁴ and of late, has begun to tie the numbers produced by a *fractal analysis* of a Pollock painting to the unique interplay of physiology and artistry that Pollock employed in making his drip paintings. Most famously however, in the employ as a consultant to the Pollock-Krasner Foundation, Taylor has used his mathematical methods as a means of authentication, finding particular fractal properties that seem characteristic of Pollock and virtually unreproducible by an artist hoping to pose as Pollock. The most newsworthy of these stylometric authentications was Taylor’s recent analysis that cast significant doubt on the provenance of a sampling of a cache of dripworks in a storage bin in Long Island, New York.¹

This paper continues the examination of the fractal methodology as a tool for Pollock stylometry. On the one hand we aim to elaborate on Taylor’s work, showing how the calculation of the “entropy dimension” of the

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painting, a quantity related to fractal dimension, may also be of interest as a means of providing yet another quantification of Pollock's drip style. On the other hand, we also perform something of a stability analysis of these fractal-based techniques. That is, any computational procedure involves choices, properties like the resolution of the digital representation or means of color extraction and it is important to ask to what degree - if any - the results depend on these choices (this is the first question a connoisseur should ask regarding the introduction of these digital technique into the arena of authentication). We believe that such analyses are necessary for pushing the field forward.

2. TECHNICAL BACKGROUND

The application of fractal techniques in arts begin with high resolution digital scans or photographs of the paintings. Key to fractal analysis is the notion of *dimension* of a fractal. The standard way to understand dimension of an object is from its scaling property. For example, we generally consider a line segment to have dimension 1, a square to have dimension 2 and a cube to have dimension 3. This translates nicely into scaling properties. If we double the size of these objects, the length of the line segment becomes 2 times the original length; the area of the square becomes $2^2 = 4$ times the original area; the volume of the cube becomes $2^3 = 8$ times the original volume. But some objects can be more intriguing. Take the Sierpinski gasket (Figure 1) as an example.* If we double its size we end up with 3 Sierpinski gaskets of the original size. So what is the dimension of it? Rather naturally we consider its dimension to be the number α such that $2^\alpha = 3$, which is $\alpha = \log 3 / \log 2 \approx 1.585$.

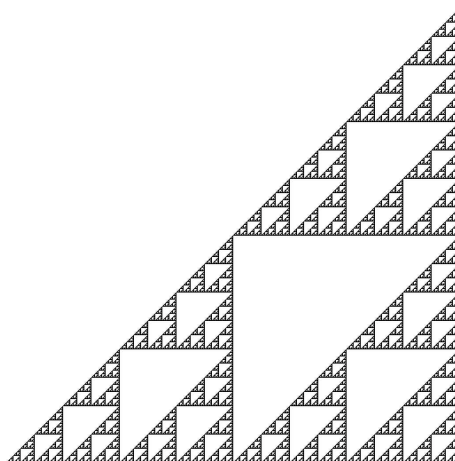


Figure 1. The Sierpinski gasket

It is not always as easy to find the dimension of an object. The most commonly used scheme for finding the dimension is the box-counting scheme. Since we are only concerned with paintings we shall demonstrate how this scheme works by looking at objects in the plane. In the box-counting scheme, the plane is divided into square grids of certain size, and we look for the number of grid squares that contain pieces of our objects. For example, assume our object is a right triangle of sides 1, 1, $\sqrt{2}$ and the size of the grid is $1/10$ as shown in Figure 2, we get 55 grid squares that contain pieces of the triangle. After this counting we compute the number $\log 55 / \log 10 \approx 1.74$. Now if we reduce the size of the grid to $1/100$ it is not hard to figure out that there are now 5050 grid squares that contain pieces of the triangle. We now have $\log 5050 / \log 100 \approx 1.85$. We can continue this process with increasingly reduced grid size, and we will notice that the number we compute gets closer and closer to 2, which is the dimension of the triangle we are computing. For example, if we make the grid size to be $1/100000$ then there are 5000050000 grid squares that contain pieces of the triangle. The number $\log 5000050000 / \log 100000$ is approximately 1.94. This box-counting scheme can be applied to the Sierpinski gasket in Figure 1. If we make the grid size to be $1/2^k$ it is not difficult to count that there are 3^k grid squares that contain pieces of the

*See http://en.wikipedia.org/wiki/Sierpinski_triangle for a detailed description of its construction.

Sierpinski gasket. The corresponding numbers are $\log 3^k / \log 2^k = \log 3 / \log 2$, regardless of k . So the dimension is $\log 3 / \log 2$.

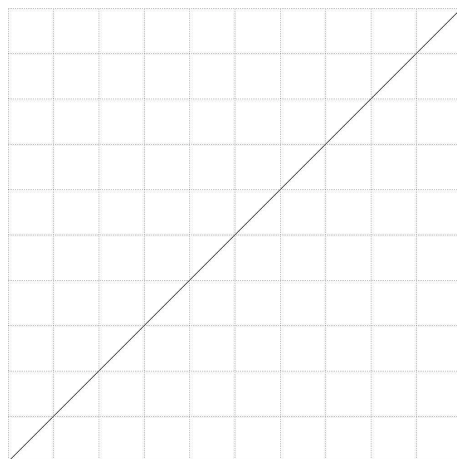


Figure 2. Basic grid showing the boxes for a single iteration of box-counting. The Sierpinski gasket would fit in the lower triangle covered by 55 boxes.

The dimension computed from the above box-counting scheme is called the *box-counting dimension*. It often yields important clues about a fractal. Unfortunately in real world applications we do not have the luxury of infinitely small scales, as the scales are limited by the resolution. This is certainly the case for digital images. Nevertheless the same box-counting scheme can be adapted to obtain the dimension of a fractal image even with limited resolution. To understand how it works, we consider the case in which our digital image is a binary black and white image, where the white pixels represent the background and the black pixels represent the fractal image to be analyzed. To compute the box-counting dimension of the fractal we first subdivide the image into squares of side length k . Let N_k denote the number of these squares that contain black pixels. We typically start from a small k and gradually increase it to a suitably large value. As we gradually increase k the points $(\log N_k, -\log k)$ are plotted. This plot is known as the *box-counting dimension plot*. A fractal image is effectively defined as one for which these points lie essentially on a straight line, and we designate the slope of the best fitting line as the box-counting dimension of the fractal. Note that if these points lie (approximately) along a line of slope d , the implication is that roughly k^d squares with a side of length $1/k$ are needed to cover the (black) image.

The box dimension of a fractal is a way to measure its overall density and distribution. A good practical example is to look at the location plot of cell phone towers. There are currently around 175,000 cell phone towers in the country. These towers are not uniformly scattered around the country. If we divide the country into 10 miles by 10 miles grids, then we divide the country into roughly 40,000 grid squares. We may count, say, 6,000 of them that contain cell phone towers. If we make the grid size to be 40 miles by 40 miles we may only count 300 grid squares that contain cell phone towers. Thus the box-counting dimension plot tells us in certain quantitative way that some places are rather densely packed with cell phone towers while other places have none. Applied to drip paintings, one may hope that the box-counting dimensions will yield quantitative information that lead to digital signatures. This turns out to be quite naive. Box-counting dimensions alone do not yield enough significant information. A much more powerful technique is to examine the box-counting dimension plots. By analyzing the breaks and other geometric properties of these plots a wealth of information can be obtained. Such is the foundation of the analysis performed by Taylor et al.^{3,5} for the drip paintings by Jackson Pollock. This analysis has ultimately led to what they believe to be digital signatures of Pollock.

While the box-counting dimension is an important measurement of the density, a fractal set can be much more intricate in the sense that the geometric properties within the fractal can vary from locality to locality. The density within a fractal may vary with locality, so at each locality we may see a different dimension, the so-called *local dimension*. Using the cell phone towers as the example again. A 10 miles by 10 miles grid square

in a major metropolitan area like New York City may contain 60 towers while a 40 miles by 40 miles grid square may contain more than 1,000 towers. By contrast, any given 40 miles by 40 miles grid square in rural areas of Montana may only contain five or less towers. Clearly if we view the location plot of cell phone towers as a fractal, the density of towers in New York City is far higher than that of in rural areas of Montana. Translated into mathematical language, the local dimension in the New York City is greater than the local dimension of any rural area of Montana. Clearly a single measurement such as the box-counting dimension does not capture these local intricacies. To overcome this limitation more sophisticated techniques are needed. "Multifractal analysis" is a broad term used to describe these more sophisticated techniques. Intuitively, we may view a fractal as an intricate weave of many "subfractals," each of which has a homogeneous local dimension. These subfractals form the so-called *multifractal*. With the cell phone tower example, fractal representing the location of towers will be decomposed into various subfractals according to local dimensions. Big metropolitan areas such as New York City and Los Angeles have high local dimensions, and these areas will be grouped together to form a subfractal. The rural areas of Montana, Wyoming and their neighbors have similarly low local dimensions, and they will be grouped together into another subfractal. Of course, there will be other local dimensions in between, which lead to other subfractals. This allows us to decompose the original fractal into subfractals, each of which contain portions that are rather homogeneous in terms of tower density. Multifractal analysis refers to the analysis of these subfractals that constitute the multifractal. It is one of the most powerful techniques in fractal analysis.

A standard part of multifractal analysis involves finding the dimensions as well as the distributions of the subfractals. These analyses yield very useful informations about the fractal that the box-counting dimension alone cannot. In the cell phone tower example, the analyses of the subfractals can tell us roughly how many towers are in the high density areas and where they are distributed, for example. Applied to drip paintings such as those by Pollock it is possible to gain some intrinsic knowledge that other techniques cannot. This comprehensive multifractal analysis, if carried out correctly, will require higher quality digital scans of the paintings both in terms of resolution and color accuracy. We plan to carry out such an analysis for Pollock drip paintings in our next project once we overcome some of the limitations. At this time, however, there are other techniques in multifractal analysis that are less constrained by the quality of the digital scans. One of them is the so-called L^q -dimension of the fractal, where q is any nonnegative real number.

We shall not discuss the mathematical theory of L^q -dimension in detail here. Those who are interested in it can find discussions in the paper⁶ and the references therein. Instead we explain how to compute the L^q -dimension for a binary image. As with the box-counting dimension, we subdivide the image into square grid of side length k . The key difference is that while for the box-counting dimension we are only interested in whether a grid square contains any black pixels, for the L^q -dimension we take into considerations how many black pixels are in a grid square. The grid squares containing more black pixels will count more than those that containing fewer. More precisely, for each grid square we first count the number of all black pixels. These pixel counts are then raised to the q -th power and they are summed up. We use $C_{q,k}$ to denote this sum. In mathematical language label the grid squares of size k as S_1, S_2, \dots, S_m . Let b_j be the number of black pixels in the square S_j . The value $C_{q,k}$ is defined as

$$C_{q,k} = \sum_{j=1}^m b_j^q.$$

Like the box-counting dimension plot, we start with a small k and gradually increase k until it is suitably large. As we increase k the points $(\log C_{q,k}, -\log k)$ are plotted. For a fractal set we expect to see these points to lie on a straight line. The slope of the best fitting straight line, divided by the number $1 - q$, is defined to be the L^q -dimension of the fractal. In practice, we actually plot the points $(\log C_{q,k}, (q - 1) \log k)$. The slope of the best fitting line will give us the L^q -dimension. This eliminates the need for division by $1 - q$.

Observe that if $q = 0$ then $C_{q,k}$ is the same as N_k in the box-counting dimension, so D_0 is exactly the box-counting dimension. Thus the box-counting dimension is a special case of the L^q -dimension with $q = 0$. Another special case is $q = 1$. Note that in this case the number $C_{q,k}$ is exactly the number of black pixels, regardless of the size k of the square grid. Furthermore, $q - 1 = 0$. Thus the L^1 -dimension plot $(\log C_{q,k}, (q - 1) \log k)$ consists of a single point, which is meaningless. To overcome this problem we modify the formula slightly. Instead of letting using $C_{1,k} = \sum_{j=1}^m b_j^1$, which is simply the total number of black pixels in the image, we consider E_k

defined by

$$E_k = \sum_{j=1}^m b_j \log(b_j).$$

Plot the points $(\log E_k, -\log k)$. The slope of the best fitting straight line of the plotted points is called the *entropy dimension*. It is used in place of the L^1 -dimension. Our experiments seem to show that entropy dimension plots constitute an intriguing alternative to box-counting dimension plots.

While the box-counting dimension plots are a quick and easy way to study paintings such as those by Pollock, the L^q -dimension plots will add a sophisticated new tool to fractal techniques in arts, as we shall demonstrate later with our analysis of known drip paintings by Pollock and those found in the Matter Estate. Strictly speaking computing the L^q -dimensions cannot be classified as a true multifractal technique. The L^q -dimension for each q , if standing alone, plays a very similar role as the box-counting dimension, i.e. it only measures certain global property of the fractal. The key is to examine the behavior of the L^q -dimension plots for a broad spectrum of values q . As the value q becomes larger the L^q -dimension will be dominated increasingly by the higher density portion of the fractal. The cell phone towers example illustrates this point. For $q = 0$ in the box-counting dimension case, all it matters is whether a grid square contains a tower. So a 10 miles by 10 miles grid square over Manhattan containing 60 towers has a count of 1, the same as a grid square in the rural areas of Montana containing one tower. However, for $q = 0.5$ the Manhattan square will have a count of $60^{0.5} \approx 7.746$ while the Montana square has a count of 1. For $q = 3$ the corresponding counts are $60^3 = 216,000$ and $1^3 = 1$, respectively. It is easy to see that the bigger q is, the more dominant the L^q -dimension is by areas covered in high density with cell phone towers. It is hoped that this transition as one varies the value of q will help to uncover informations about the fractal that are not available through analysis of the box-counting dimension alone.

This approach was first adopted by Mureika et al.^{7,8} to study paintings by Pollock and the group Les Automatistes. In their work the L^q -dimensions of some paintings were compared, although no digital signature was found. Our approach differs significantly from theirs in our emphasis: While theirs focused on finding the dimensions, ours focus on the geometric properties of L^q -dimension plots, particularly the entropy dimension plots. We believe these geometric properties offer a more robust way to uncover digital signatures, since the value of the dimensions D_q can not be calculated precisely as a result of imperfections in the scanning and processing, as well as human subjectiveness in the mathematical model. Our view seems to be supported in part by the work of Taylor et al.,⁵ in which by looking at the properties of the box-counting dimension plots a set of digital signatures for Pollock's drip paintings was obtained.

3. MULTIFRACTAL ANALYSIS OF POLLOCK'S DRIP PAINTINGS

Courtesy of the New York Museum of Modern Art we have obtained high resolution digital images of several Pollock drip paintings. They are *Number 1, 1948 (1948)*, *White Light (1954)*, *Gothic (1944)*, *Full Fathom Five (1947)*, *Echo Number 25 (1951)* and *Number 1 (1950)*. These digital photos were captured using a 16 MP Phase One medium format digital camera.

Our multifractal analysis of Pollock paintings involves the study of L^q -dimension plots. We first crop out the unwanted areas in these images. Since each Pollock painting uses a small number of different paints, like Taylor et al we first decompose each image into several *color layers*. Each color layer is itself a black and white binary image but contains only the pixels of a particular color. These color layers are extracted automatically by looking at clusterings in the RGB color space. Figure 4 shows the extraction of a white layer from "Number 1, 1948." Note that in the extraction, the white becomes black and all pixels not extracted as white as turned into white.

The L^q -dimension plots are then made for a broad range of q , starting from box-counting dimension $q = 0$ to a rather large value of q . We have paid particular attention to the entropy dimension plots. In each L^q -dimension plot we let the size of squares be $5 \cdot (1.1)^n$ where n goes from 0 to around 50, depending on the size of the size of the digital image.[†]

[†]In principle, all color layer extractions could be accomplished in any standard image processing package. All of our image manipulation and subsequent calculations were accomplished using software written by J. Elton. We are happy to provide the analysis code upon request.

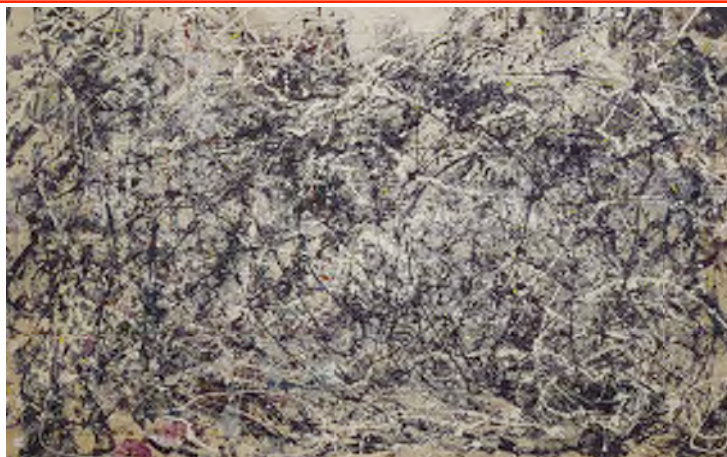


Figure 3. "Number 1, 1948," J. Pollock

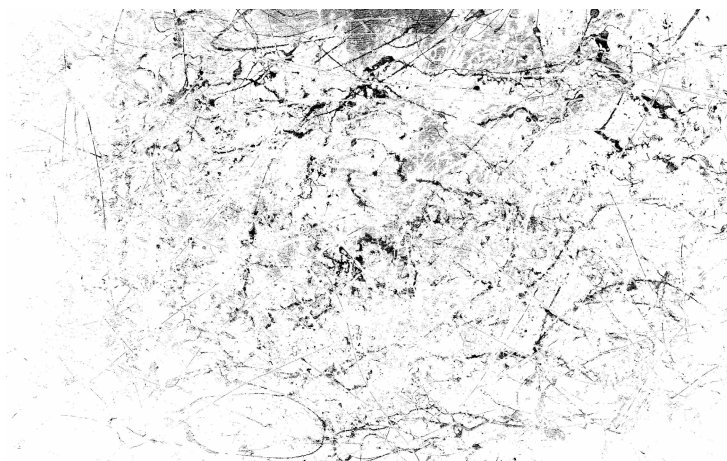


Figure 4. White layer extraction from "Number 1, 1948," J. Pollock

With the exception of the entropy dimension plots, the L^q -dimension plots all show essentially a straight line within the scale range we have chosen for the broad range of q 's tested. The L^q -dimensions, using the least square best fit straight lines, range rather widely. There does not seem to have a particular signature from the values of the L^q -dimensions, at least none we could see.

The entropy dimension plots are much more intriguing. They are all essentially on a straight line until the size of the square reaches a certain threshold, after which the points start to oscillate. These oscillations seem to follow a very similar pattern. Our study suggests that this pattern of oscillation may be a digital signature of Pollock. Figure 5 shows a representative collection of plots from "Number 1, 1948."

In May 2005 it was announced that Alex Matter, son of friends of Jackson Pollock, has discovered a cache of over 30 paintings given by Pollock to his family. The authenticity of these paintings have not been settled. Taylor, using his fractal technique, has declared these paintings to be fakes, but his claim has not been universally accepted.

To see whether our multifractal techniques can shed some light in this dispute, we used two of these paintings published in the New York Times to compare with the known Pollocks we have analyzed. In addition, we also analyzed a drip painting by Pollock's wife Lee Krasner, the image of which we have obtained along with those of Pollock's from MoMA. There were some initial concerns about the paintings from the Matter Estate because we scanned them from the NYT. However, since the oscillations in the entropy dimension plots appear at a

relatively large scale, we feel sufficiently comfortable with these plots.

As one can see in Figure 6, the oscillations in the entropy dimension plots for both the Krasner and the paintings from the Matter Estate differ substantially from that in Pollock paintings. Our analysis seems to support Taylor's analysis, that these paintings are fakes.

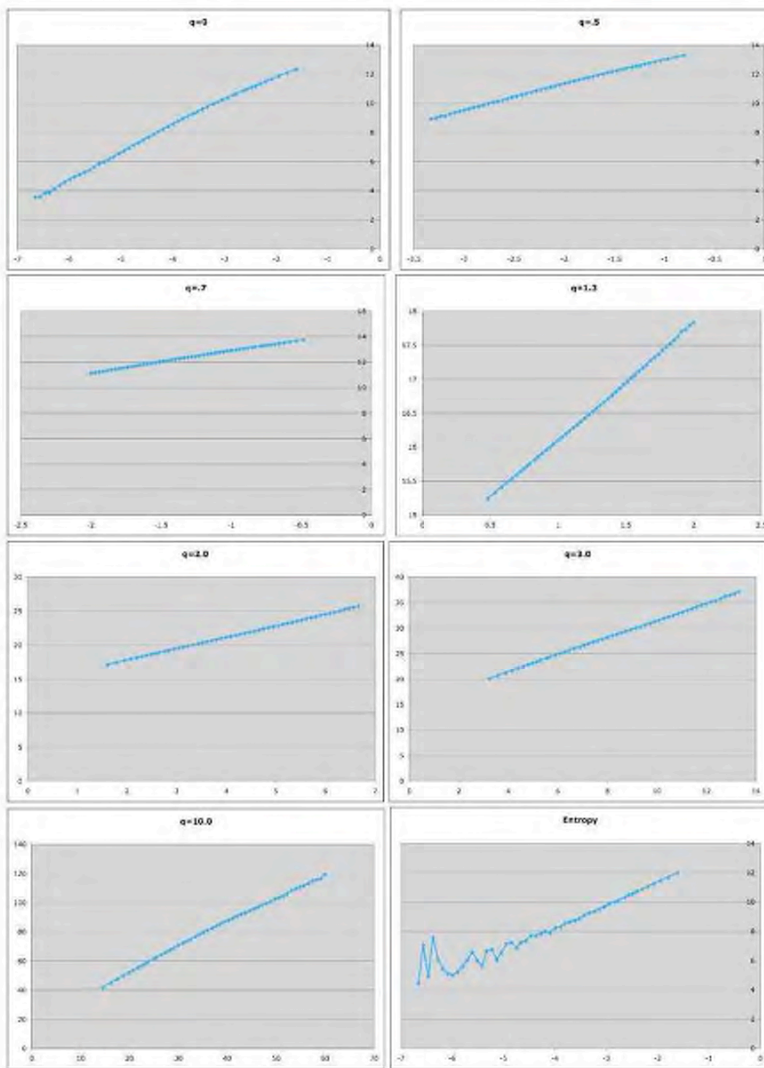


Figure 5. "Box-counting" plots from a white layer of "Number 1, 1948," J. Pollock. Values for q range (left to right and top to bottom) among 0, 0.5, 0.7, 1.3, 2.0, 3.0, 10.0 and the entropy plot. The generally linear form of the plots indicates a well-defined limit and hence fractal dimension for each value of q although it is possible to find a "breakpoint" in the line as suggested in.³ Note that the entropy plot displays interesting initial "jitter" until settling down to linear behavior. The general form of these plots was found over a range of layers and paintings.

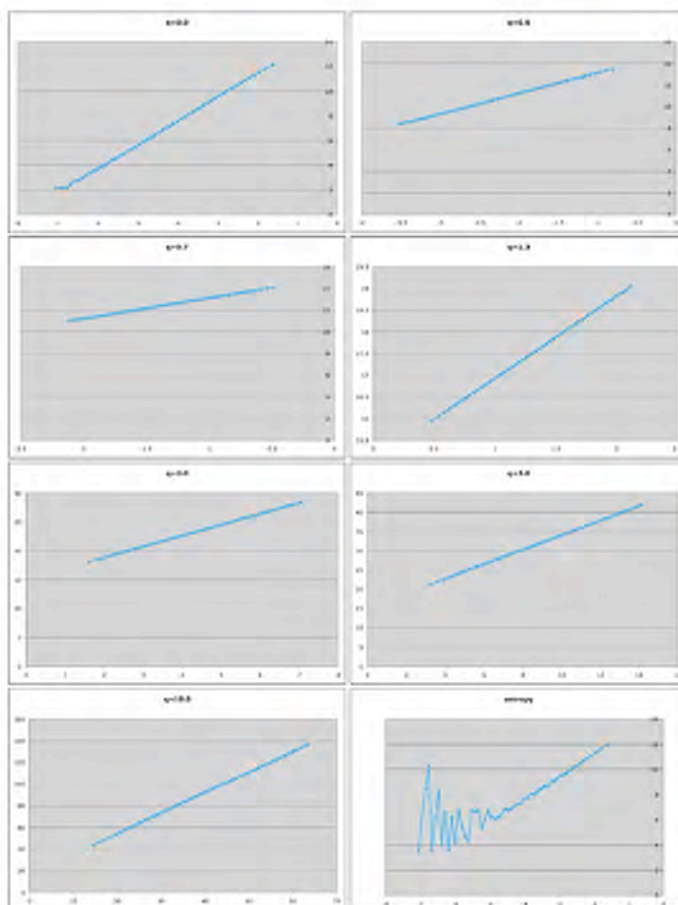


Figure 6. "Box-counting" plots from the black layer of a widely circulated image of one of the "Matter-Pollocks" (see eg., The New York Times...). Values for q range (left to right and top to bottom) among 0, 0.5, 0.7, 1.3, 2.0, 3.0, 10.0 and the entropy plot. The generally linear form of the plots indicates a well-defined limit and hence fractal dimension for each value of q although it is possible to find a "breakpoint" in the line as suggested in.³ Note that the entropy plot displays a much more widely varying initial "jitter" until settling down to linear behavior. It is in particular, this entropy plot where we see a dramatic distinction between a secure Pollock and this drip painting found by Alex Matter.

4. SLOPE STATISTICS

In³ there is an indication that the box counts that form the initial data for the calculation of fractal dimensions may in fact indicate two distinct regimes of fractal dimension, perhaps reflecting a two-step process of composition of background and detail. In this section we explore this idea, taking the box count statistics given in Section 3 for the white layer of One 1948 (1948) by Pollock and the black layer from a scanned photograph of an image of one of the Matter Pollock drip paintings obtained from a New York Times newspaper article.

Given the box count data in Figures 5 and 6 there are potentially many ways in which one might look for a natural fit by two slopes. Rather than prejudice ourselves to look for such a phenomena we looked at the variation in slope of the log-log plot of the box count/box size data using a sliding window of some fixed number of data points. That is, as the window is moved along, at each point a best (least-squares fit) line is computed and the slope is calculated. We tried windows of 5, 7, and 11 data points. As the number of points increases, the effect is one of a greater amount of smoothing, thus making it less likely that a significant change in slope would be computed/detected. Figure 7 presents some examples of the results of this experiment for the Pollock and Matter layer extraction for varying q .

In fact it turns out that varying the window length (at least among the values 5, 7, and 11) does not have much effect in terms of where the “break” is located or in the slope calculations of the fit lines. Nonetheless, with this methodology it does not appear that for $q > 0$ there is an obvious regime change for the slope, in either the Pollock layer or Matter-Pollock layer. For $q = 0$, there does not appear to be a sharp discontinuity in the slope, but there does appear to be a place where the rise in slope is most rapid, near box size 23 for the Matter Pollock image, and near box size 37 for the Pollock image. We tried fitting separate least-squares line segments to points to the left of a break point and points to the right, for various break points. When we did this, we got the best overall fit when the breakpoint was at 23 for the Matter-Pollock image, and the slopes were 1.80 and 1.94 for the two “regimes”. For the layer from Pollock’s “Number 1, 1948,” the optimal breakpoint was at box size 37, and the slopes for the two assumed “regimes” were 1.24 and 1.81. The rise was not really sharp enough to justify calling this a regime change, and so we are a little unjustified in fitting two line segments this way; but there does seem to be an observable S-shape to the slope curve for $q = 0$.

5. ROBUSTNESS

Calculations of box dimension as described above can be done for any binary image. If the image is not fractal-like then the points may not lie on a straight line. Even more, digital images, unlike pure mathematical fractal constructs like “Sierpinski’s gasket” (cf. Figure 1) do not permit the luxury of infinite scalability that would allow box-counting to be performed at an arbitrarily fine resolution. In the real world case of finite digital imagery what we have is at best an approximation of a true fractal, and the box-counting dimension from the plotted points can easily be affected by our computational model, image scanning and processing errors, as well as human bias. Nevertheless, box-counting dimension plots often yield important subtle clues about artists and their paintings. In fact, some of these clues are distinct enough to form digital signatures of an artist. This is precisely the findings of Taylor et al³ on Pollock and his drip paintings.

To take a small step toward making a science of the use of digital techniques for stylistic identification in the visual arts, i.e., an expansion of stylometry to include the visual arts, it is therefore important to begin to study the effects, or the lack thereof, of the choices made along the way in the computational process. To this end, using we considered the step of layer extraction. For this, a given color layer is extracted by first selecting a pixel whose corresponding RGB value is meant to be representative of the color layer to be extracted and then choosing a “tolerance” for the pixel value. The tolerance is simply the range of colors that are to be included in that layer. For example, if a pixel with RGB values (20,55,109) were selected with a tolerance of 10, then the extracted layer would consist of all pixels whose RGB coordinates fell within the cube (in RGB space) whose R value was between 10 and 30, G value was between 10 and 30, and B value was between 99 and 119. While it might seem natural to select a tolerance of 0, the fact is that many factors may cause the same paint used in different parts of the work to be captured digitally at slightly different values.

Using our digital capture of “Number 1, 1948,” currently in the collection of the New York Museum of Modern Art (see Figure 3), we performed layer extractions of a black layer, using the same base pixel with three tolerance

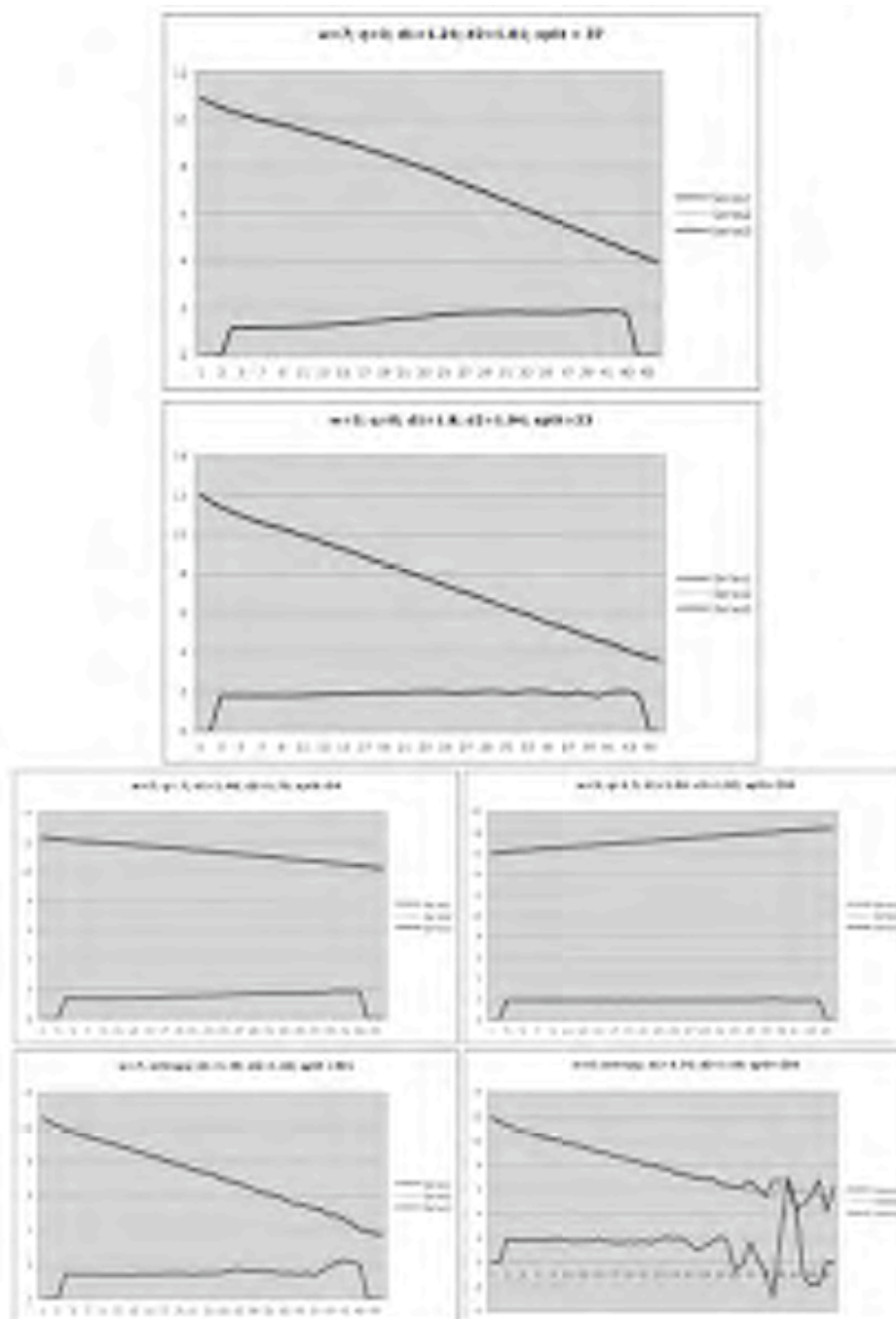


Figure 7. Here we show some exemplary results from the process of trying to fit the box-counting data by a two-piece piecewise linear curve. Data from the white layer of “Number 1, 1948,” J. Pollock is used for the top graph as well as the two lower graphs on the left, while the data from the black layer of the “Matter-Pollock” discussed herein is used for the other graphs. Results from using windows of seven boxes for the Pollock and five boxes for the Matter-Pollock are shown. The bottom plot in each curve is the running calculation of the slope and the top curve is actually two curves in each case, one the linear interpolation of the (log) box counting data and the other the piecewise linear fit. The two top graphs are for $q = 0$. This has the most pronounced change in slope. The graphs on the third row are $q = .7$ for Pollock and $q = 1.3$ for Matter-Pollock. The pairs of plots in each graph are virtually indistinguishable with negligible change in slope. The last plot is of entropy for both images.

values 10, 15, 20, and 25 and then extracted a white layer using a base pixel and tolerances 15, 23, and 30. The white layer shown in Figure 4 shows a white layer from “Number 1, 1948” extracted using a representative white pixel and tolerance of 15. Figure 8 shows the white layer extracted with a tolerance of 15 and for comparison, the white layer using the same base pixel but with a tolerance of 30.

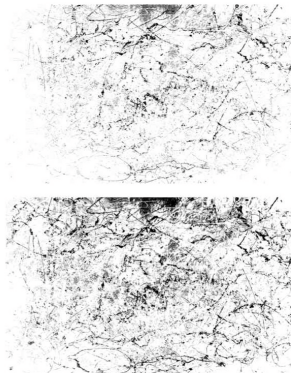


Figure 8. White layers extracted from “Number 1, 1948,” J. Pollock using the same base pixel but with differing tolerances: 15 (top) and 30 (bottom).

In all cases we found that for each value of q , the variation in the corresponding calculation of the fractal q -dimension was less than 0.1, but was not constant. This in turn suggests that any statistical consideration of Pollock’s work that depends on fractal dimension should allow for a degree of variability within one tenth or so.

6. CONCLUSION

The method of “box-counting” and more generally, fractal geometry has begun to play an important role in the authentication of the work of Jackson Pollock. In this paper we suggest a related calculation, the so-called “entropy dimension” that is an extension of multifractal methods, that may also be a useful statistic for authentication. We further take a small step toward building a body of robustness studies for the use of this particular digital technique by showing that in a small, but representative test set of Pollock works, that the calculation of box-counting dimension and more generally, the L^q dimension (related to the computation of a multifractal signature) is partially dependent on the process of color layer extraction, but only up to about a value of one tenth or so, which has implications for the degree of accuracy that one can expect in using box-counting for authentication. If digital techniques are going to be more widely accepted in authentication and visual stylometry in general, related robustness and stability studies must be performed.

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